## Second Edition

# SQL and Relational Theory 

 How to Write Accurate SOL Code
## SQL and Relational Theory

SQL is full of difficulties and traps for the unwary. You can avoid them if you understand relational theory, but only if you know how to put the theory into practice. In this insightful book, author C. J. Date explains relational theory in depth, and demonstrates through numerous examples and exercises how you can apply it directly to your use of SQL.

This second edition includes new material on recursive queries, "missing information" without nulls, new update operators, and topics such as aggregate operators, grouping and ungrouping, and view updating. If you have a modest-to-advanced background in SQL, you'll learn how to deal with a host of common SQL dilemmas.

- Why is proper column naming so important?
- Nulls in your database are causing you to get wrong answers. Why? What can you do about it?
- Is it possible to write an SQL query to find employees who have never been in the same department for more than six months at a time?
- SQL supports "quantified comparisons," but they're better avoided. Why? How do you avoid them?
- Constraints are crucially important, but most SQL products don't support them properly. What can you do to resolve this situation?

Database theory and practice have evolved since the relational model was developed more than 40 years ago. SQL and Relational Theory draws on decades of research to present the most up-to-date treatment of SQL available.
C. J. Date has a stature that is unique within the database industry. A prolific writer well known for the bestselling textbook An Introduction to Database Systems (Addison-Wesley), he has an exceptionally clear style when writing about complex principles and theory.


Twitter: @oreillymedia facebook.com/oreilly
oreilly.com

## Relational Theory

How to Write Accurate SQL Code
C. J. Date

# SQL and Relational Theory: How to Write Accurate SQL Code (2 ${ }^{\text {nd }}$ edition) 

by C. J. Date

Copyright © 2012 C. J. Date. All rights reserved.
Printed in the United States of America.

Published by O'Reilly Media, Inc., 1005 Gravenstein Highway North, Sebastopol, CA95472.

O'Reilly books may be purchased for educational, business, or sales promotional use. Online editions are also available for most titles (http://my.safaribooksonline.com). For more information, contact our corporate/institutional sales department: (800) 998-9938 or corporate@oreilly.com.

## Printing History:

January 2009: First Edition.
December 2011: Second Edition.

## Revision History:

2011-12-08 First release
See http://oreilly.com/catalog/errata.csp?isbn $=9781449316402$ for release details.

Nutshell Handbook, the Nutshell Handbook logo, and the O'Reilly logo are registered trademarks of O'Reilly Media, Inc. SQL and Relational Theory: How to Write Accurate SQL Code and related trade dress are trademarks of O'Reilly Media, Inc.

Many of the designations used by manufacturers and sellers to distinguish their products are claimed as trademarks. Where those designations appear in this book, and O'Reilly Media, Inc., was aware of a trademark claim, the designations have been printed in caps or initial caps.

While every precaution has been taken in the preparation of this book, the publisher and authors assume no responsibility for errors or omissions, or for damages resulting from the use of the information contained herein.

> Those who are enamored of practice without theory are like a pilot who goes into a ship without rudder or compass and never has any certainty where he is going

> Practice should always be based upon a sound knowledge of theory.

-Leonardo da Vinci (1452-1519)

The trouble with people is not that they don't know but that they know so much that ain't so.
-Josh Billings (1818-1885)

Languages die... mathematical ideas do not.
-G. H. Hardy (1877-1947)

Unfortunately, the gap between theory and practice is not as wide in theory as it is in practice.
-Anon.

These are my principles.
If you don't like them, I have others.
-Groucho Marx (1890-1977)

There is no royal road to geometry.
-Euclid (c. 365-275 BCE), attrib.

To all those who think an exercise like this one is worthwhile, and in particular to the memory of Lex de Haan, who is very much missed

## About the Author

C. J. Date is an independent author, lecturer, researcher, and consultant, specializing in relational database technology. He is best known for his book An Introduction to Database Systems, 8th edition (Addison-Wesley, 2004), which has sold some 850,000 copies at the time of writing and is used by several hundred colleges and universities worldwide. He is also the author of many other books on database management, including most recently:

- From Addison-Wesley: Databases, Types, and the Relational Model: The Third Manifesto, 3rd edition (coauthored with Hugh Darwen, 2006)

■ From Apress: Date on Database: Writings 2000-2006 (2006)

- From Trafford: Logic and Databases: The Roots of Relational Theory (2007)
- From Apress: The Relational Database Dictionary, Extended Edition (2008)

■ From Trafford: Database Explorations: Essays on The Third Manifesto and Related Topics (coauthored with Hugh Darwen, 2010)

- From Ventus: Go Faster! The TransRelational ${ }^{T M}$ Approach to DBMS Implementation (2011)

Another book, Normal Forms and All That Jazz: A Database Professional's Guide to Database Design Theory (a companion to the present book), is also due for publication in the near future.

Mr. Date was inducted into the Computing Industry Hall of Fame in 2004. He enjoys a reputation that is second to none for his ability to explain complex technical subjects in a clear and understandable fashion.

## Contents

Preface to the First Edition ..... xi
Preface to the Second Edition ..... xvi
Chapter 1 Setting the Scene 1
The relational model is much misunderstood ..... 1
Some remarks on terminology ..... 2
Principles not products ..... 4
A review of the original model ..... 5
Model vs. implementation ..... 12
Properties of relations ..... 14
Base vs. derived relations ..... 18
Relations vs. relvars ..... 19
Values vs. variables ..... 21
Concluding remarks ..... 22
Exercises ..... 23
Chapter 2 Types and Domains ..... 25
Types and relations ..... 25
Equality comparisons ..... 26
Data value atomicity ..... 31
What's a type? ..... 34
Scalar vs. nonscalar types ..... 37
Scalar types in SQL ..... 39
Type checking and coercion in SQL ..... 40
Collations in SQL ..... 42
Row and table types in SQL ..... 43
Concluding remarks ..... 45
Exercises ..... 46
Chapter 3 Tuples and Relations, Rows and Tables ..... 49
What's a tuple? ..... 49
Rows in SQL ..... 53
What's a relation? ..... 55
Relations and their bodies ..... 57
Relations are $n$-dimensional ..... 58
Relational comparisons ..... 58
TABLE_DUM and TABLE_DEE ..... 59
Tables in SQL ..... 60
Column naming in SQL ..... 62
Concluding remarks ..... 64
Exercises ..... 64
Chapter 4 No Duplicates, No Nulls ..... 67
What's wrong with duplicates? ..... 67
Duplicates: further issues ..... 72
Avoiding duplicates in SQL ..... 72
What's wrong with nulls? ..... 74
Avoiding nulls in SQL ..... 77
A remark on outer join ..... 79
Concluding remarks ..... 80
Exercises ..... 80
Chapter 5 Base Relvars, Base Tables ..... 85
Updating is set level ..... 86
Relational assignment ..... 88
More on candidate keys ..... 92
More on foreign keys ..... 94
Relvars and predicates ..... 97
Relations vs. types ..... 99
Exercises ..... 101
Chapter 6 SQL and Relational Algebra I: The Original Operators ..... 105
Some preliminaries ..... 105
More on closure ..... 108
Restriction ..... 110
Projection ..... 111
Join ..... 112
Union, intersection, and difference ..... 116
Which operators are primitive? ..... 119
Formulating expressions one step at a time ..... 119
What do relational expressions mean? ..... 121
Evaluating SQL table expressions ..... 122
Expression transformation ..... 123
The reliance on attribute names ..... 125
Exercises ..... 127
Chapter 7 SQL and Relational Algebra II: Additional Operators ..... 131
Exclusive union ..... 131
Semijoin and semidifference ..... 132
Extend ..... 133
Image relations ..... 135
Divide ..... 138
Aggregate operators ..... 139
Image relations bis ..... 144
Summarization ..... 146
Summarization bis ..... 150
Group, ungroup, and relation valued attributes ..... 152
"What if" queries ..... 157
A note on recursion ..... 159
What about ORDER BY? ..... 163
Exercises ..... 164
Chapter 8 SQL and Constraints ..... 169
Type constraints ..... 169
Type constraints in SQL ..... 173
Database constraints ..... 174
Database constraints in SQL ..... 178
Transactions ..... 180
Why database constraint checking must be immediate ..... 180
But doesn't some checking have to be deferred? ..... 182
Constraints and predicates ..... 185
Miscellaneous issues ..... 186
Exercises ..... 188
Chapter 9 SQL and Views ..... 193
Views are relvars ..... 194
Views and predicates ..... 197
Retrieval operations ..... 198
Views and constraints ..... 199
Update operations ..... 203
What are views for? ..... 211
Views and snapshots ..... 212
Exercises ..... 213
Chapter 10 SQL and Logic ..... 215
Why do we need logic? ..... 216
Simple and compound propositions ..... 217
Simple and compound predicates ..... 222
Quantification ..... 223
Relational calculus ..... 227
More on quantification ..... 234
Some equivalences ..... 241
Concluding remarks ..... 244
Exercises ..... 244
Chapter 11 Using Logic to Formulate SQL Expressions ..... 247
Some transformation laws ..... 247
Example 1: Logical implication ..... 250
Example 2: Universal quantification ..... 251
Example 3: Implication and universal quantification ..... 252
Example 4: Correlated subqueries ..... 254
Example 5: Naming subexpressions ..... 255
Example 6: More on naming subexpressions ..... 258
Example 7: Dealing with ambiguity ..... 259
Example 8: Using COUNT ..... 261
Example 9: Join queries ..... 262
Example 10: UNIQUE quantification ..... 263
Example 11: ALL or ANY comparisons ..... 265
Example 12: GROUP BY and HAVING ..... 269
Exercises ..... 270
Chapter 12 Miscellaneous SQL Topics ..... 273
SELECT * ..... 273
Explicit tables ..... 274
Name qualification ..... 274
Range variables ..... 275
Subqueries ..... 277
"Possibly nondeterministic" expressions ..... 280
Empty sets ..... 281
A simplified BNF grammar ..... 281
Exercises ..... 285
Appendix A The Relational Model ..... 287
The relational model vs. others ..... 288
The significance of theory ..... 291
The relational model defined ..... 293
Database variables ..... 298
Objectives of the relational model ..... 299
Some database principles ..... 300
What remains to be done? ..... 301
Appendix B SQL Departures from the Relational Model ..... 305
Appendix C A Relational Approach to Missing Information ..... 307
Vertical decomposition ..... 308
Horizontal decomposition ..... 309
What do the shaded entries mean? ..... 311
Constraints ..... 313
Queries ..... 314
More on predicates ..... 317
Exercises ..... 320
Appendix D A Tutorial D Grammar ..... 321
Appendix E Summary of Recommendations ..... 325
Appendix F Answers to Exercises ..... 329
Chapter 1 ..... 329
Chapter 2 ..... 335
Chapter 3 ..... 341
Chapter 4 ..... 346
Chapter 5 ..... 352
Chapter 6 ..... 358
Chapter 7 ..... 366
Chapter 8 ..... 379
Chapter 9 ..... 389
Chapter 10 ..... 395
Chapter 11 ..... 403
Chapter 12 ..... 405
Appendix C ..... 407
Appendix G Suggestions for Further Reading ..... 409
Index ..... 419

## Preface to the First Edition

SQL is ubiquitous. But SQL is hard to use: It's complicated, confusing, and error prone (much more so, I venture to suggest, than its apologists would have you believe). In order to have any hope of writing SQL code that you can be sure is accurate, therefore - meaning it does exactly what it's supposed to do, no more and no less-you must follow some appropriate discipline. And it's the thesis of this book that using SQL relationally is the discipline you need. But what does this mean? Isn't SQL relational anyway?

Well, it's true that SQL is the standard language for use with relational databases-but that fact in itself doesn't make it relational. The sad truth is, SQL departs from relational theory in all too many ways; duplicate rows and nulls are two obvious examples, but they're not the only ones. As a consequence, the language gives you rope to hang yourself with, as it were. So if you don't want to hang yourself, you need to understand relational theory (what it is and why); you need to know about SQL's departures from that theory; and you need to know how to avoid the problems they can cause. In a word, you need to use SQL relationally. Then you can behave as if SQL truly were relational, and you can enjoy the benefits of working with what is, in effect, a truly relational system.

Now, a book like this wouldn't be needed if everyone was using SQL relationally already-but they aren't. On the contrary, I observe much bad practice in current SQL usage. I even observe such practice being recommended, in textbooks and similar publications, by writers who really ought to know better (no names, no pack drill); in fact, a review of the literature in this regard is a pretty dispiriting exercise. The relational model first saw the light of day in 1969, and yet here we are, over 40 years later, and it still doesn't seem to be very well understood by the database community at large. Partly for such reasons, this book uses the relational model itself as an organizing principle; it explains various features of the model in depth, and shows in every case how best to use SQL in order to comply with the feature in question.

## Prerequisites

I assume you're a database practitioner and therefore reasonably familiar with SQL already. To be specific, I assume you have a working knowledge of either the SQL standard or (perhaps more likely in practice) at least one SQL product. However, I don't assume you have a deep knowledge of relational theory as such (though I do hope you understand that the relational model is a good thing in general, and adherence to it wherever possible is a desirable goal). In order to avoid misunderstandings, therefore, I'll be describing various features of the relational model in detail, as well as showing how to use SQL to conform to those features. But what I won't do is attempt to justify all of those features; rather, I'll assume you're sufficiently experienced in database matters to understand why, e.g., the notion of a key makes sense, or why you sometimes need to do a join, or why many to many relationships need to be supported. (If I were to include such justifications, this would be a very different book-quite apart from anything else, it would be much bigger than it already is-and in any case, that book has already been written.)

I've said I expect you to be reasonably familiar with SQL. However, I should add that I'll be explaining certain aspects of SQL in detail anyway-especially aspects that might be encountered less frequently in practice. (The SQL notion of possibly nondeterministic expressions is a case in point here. See Chapter 12.)

## Database in Depth

This book is based on, and intended to replace, an earlier one with the title Database in Depth: Relational Theory for Practitioners (O'Reilly Media Inc., 2005). My aim in that earlier book was as follows (this is a quote from the preface):

## xii Preface to the First Edition

After many years working in the database community in various capacities, I've come to realize there's a real need for a book for practitioners (not novices) that explains the basic principles of relational theory in a way not tainted by the quirks and peculiarities of existing products, commercial practice, or the SQL standard. I wrote this book to fill that need. My intended audience is thus experienced database practitioners who are honest enough to admit they don't understand the theory underlying their own field as well as they might, or should. That theory is, of course, the relational modeland while it's true that the fundamental ideas of that theory are all quite simple, it's also true that they're widely misrepresented, or underappreciated, or both. Often, in fact, they don't seem to be understood at all. For example, here are a few relational questions ... How many of them can you answer? ${ }^{1}$

```
What exactly is first normal form?
What's the connection between relations and predicates?
What's semantic optimization?
What's an image relation?
Why is semidifference important?
Why doesn't deferred integrity checking make sense?
What's a relation variable?
What's prenex normal form?
Can a relation have an attribute whose values are relations?
Is SQL relationally complete?
Why is The Information Principle important?
How does XML fit with the relational model?
```

This book provides answers to these and many related questions. Overall, it's meant to help database practitioners understand relational theory in depth and make good use of that understanding in their professional day-to-day activities.

As the final sentence in this extract indicates, it was my hope that readers of that book would be able to apply its ideas for themselves, without further assistance from me as it were. But I've since come to realize that, contrary to popular opinion, SQL is such a difficult language that it can be far from obvious how to use it without violating relational principles. I therefore decided to expand the original book to include explicit, concrete advice on exactly that issue (how to use SQL relationally, I mean). So my aim in the present book is still the same as before-I want to help database practitioners understand relational theory in depth and make good use of that understanding in their professional activities-but I've tried to make the material a little easier to digest, perhaps, and certainly easier to apply. In other words, I've included a great deal of SQL-specific material (and it's this fact, more than anything else, that accounts for the increase in size over the previous book).

## Further Remarks on the Text

I need to take care of several further preliminaries. First of all, my own understanding of the relational model has evolved over the years, and continues to do so. This book represents my very latest thinking on the subject; thus, if you detect any technical discrepancies-and there are a few-between this book and other books you might have seen by myself (including in particular the one the present book is meant to replace), the present book should be taken as superseding. Though I hasten to add that such discrepancies are mostly of a fairly minor nature; what's more, I've taken care always to relate new terms and concepts to earlier ones, wherever I felt it was necessary to do so.

Second, I will, as advertised, be talking about theory-but it's an article of faith with me that theory is practical. I mention this point explicitly because so many seem to believe the opposite: namely, that if something's

[^0]theoretical, it can't be practical. But the truth is that theory (at least, relational theory, which is what I'm talking about here) is most definitely very practical indeed. The purpose of that theory is not just theory for its own sake; the purpose of that theory is to allow us to build systems that are 100 percent practical. Every detail of the theory is there for solid practical reasons. As Stéphane Faroult, a reviewer of the earlier book, wrote: "When you have a bit of practice, you realize there's no way to avoid having to know the theory." What's more, that theory is not only practical, it's fundamental, straightforward, simple, useful, and it can be fun (as I hope to demonstrate in the course of this book).

Of course, we really don't have to look any further than the relational model itself to find the most striking possible illustration of the foregoing thesis. Indeed, it really shouldn't be necessary to have to defend the notion that theory is practical, in a context such as ours: namely, a multibillion dollar industry totally founded on one great theoretical idea. But I suppose the cynic's position would be "Yes, but what has theory done for me lately?" In other words, those of us who do think theory is important must continually be justifying ourselves to our criticswhich is another reason why I think a book like this one is needed.

Third, as I've said, the book does go into a fair amount of detail regarding features of SQL or the relational model or both. (It deliberately has little to say on topics that aren't particularly relational; for example, there isn't much on transactions.) Throughout, I've tried to make it clear when the discussions apply to SQL specifically, when they apply to the relational model specifically, and when they apply to both. I should emphasize, however, that the SQL discussions in particular aren't meant to be exhaustive. SQL is such a complex language, and provides so many different ways of doing the same thing, and is subject to so many exceptions and special cases, that to be exhaustive-even if it were possible, which I tend to doubt-would be counterproductive; certainly it would make the book much too long. So I've tried to focus on what I think are the most important issues, and I've tried to be as brief as possible on the issues I've chosen to cover. And I'd like to claim that if you do everything I tell you, and don't do anything I don't tell you, then to a first approximation you'll be safe: You'll be using SQL relationally. But whether that claim is justified, or to what extent it is, must be for you to judge.

To the foregoing I have to add that, unfortunately, there are some situations in which SQL just can't be used relationally. For example, some SQL integrity checking simply has to be deferred (usually to commit time), even though the relational model explicitly rejects such checking as logically flawed. The book does offer advice on what to do in such cases, but I fear it often boils down to just Do the best you can. At least I hope you'll understand the risks involved in departing from the model.

I should say too that some of the recommendations offered aren't specifically relational anyway but are, rather, just matters of general good practice-though sometimes there are relational implications (implications that can be a little unobvious, too, perhaps I should add). Avoid coercions is a good example here.

Fourth, please note that I use the term $S Q L$ throughout the book to mean the standard version of that language exclusively, not some proprietary dialect, barring explicit statements to the contrary. In particular, I follow the standard in assuming the pronunciation "ess cue ell," not "sequel" (though this latter is common in the field), thereby saying things like an SQL table, not $a$ SQL table.

Fifth, the book is meant to be read in sequence, pretty much, except as noted here and there in the text itself (most of the chapters do rely to some extent on material covered in earlier ones, so you shouldn't jump around too much). Also, each chapter includes a set of exercises. You don't have to do those exercises, of course, but I think it's a good idea to have a go at some of them at least. Answers, often giving more information about the subject at hand, are given in Appendix F.

Finally, I'd like to mention that I have some live seminars available based on the material in this book. See www.justsql.co.uk/chris_date/chris_date.htm or www.thethirdmanifesto.com for further details. An online version of one of those seminars is available too, at http://oreilly.com/catalog/0636920010005/.

## Using Code Examples

This book is here to help you get your job done. In general, you may use the code in this book in your programs and documentation. You do not need to contact us for permission unless you're reproducing a significant portion of the code. For example, writing a program that uses several chunks of code from this book does not require permission. Selling or distributing a CD-ROM of examples from O'Reilly books does require permission. Answering a question by citing this book and quoting example code does not require permission. Incorporating a significant amount of example code from this book into your product's documentation does require permission.

We appreciate, but do not require, attribution. An attribution usually includes the title, author, publisher, and ISBN. For example: "SQL and Relational Theory, Second Edition, by C.J. Date (O'Reilly). Copyright 2012 C.J. Date, 9781449316402."

If you feel your use of code examples falls outside fair use or the permission given above, feel free to contact us at permissions@oreilly.com.

## Comments and Questions

Please address comments and questions concerning this book to the publisher:
O'Reilly Media, Inc. 1005 Gravenstein Highway North
Sebastopol, CA 95472
(800) 998-9938 (in the United States or Canada)
(707) 829-0515 (international or local)
(707) 829-0104 (fax)

We have a web page for this book, where we list errata, examples, and any additional information. You can access this page at http://shop.oreilly.com/product/0636920022879.do.

To comment or ask technical questions about this book, send email to bookquestions@oreilly.com.
For more information about our books, courses, conferences, and news, see our website at http://www.oreilly.com.
Find us on Facebook: http:///facebook.com/oreilly.
Follow us on Twitter: http://twitter.com/oreillymedia.
Watch us on YouTube: http://www.youtube.com/oreillymedia.

## Safari ${ }^{\circledR 8}$ Books Online

Safari Books Online is an on-demand digital library that lets you easily search over 7,500 technology and creative reference books and videos to find the answers you need quickly.

With a subscription, you can read any page and watch any video from our library online. Read books on your cell phone and mobile devices. Access new titles before they are available for print, and get exclusive access to manuscripts in development and post feedback for the authors. Copy and paste code samples, organize your favorites, download chapters, bookmark key sections, create notes, print out pages, and benefit from tons of other time-saving features.

O'Reilly Media has uploaded this book to the Safari Books Online service. To have full digital access to this book and others on similar topics from O'Reilly and other publishers, sign up for free at http://my.safaribooksonline.com.

## Acknowledgments

I'd been thinking for some time about revising the earlier book to include more on SQL in particular, but the spur that finally got me down to it was sitting in on a class, late in 2007, for database practitioners. The class was taught by Toon Koppelaars and was based on the book he wrote with Lex de Haan (see Appendix G of the present book), and very good it was, too. But what struck me most about that class was seeing firsthand the kinds of difficulties the attendees had in applying relational and logical principles to their use of SQL. Now, I do assume those attendees had some knowledge of those topics-they were database practitioners, after all-but it seemed to me they really needed some guidance in the application of those ideas to their daily database activities. And so I put this book together. So I'm thankful, first of all, to Toon and Lex for providing me with the necessary impetus to get started on this project. I'm grateful also to my reviewers Herb Edelstein, Sheeri Ktitzer, Andy Oram, Peter Robson, and Baron Schwartz for their comments on earlier drafts, and Hugh Darwen and Jim Melton for other technical assistance. Next, I'd like to thank my wife Lindy, as always, for her support throughout this and all of my other database projects over the years. Finally, I'm grateful to everyone at O'Reilly-especially Isabel Kunkle and Andy Oram-for their encouragement, contributions, and support throughout the production of this book.

C. J. Date<br>Healdsburg, California<br>2008

## Preface to the Second Edition

This edition differs from its predecessor in a number of ways. The overall objective remains the same, of courseusing SQL relationally is still the emphasis-but the text has been revised throughout to reflect, among other things, experience gained from teaching live seminars based on the first edition.

One significant change is a deletion: The appendix on design theory has gone. There are two reasons for this change. First, design theory as such never really did have all that much to do with the book's main message, anyway; second, the appendix was getting so extensive that it threatened to overwhelm the rest of the text. (It was already longer than any chapter or any other appendix in the book. In fact, I've since expanded the material into a separate book in its own right. That book-Normal Forms and All That Jazz: A Database Professional's Guide to Database Design Theory-is due to be published soon by O'Reilly. It can be seen as a companion, or perhaps a sequel, to the present book.)

On the positive side, a lot of new material has been added (including, importantly, a discussion of how to deal with missing information without using nulls); examples, exercises, and answers have been expanded and improved in various respects; and the treatment of SQL has been upgraded to cover recent changes to the SQL standard. A variety of corrections and numerous cosmetic improvements have also been made. ${ }^{2}$ (In particular, the Tutorial D examples-Tutorial D being the language I use to illustrate relational concepts-have been upgraded to reflect several recent improvements to that language. See Appendix D.) The net effect is to make the text rather more comprehensive-but, sadly, some 25 percent bigger-than its predecessor.

Talking of the text, I'd like to say something about my use of footnotes. Frankly, I'm rather embarrassed at how many footnotes there are; I'm well aware how annoying they can be-indeed, they can seriously impede readability. But any text dealing with SQL is more or less forced into a heavy use of footnotes, at least if it wants to be tutorial in nature and yet reasonably comprehensive at the same time. The reason is that SQL involves so many inconsistencies, exceptions, and special cases that treating everything "in line"-i.e., at the same level of description-makes it very difficult to see the forest for the trees. (Indeed, this is one reason why the SQL standard itself is so difficult to understand.) Thus, there are numerous places in the book where the major idea is described "in line" in the main body of the text, and exceptions and the like (which must at least be mentioned, for reasons of accuracy and completeness) are relegated to a footnote. It might be best simply to ignore all footnotes on a first reading.

C. J. Date<br>Healdsburg, California<br>2012

[^1]
## Chapter 1

## Setting the Scene

> My soul, sit thou a patient looker-on; Judge not the play before the play is done;
> Her plot hath many changes; every day Speaks a new scene; the last act crowns the play.
> $\quad-$ Francis Quarles: Emblems (1635)

A relational approach to SQL: That's the theme, or one of the themes, of this book. Of course, to treat such a topic adequately, I need to cover relational issues as well as issues of SQL per se-and while this remark obviously applies to the book as a whole, it applies to this first chapter with special force. As a consequence, this chapter has comparatively little to say about SQL as such. What I want to do is review material that for the most part, at any rate, I hope you already know. My intent is to establish a point of departure, as it were: in other words, to lay some groundwork on which the rest of the book can build. But even though $I$ hope you're familiar with most of what $I$ have to say in this chapter, I'd like to suggest, respectfully, that you not skip it. You need to know what you need to know (if you see what I mean); in particular, you need to be sure you have the prerequisites needed to understand the material to come in later chapters. In fact I'd like to recommend, politely, that throughout the book you not skip the discussion of some topic just because you think you're familiar with that topic already. For example, are you absolutely sure you know what a key is, in relational terms? Or a join? ${ }^{1}$

## THE RELATIONAL MODEL IS MUCH MISUNDERSTOOD

Professionals in any discipline need to know the foundations of their field. So if you're a database professional, you need to know the relational model, because the relational model is the foundation (or a large part of the foundation, at any rate) of the database field in particular. Now, every course in database management, be it academic or commercial, does at least pay lip service to the idea of teaching the relational model-but most of that teaching seems to be done very badly, if results are anything to go by. Certainly the model isn't well understood in the database community at large. Here are some possible reasons for this state of affairs:

- The model is taught in a vacuum. That is, for beginners at least, it's hard to see the relevance of the material, or it's hard to understand the problems it's meant to solve, or both.
- The instructors themselves don't fully understand or appreciate the significance of the material.

[^2]
## 2 Chapter 1 / Setting the Scene

Perhaps most likely in practice, the model as such isn't taught at all-the SQL language, or some specific dialect of that language, such as the Oracle dialect, is taught instead.

So this book is aimed at database practitioners in general, and SQL practitioners in particular, who have had some exposure to the relational model but don't know as much about it as they ought to, or would like to. It's definitely not meant for beginners; however, it isn't just a refresher course, either. To be more specific, I'm sure you know something about SQL; but-and I apologize for the possibly offensive tone here-if your knowledge of the relational model derives only from your knowledge of SQL, then I'm afraid you won't know the relational model as well as you should, and you'll probably know "some things that ain't so." I can't say it too strongly: SQL and the relational model aren't the same thing. Here by way of illustration are some relational issues that SQL isn't too clear on (to put it mildly):

- What databases, relations, and tuples really are
- The difference between relation values and relation variables
- The relevance of predicates and propositions
- The importance of attribute names
- The crucial role of integrity constraints
- The Information Principle and its significance
and so on (this isn't an exhaustive list). All of these issues, and many others, are addressed in this book.
I say again: If your knowledge of the relational model derives only from your knowledge of SQL, then you might know "some things that ain't so." One consequence is that you might find, in reading this book, that you have to do some unlearning-and unlearning, unfortunately, is very hard to do.


## SOME REMARKS ON TERMINOLOGY

You probably noticed right away, in that bullet list of relational issues in the previous section, that I used the formal terms relation, tuple (usually pronounced to rhyme with couple), and attribute. SQL doesn't use these terms, of course-it uses the more "user friendly" terms table, row, and column instead. And I'm generally sympathetic to the idea of using more user friendly terms, if they can help make the ideas more palatable. In the case at hand, however, it seems to me that, regrettably, they don't make the ideas more palatable; instead, they distort them, and in fact do the cause of genuine understanding a grave disservice. The truth is, a relation is not a table, a tuple is not a row, and an attribute is not a column. And while it might be acceptable to pretend otherwise in informal contexts-indeed, I often do so myself-I would argue that it's acceptable only if we all understand that the more user friendly terms are just an approximation to the truth and fail overall to capture the essence of what's really going on. To put it another way: If you do understand the true state of affairs, then judicious use of the user friendly terms can be a good idea; but in order to learn and appreciate that true state of affairs in the first place, you really do need to come to grips with the formal terms. In this book, therefore, I'll tend to use those formal terms (at least when I'm talking about the relational model as opposed to SQL), and I'll give precise definitions for them at the relevant juncture. In SQL contexts, by contrast, I'll use SQL's own terms.

And another point on terminology: Having said that SQL tries to simplify one set of terms, I must say too that it does its best to complicate another. I refer to its use of the terms operator, function, procedure, routine, and
method, all of which denote essentially the same thing (with, perhaps, very minor differences). In this book I'll use the term operator throughout; thus, for example, I'll refer to " $=$ " (equality comparison), ":=" (assignment), " + " (addition), DISTINCT, JOIN, SUM, GROUP BY (etc., etc.) all as operators specifically.

Talking of SQL, incidentally, let me remind you that (as stated in the preface) I use that term to mean the standard version of the language exclusively, except in a few places where the context demands otherwise. ${ }^{2}$ However:

- The standard's use of terminology is sometimes not very apt. In such situations, I generally prefer to use terminology of my own. For example, I use the term table expression in place of the standard term query expression, for the following reasons among others: First, the value such expressions denote is indeed a table and not a query; second, queries aren't the only context in which such expressions are used anyway. (As a matter of fact the standard does use the term table expression, but again it does so quite inappropriately; to be specific, it uses it to refer to what comes after the SELECT clause in a SELECT expression.)
- Following on from the previous point, I should add that not all table expressions-in either my sense or the standard's-are legal in SQL in all contexts where they might be expected to be. In particular, an explicit JOIN invocation, although it certainly does denote a table, can't appear as a "stand alone" table expression (i.e., at the outermost level of nesting), nor can it appear as the table expression in parentheses that constitutes a subquery (see Chapter 12). ${ }^{3}$ Please note that these remarks apply to many of the individual discussions in the body of the book; it would be very tedious to keep on repeating them, however, and I won't. (They're reflected in the BNF grammar in Chapter 12, however.)
- I ignore aspects of the standard that might be regarded as a trifle esoteric-especially if they aren't part of what the standard calls Core SQL or don't have much to do with relational processing as such. Examples here include the so called analytic or window (OLAP) functions; dynamic SQL; temporary tables; and details of user defined types.

For reasons that aren't important here, I use a style for comments that differs from that of the standard. To be specific, I show comments as text strings in italics, bracketed by "/*" and "*/" delimiters.

Be aware, however, that all SQL products include features that aren't part of the standard per se. Row IDs provide a common example. My general advice regarding such features is: By all means use them if you want to-but not if they violate relational principles (after all, what I'm advocating is supposed to be a relational approach to SQL). For example, row IDs in particular are likely to violate either The Principle of Interchangeability (see Chapter 9) or The Information Principle (see Appendix A) or both; and if they do, then I certainly wouldn't use them. But, here and everywhere, the overriding rule is: You can do what you like, so long as you know what you're doing.

[^3]
## 4 Chapter 1 / Setting the Scene

## PRINCIPLES NOT PRODUCTS

It's worth taking a few moments to examine the question of why, as I claimed earlier, you as a database professional need to know the relational model. The reason is that the relational model isn't product specific; instead, it's concerned with principles. What do I mean by principles? Well, here's a definition (from Chambers Twentieth Century Dictionary):
principle: a source, root, origin: that which is fundamental: essential nature: theoretical basis: a fundamental truth on which others are founded or from which they spring

The point about principles is: They endure. By contrast, products and technologies (and the SQL language, come to that) change all the time-but principles don't. For example, suppose you know Oracle; in fact, suppose you're an expert on Oracle. But if Oracle is all you know, then your knowledge is not necessarily transferable to, say, a DB2 or SQL Server environment (it might even make it harder to make progress in that new environment). But if you know the underlying principles-in other words, if you know the relational model-then you have knowledge and skills that will be transferable: knowledge and skills that you'll be able to apply in every environment and will never be obsolete.

In this book, therefore, we'll be concerned with principles, not products, and foundations, not fashion or fads. But I do realize you sometimes have to make compromises and tradeoffs in the real world. For one example, sometimes you might have good pragmatic reasons for not designing the database in the theoretically optimal way. For another, consider SQL once again. Although it's certainly possible to use SQL relationally (for the most part, at any rate), sometimes you'll find-because existing implementations are so far from perfect-that there are severe performance penalties for doing so ... in which case you might be more or less forced into doing something not "truly relational" (like writing a query in some unnatural way to force the implementation to use an index). However, I believe very firmly that you should always make such compromises and tradeoffs from a position of conceptual strength. That is:

You should understand what you're doing when you do decide to make such a compromise.

- You should know what the theoretically correct situation is, and you should have strong reasons for departing from it.
- You should document those reasons, too, so that if they cease to be valid at some future time (for example, because a new release of the product you're using does a better job in some respect), then it might be possible to back off from the original compromise.

The following quote-which is due to Leonardo da Vinci (1452-1519) and is thus some 500 years old-sums up the situation admirably:

Those who are enamored of practice without theory are like a pilot who goes into a ship without rudder or compass and never has any certainty where he is going. Practice should always be based on a sound knowledge of theory.
(OK, I added the italics.)

## A REVIEW OF THE ORIGINAL MODEL

The purpose of this section is to serve as a kickoff point for subsequent discussions; it reviews some of the most basic aspects of the relational model as originally defined. Note that qualifier-"as originally defined"! One widespread misconception about the relational model is that it's a totally static thing. It's not. It's like mathematics in that respect: Mathematics too is not a static thing but changes over time. In fact, the relational model can itself be seen as a small branch of mathematics; as such, it evolves over time as new theorems are proved and new results discovered. What's more, those new contributions can be made by anyone who's competent to do so; like other branches of mathematics, the relational model, though originally invented by one man, has become a community effort and now belongs to the world.

By the way, in case you don't know, that one man was E. F. Codd, at the time a researcher at IBM (E for Edgar and F for Frank-but he always signed with his initials; to his friends, among whom I was proud to count myself, he was Ted). It was late in 1968 that Codd, a mathematician by training, first realized that the discipline of mathematics could be used to inject some solid principles and rigor into a field, database management, that prior to that time was all too deficient in any such qualities. His original definition of the relational model appeared in an IBM Research Report in 1969, and I'll have a little more to say about that paper in Appendix G.

## Structural Features

The original model had three major components-structure, integrity, and manipulation-and I'll briefly describe each in turn. Please note right away, however, that all of the "definitions" I'll be giving here are very loose; I'll make them more precise as and when appropriate in later chapters.

First of all, then, structure. The principal structural feature is, of course, the relation itself, and as everybody knows it's usual to picture relations on paper as tables (see Fig. 1.1 below for a self-explanatory example). Relations are defined over types (also known as domains); a type is basically a conceptual pool of values from which actual attributes in actual relations take their actual values. With reference to the simple departments-andemployees database of Fig. 1.1, for example, there might be a type called DNO ("department numbers"), which is the set of all valid department numbers, and then the attribute called DNO in the DEPT relation and the attribute called DNO in the EMP relation would both contain values from that conceptual pool. (By the way, it isn't necessary-though it's often a good idea-for attributes to have the same name as the corresponding type, and frequently they won't. We'll see plenty of counterexamples later.)


Fig. 1.1: The departments-and-employees database-sample values
As I've said, tables like those in Fig. 1.1 depict relations: $n$-ary relations, to be precise. An $n$-ary relation can be pictured as a table with $n$ columns; the columns in that picture represent attributes of the relation and the rows represent tuples. The value $n$ can be any nonnegative integer. A 1-ary relation is said to be unary; a 2-ary relation, binary; a 3-ary relation, ternary; and so on.

The relational model also supports various kinds of keys. To begin with-and this point is crucial!-every relation has at least one candidate key. ${ }^{4}$ A candidate key is just a unique identifier; in other words, it's a combination of attributes-often but not always a "combination" consisting of just a single attribute-such that every tuple in the relation has a unique value for the combination in question. In Fig. 1.1, for example, every department has a unique department number and every employee has a unique employee number, so we can say that $\{\mathrm{DNO}\}$ is a candidate key for DEPT and $\{E N O\}$ is a candidate key for EMP. Note the braces, by the way; to repeat, candidate keys are always combinations, or sets, of attributes (even when the set in question contains just one attribute), and the conventional representation of a set on paper is as a commalist of elements enclosed in braces.

Aside: This is the first time I've mentioned the useful term commalist, but I'll be using it a lot in the pages ahead. It can be defined as follows: Let $x y z$ be some syntactic construct (for example, "attribute name"). Then the term $x y z$ commalist denotes a sequence of zero or more $x y z$ 's in which each pair of adjacent $x y z$ 's is separated by a comma (as well as, optionally, one or more spaces either before or after the comma or both). For example, if $A, B$, and $C$ are attribute names, then the following are all attribute name commalists:

```
A,B,C
C,A,B
B
A , C
```

So too is the empty sequence of attribute names.
Moreover, when some commalist is enclosed in braces and thereby denotes a set, then (a) the order in which the elements appear within that commalist is immaterial (because sets have no ordering to their elements), and (b) if an element appears more than once, it's treated as if it appeared just once (because sets don't contain duplicate elements). End of aside.

Next, a primary key is a candidate key that's been singled out for special treatment in some way. Now, if the relation in question has just one candidate key, then it doesn't make any real difference if we decide to call that key "primary." But if that relation has two or more candidate keys, then it's usual to choose one of them as primary, meaning it's somehow "more equal than the others." Suppose, for example, that every employee always has both a unique employee number and a unique employee name-not a very realistic example, perhaps, but good enough for present purposes-so that $\{E N O\}$ and $\{E N A M E\}$ are both candidate keys for EMP. Then we might choose $\{E N O\}$, say, to be the primary key.

Observe that I said it's usual to choose a primary key. Indeed it is usual-but it's not 100 percent necessary. If there's just one candidate key, then there's no choice and no problem; but if there are two or more, then having to choose one and make it primary smacks a little bit of arbitrariness (at least to me). Certainly there are situations where there don't seem to be any good reasons for making such a choice. In this book, therefore, I usually will follow the primary key discipline-and in pictures like Fig. 1.1 I'll indicate primary key attributes by double underlining ${ }^{5}$-but I want to stress the fact that it's really candidate keys, not primary keys, that are significant from a relational point of view. Partly for that reason, from this point forward I'll use the term key, unqualified, to mean

[^4]any candidate key, regardless of whether the candidate key in question has additionally been designated as "primary." (In case you were wondering, the "special treatment" enjoyed by primary keys over other candidate keys is mainly syntactic in nature, anyway; it isn't fundamental, and it isn't very important.)

Finally, a foreign key is a combination, or set, of attributes $F K$ in some relation $r 2$ such that each $F K$ value is required to be equal to some value of some key $K$ in some relation $r l$ ( $r l$ and $r 2$ not necessarily distinct). ${ }^{6}$ With reference to Fig. 1.1, for example, $\{\mathrm{DNO}\}$ is a foreign key in EMP whose values are required to match values of the key $\{\mathrm{DNO}\}$ in DEPT (as I've tried to suggest by means of a suitably labeled arrow in the figure). By required to match here, I mean that if, for example, EMP contains a tuple in which the DNO attribute has the value D2, then DEPT must also contain a tuple in which the DNO attribute has the value D2-for otherwise EMP would show some employee as being in a nonexistent department, and the database wouldn't be "a faithful model of reality."

## Integrity Features

An integrity constraint (constraint for short) is basically just a boolean expression that must evaluate to TRUE. In the case of departments and employees, for example, we might have a constraint to the effect that SALARY values must be greater than zero. Now, any given database will be subject to numerous constraints; however, all of those constraints will necessarily be specific to that database and will thus be expressed in terms of the relations in that database. By contrast, the relational model as originally formulated includes two generic constraints-generic, in the sense that they apply to every database, loosely speaking. One has to do with primary keys and the other with foreign keys. Here they are:

- The entity integrity rule: Primary key attributes don't permit nulls.
- The referential integrity rule: There mustn't be any unmatched foreign key values.

I'll explain the second rule first. By the term unmatched foreign key value, I mean a foreign key value for which there doesn't exist an equal value of the pertinent candidate key (the "target key"); thus, for example, the departments-and-employees database would be in violation of the referential integrity rule if it included an EMP tuple with a DNO value of D2, say, but no DEPT tuple with that same DNO value. So the referential integrity rule simply spells out the semantics of foreign keys; the name "referential integrity" derives from the fact that a foreign key value can be regarded as a reference to the tuple with that same value for the corresponding target key. In effect, therefore, the rule just says: If $B$ references $A$, then $A$ must exist.

As for the entity integrity rule, well, here I have a problem. The fact is, I reject the concept of "nulls" entirely; that is, it's my very strong opinion that nulls have no place in the relational model. (Codd thought otherwise, obviously, but I have strong reasons for taking the position I do.) In order to explain the entity integrity rule, therefore, I need to suspend disbelief, as it were (at least for a few moments). Which I'll now proceed to do ... but please understand that I'll be revisiting the whole issue of nulls in Chapters 3 and 4.

In essence, then, a null is a "marker" that means value unknown. Crucially, it's not itself a value; it is, to repeat, a marker, or flag. For example, suppose we don't know employee E2's salary. Then, instead of entering some real SALARY value in the tuple for employee E2 in relation EMP—we can't enter a real value, by definition, precisely because we don't know what that value should be-we mark the SALARY position within that tuple as null, as indicated here:

[^5]
## 8 Chapter 1 / Setting the Scene

| ENO | ENAME | DNO | SALARY |
| :---: | :---: | :---: | :---: |
| E2 | Cheng | D1 | <</m, |

Now, it's important to understand that this tuple contains nothing at all in the SALARY position. But it's very hard to draw pictures of nothing at all! I've tried to show the SALARY position is empty in the picture above by shading it, but it would be more accurate not to show that position at all. Be that as it may, I'll use this same convention of representing empty positions by shading elsewhere in this book-but that shading does not, to repeat, represent any kind of value at all. You can think of it (the shading, that is) as constituting the null "marker," or flag, if you like.

To get back to the entity integrity rule: In terms of relation EMP, then, that rule says, loosely, that a given employee tuple might have an unknown name, or an unknown department number, or an unknown salary-but it can't have an unknown employee number. The justification, such as it is, for this state of affairs is that if the employee number were unknown, we wouldn't even know which "entity" (i.e., which employee) we were talking about.

That's all I want to say about nulls for now. Please forget about them until further notice.

## Manipulative Features

The manipulative part of the model in turn divides into two parts:

- The relational algebra, which is a collection of operators (e.g., difference, or MINUS) that can be applied to relations
- A relational assignment operator, which allows the value of some relational expression (e.g., $r 1$ MINUS $r 2$, where $r 1$ and $r 2$ are relations) to be assigned to some relation

The relational assignment operator is fundamentally how updates are done in the relational model, and I'll have more to say about it later, in the section "Relations vs. Relvars." Note: I follow the usual convention throughout this book in using the generic term update to refer to the INSERT, DELETE, and UPDATE (and assignment) operators considered collectively. When I want to refer to the UPDATE operator specifically, I'll set it in all caps as just shown.

As for the relational algebra, it consists of a set of operators that-speaking very loosely-allow us to derive "new" relations from "old" ones. Each such operator takes one or more relations as input and produces another relation as output; for example, difference (MINUS) takes two relations as input and "subtracts" one from the other, to derive another relation as output. And it's very important that the output is another relation: That's the well known closure property of the relational algebra. The closure property is what lets us write nested relational expressions; since the output from every operation is the same kind of thing as the input, the output from one operation can become the input to another. For example, we can take the difference $r 1$ MINUS $r 2$, feed the result as input to a union with some relation $r 3$, feed that result as input to an intersection with some relation $r 4$, and so on.

Now, any number of operators can be defined that fit the simple definition of "one or more relations in, exactly one relation out." Here I'll briefly describe what are usually thought of as the original operators (essentially the ones that Codd defined in his earliest papers); ${ }^{7}$ I'll give more details in Chapter 6, and in Chapter 7 I'll describe a number of additional operators as well. Fig. 1.2 is a pictorial representation of those original operators.

[^6]Note: If you're unfamiliar with these operators and find the descriptions a little hard to follow, don't worry about it; as I've already said, I'll be going into much more detail, with lots of examples, in later chapters.

union

difference


Fig. 1.2: The original relational algebra

## Restrict

Returns a relation containing all tuples from a specified relation that satisfy a specified condition. For example, we might restrict relation EMP to just those tuples where the DNO value is D2.

## 10 Chapter 1 / Setting the Scene

## Project

Returns a relation containing all (sub)tuples that remain in a specified relation after specified attributes have been removed. For example, we might project relation EMP on just the ENO and SALARY attributes (thereby removing the ENAME and DNO attributes).

## Product

Returns a relation containing all possible tuples that are a combination of two tuples, one from each of two specified relations. Note: This operator is also known variously as cartesian product (sometimes extended or expanded cartesian product), cross product, cross join, and cartesian join, in fact, it's really just a special case of join, as we'll see in Chapter 6.

## Intersect

Returns a relation containing all tuples that appear in both of two specified relations. (Actually intersect, like product, is also a special case of join, as we'll see in Chapter 6.)

## Union

Returns a relation containing all tuples that appear in either or both of two specified relations.

## Difference

Returns a relation containing all tuples that appear in the first and not the second of two specified relations.

## Join

Returns a relation containing all possible tuples that are a combination of two tuples, one from each of two specified relations, such that the two tuples contributing to any given result tuple have a common value for the common attributes of the two relations (and that common value appears just once, not twice, in that result tuple). Note: This kind of join was originally called the natural join, to distinguish it from various other kinds to be discussed later in this book. Since natural join is far and away the most important kind, however, it's become standard practice to take the unqualified term join to mean the natural join specifically, and I'll follow that practice in this book.

One last point to close this subsection: As you probably know, there's also something called the relational calculus. The relational calculus can be regarded as an alternative to the relational algebra; that is, instead of saying the manipulative part of the relational model consists of the relational algebra (plus relational assignment), we can equally well say it consists of the relational calculus (plus relational assignment). The two are equivalent and interchangeable, in the sense that for every algebraic expression there's a logically equivalent expression of the calculus and vice versa. I'll have more to say about the calculus later, mostly in Chapters 10 and 11.

## The Running Example

I'll finish up this brief review by introducing the example I'll be using as a basis for most if not all of the discussions in the rest of the book: the familiar-not to say hackneyed-suppliers-and-parts database. (I apologize for dragging out this old warhorse yet one more time, but I believe that using the same example in a variety of books and other publications can help, not hinder, learning.) Sample values are shown in Fig. 1.3. To elaborate:

S

| SNO | SNAME | STATUS | CITY |
| :--- | :--- | ---: | :--- |
| S1 | Smith | 20 | London |
| S2 | Jones | 10 | Paris |
| S3 | Blake | 30 | Paris |
| S4 | Clark | 20 | London |
| S5 | Adams | 30 | Athens |

P

| PNO | PNAME | COLOR | WEIGHT | CITY |
| :--- | :--- | :--- | ---: | :--- |
| P1 | Nut | Red | 12.0 | London |
| P2 | Bolt | Green | 17.0 | Paris |
| P3 | Screw | Blue | 17.0 | Oslo |
| P4 | Screw | Red | 14.0 | London |
| P5 | Cam | Blue | 12.0 | Paris |
| P6 | Cog | Red | 19.0 | London |

SP

| SNO | PNO | QTY |
| :---: | :---: | :---: |
| $=\overline{\text { S1 }}$ | $=\overline{\text { P1 }}$ | 300 |
| S1 | P2 | 200 |
| S1 | P3 | 400 |
| S1 | P4 | 200 |
| S1 | P5 | 100 |
| S1 | P6 | 100 |
| S2 | P1 | 300 |
| S2 | P2 | 400 |
| S3 | P2 | 200 |
| S4 | P2 | 200 |
| S4 | P4 | 300 |
| S4 | P5 | 400 |

Fig. 1.3: The suppliers-and-parts database-sample values

## Suppliers

Relation S denotes suppliers (more accurately, suppliers under contract). Each supplier has one supplier number (SNO), unique to that supplier (as you can see from the figure, I've made $\{\mathrm{SNO}\}$ the primary key); one name (SNAME), not necessarily unique (though the SNAME values in Fig. 1.3 do happen to be unique); one status value (STATUS), representing some kind of ranking or preference level among available suppliers; and one location (CITY).

## Parts

Relation P denotes parts (more accurately, kinds of parts). Each kind of part has one part number (PNO), which is unique ( $\{\mathrm{PNO}\}$ is the primary key); one name (PNAME); one color (COLOR); one weight (WEIGHT); and one location where parts of that kind are stored (CITY).

## Shipments

Relation SP denotes shipments (it shows which parts are supplied, or shipped, by which suppliers). Each shipment has one supplier number (SNO), one part number (PNO), and one quantity (QTY). For the sake of the example, I assume there's at most one shipment at any given time for a given supplier and a given part ( $\{\mathrm{SNO}, \mathrm{PNO}\}$ is the primary key; also, $\{\mathrm{SNO}\}$ and $\{\mathrm{PNO}\}$ are both foreign keys, corresponding to the primary keys of $S$ and $P$, respectively). Notice that the database of Fig. 1.3 includes one supplier, supplier S5, with no shipments at all.

## MODEL vs. IMPLEMENTATION

Before going any further, there's an important point I need to explain, because it underpins everything else to be discussed in this book. The relational model is, of course, a data model. Unfortunately, however, this latter term has two quite distinct meanings in the database world. The first and more fundamental one is this:

Definition: A data model (first sense) is an abstract, self-contained, logical definition of the data structures, data operators, and so forth, that together make up the abstract machine with which users interact.

This is the meaning we have in mind when we talk about the relational model in particular. And, armed with this definition, we can usefully, and importantly, go on to distinguish a data model in this first sense from its implementation, which can be defined as follows:

Definition: An implementation of a given data model is a physical realization on a real machine of the components of the abstract machine that together constitute that model.

Let me illustrate these definitions in terms of the relational model specifically. First of all, consider the concept relation itself. That concept is part of the model: Users have to know what relations are, they have to know they're made up of tuples and attributes, they have to know how to interpret them, and so on. All that's part of the model. But they don't have to know how relations are physically stored on the disk, or how individual data values are physically encoded, or what indexes or other access paths exist; all that's part of the implementation, not part of the model.

Or consider the concept join: Users have to know what a join is, they have to know how to invoke a join, they have to know what the result of a join looks like, and so on. Again, all that's part of the model. But they don't have to know how joins are physically implemented, or what expression transformations take place under the covers, or what indexes or other access paths are used, or what physical I/O operations occur; all that's part of the implementation, not part of the model.

And one more example: Candidate keys (keys for short) are, again, part of the model, and users definitely have to know what keys are; in particular, they have to know that such keys have the property of uniqueness. Now, key uniqueness is typically enforced in today's systems by means of what's called a "unique index"; but indexes in general, and unique indexes in particular, aren't part of the model, they're part of the implementation. Thus, a unique index mustn't be confused with a key in the relational sense, even though the former might be used to implement the latter (more precisely, to implement some key constraint-see Chapter 8).

In a nutshell, then:

- The model (first meaning) is what the user has to know.
- The implementation is what the user doesn't have to know.

Please understand that I'm not saying here that users aren't allowed to know about the implementation; I'm just saying they don't have to. In other words, everything to do with implementation should be, at least potentially, hidden from the user.

Here are some important consequences of the foregoing definitions. First of all, observe that everything to do with performance is fundamentally an implementation issue, not a model issue. This point is widely misunderstood! For example, we often hear remarks to the effect that "joins are slow." But such remarks simply make no sense. Join is part of the model, and the model as such can't be said to be either fast or slow; only implementations can be said to possess any such quality. Thus, we might reasonably say that some specific product
$X$ has a faster or slower implementation of some specific join, on some specific data, than some other specific product $Y$ does-but that's about all.

Now, I don't want to give the wrong impression here. It's true that performance is fundamentally an implementation issue; however, that doesn't mean a good implementation will perform well if you use the model badly. Indeed, that's precisely one of the reasons why you need to know the model: so you won't use it badly. If you write an expression such as S JOIN SP, you're within your rights to expect the system to implement it efficiently; but if you insist on, in effect, hand coding the join yourself, perhaps like this (pseudocode)-

```
do for all tuples in S ;
    fetch S tuple into TS , TN , TT , TC ;
    do for all tuples in SP with SNO = TS ;
        fetch SP tuple into TS , TP , TQ ;
        emit TS, TN , TT , TC , TP , TQ ;
    end ;
end ;
```

-then there's no way you're going to get good performance. Recommendation: Don't do this. Relational systems shouldn't be used like simple access methods. ${ }^{8}$

By the way, these remarks about performance apply to SQL too. Like the relational operators (join and the rest), SQL as such can't be said to be fast or slow-only implementations can sensibly be described in such terms-but it's also possible to use SQL in such a way as to guarantee bad performance. Although I'll generally have little to say about performance in this book, therefore, I will occasionally point out certain performance implications of what I'm recommending.

Aside: I'd like to elaborate for a moment on this matter of performance. By and large, my recommendations in this book are never based on performance as a prime motivator; after all, it has always been an objective of the relational model to take performance concerns out of the hands of the user and put them into the hands of the system instead. However, it goes without saying that this objective hasn't yet been fully achieved, and so (as I've already said) the goal of using SQL relationally must sometimes be compromised in the interest of achieving satisfactory performance. That's another reason why, as I said earlier in this chapter, the overriding rule has to be: You can do what you like, so long as you know what you're doing. End of aside.

Back to model vs. implementation, and points arising from that distinction: The second point is that, as you probably realize, it's precisely the separation of model and implementation that allows us to achieve physical data independence. Physical data independence-not a great term, by the way, but we seem to be stuck with it-means we have the freedom to make changes in the way the data is physically stored and accessed without having to make corresponding changes in the way the data is perceived by the user. Now, the reason we might want to change those storage and access details is, typically, performance; and the fact that we can make such changes without having to change the way the data looks to the user means that existing programs, queries, and the like can all still work after the change. Very importantly, therefore, physical data independence means protecting investment in user training and applications (investment in logical database design also, I might add).

It follows from all of the above that, as previously indicated, indexes, and indeed physical access paths of any kind, are properly part of the implementation, not the model; they belong under the covers and should be hidden from the user. (Note that access paths as such are nowhere mentioned in the relational model.) For the same reasons, they should be rigorously excluded from SQL also. Recommendation: Avoid the use of any SQL

[^7]construct that violates this precept. (Actually there's nothing in the standard that does, so far as I'm aware, but I know the same isn't true of certain SQL products.)

Anyway, as you can see from the foregoing definitions, the distinction between model and implementation is really just a special case-a very important special case-of the familiar distinction between logical and physical considerations in general. Sadly, however, most of today's SQL systems don't make those distinctions as clearly as they should. As a direct consequence, they deliver far less physical data independence than they should, and far less than, in principle, relational systems are capable of. I'll come back to this issue in the next section.

Now I turn to the second meaning of the term data model, which I dare say you're very familiar with. It can be defined thus:

Definition: A data model (second sense) is a model of the data-especially the persistent data-of some particular enterprise.

In other words, a data model in the second sense is just a (logical, and possibly somewhat abstract) database design. For example, we might speak of the data model for some bank, or some hospital, or some government department.

Having explained these two different meanings, I'd like to draw your attention to an analogy that I think nicely illuminates the relationship between them:

- A data model in the first sense is like a programming language, whose constructs can be used to solve many specific problems but in and of themselves have no direct connection with any such specific problem.
- A data model in the second sense is like a specific program written in that language-it uses the facilities provided by the model, in the first sense of that term, to solve some specific problem.

By the way, it follows from all of the above that if we're talking about data models in the second sense, then we might reasonably speak of "relational models" in the plural, or "a" relational model (with an indefinite article). But if we're talking about data models in the first sense, then there's only one relational model, and it's the relational model (with the definite article). I'll have more to say on this latter point in Appendix A.

For the remainder of this book I'll use the term data model, or more usually just model for short, exclusively in its first sense.

## PROPERTIES OF RELATIONS

Now let's get back to our examination of basic relational concepts. In this section, I want to focus on some specific properties of relations themselves. First of all, every relation has a heading and a body: The heading is a set of attributes (where by the term attribute I mean, very specifically, an attribute-name/type-name pair, and no two attributes in the same heading have the same attribute name), and the body is a set of tuples that conform to that heading. In the case of the suppliers relation in Fig. 1.3, for example, there are four attributes in the heading and five tuples in the body. Note, therefore, that a relation doesn't really contain tuples-it contains a body, and that body in turn contains the tuples-but we do usually talk as if relations contained tuples directly, for simplicity.

By the way, although it's strictly correct to say the heading consists of attribute-name/type-name pairs, it's usual to omit the type names in pictures like Fig. 1.3 and hence to pretend the heading is just a set of attribute names. For example, the STATUS attribute does have a type-INTEGER, let's say-but I didn't show it in Fig. 1.3. But you should never forget it's there!

Next, the number of attributes in the heading is the degree (sometimes the arity), and the number of tuples in the body is the cardinality. For example, relation S in Fig. 1.3 has degree 4 and cardinality 5; likewise, relation P in
that figure has degree 5 and cardinality 6 , and relation SP in that figure has degree 3 and cardinality 12 . Note: The term degree is used in connection with tuples also. ${ }^{9}$ For example, the tuples in relation S are (like relation S itself) all of degree 4 .

Next, relations never contain duplicate tuples. This property follows because a body is defined to be a set of tuples, and sets in mathematics don't contain duplicate elements. Now, SQL fails here, as I'm sure you know: SQL tables are allowed to contain duplicate rows and thus aren't relations, in general. Please understand, therefore, that throughout this book I always use the term "relation" to mean a relation-without duplicate tuples, by definition-and not an SQL table. Please understand too that relational operations always produce a result without duplicate tuples, again by definition. For example, projecting the suppliers relation of Fig. 1.3 on CITY produces the result shown here on the left and not the one on the right:

(The result on the left can be obtained via the SQL query SELECT DISTINCT CITY FROM S. Omitting that DISTINCT leads to the nonrelational result on the right. Note in particular that the table on the right has no double underlining; that's because it has no key, and hence no primary key a fortiori.)

Next, the tuples of a relation are unordered, top to bottom. This property follows because, again, a body is defined to be a set, and sets in mathematics have no ordering to their elements (thus, for example, $\{a, b, c\}$ and $\{c, a, b\}$ are the same set in mathematics, and a similar remark naturally applies to the relational model). Of course, when we draw a relation as a table on paper, we do have to show the rows in some top to bottom order, but that ordering doesn't correspond to anything relational. In the case of the suppliers relation as depicted in Fig. 1.3, for example, I could have shown the rows in any order-say supplier S 3 , then S 1 , then S 5 , then S 4 , then S 2 -and the picture would still represent the same relation. Note: The fact that relations have no ordering to their tuples doesn't mean queries can't include an ORDER BY specification, but it does mean such queries produce a result that's not a relation. ORDER BY is useful for displaying results, but it isn't a relational operator as such.

In similar fashion, the attributes of a relation are also unordered, left to right, because a heading too is a mathematical set. Again, when we draw a relation as a table on paper, we have to show the columns in some left to right order, but that ordering doesn't correspond to anything relational. In the case of the suppliers relation as depicted in Fig. 1.3, for example, I could have shown the columns in any left to right order-say STATUS, SNAME, CITY, SNO-and the picture would still represent the same relation in the relational model. Incidentally, SQL fails here too: SQL tables do have a left to right ordering to their columns (another reason why SQL tables aren't relations, in general). For example, these two pictures represent the same relation but different SQL tables:

[^8]| SNO | CITY |
| :--- | :--- |
| S1 | London |
| S2 | Paris |
| S3 | Paris |
| S4 | London |
| S5 | Athens |


| CITY | SNO |
| :--- | :---: |
| London | S1 |
| Paris | S2 |
| Paris | S3 |
| London | S4 |
| Athens | S5 |

(The corresponding SQL queries are SELECT SNO, CITY FROM S and SELECT CITY, SNO FROM S, respectively. Now, you might be thinking that the differences between these two queries, and between these two tables, are hardly very significant; in fact, however, they have some serious consequences, some of which I'll be touching on in later chapters. See, for example, the discussion of SQL's explicit JOIN operator in Chapter 6.)

Finally, relations are always normalized (equivalently, they're in first normal form, 1NF). ${ }^{10}$ Informally, what this means is that, in terms of the tabular picture of a relation, at every row and column intersection we always see just a single value. More formally, it means that every tuple in every relation contains just a single value, of the appropriate type, in every attribute position. Note: I'll have quite a lot more to say on this particular issue in the next chapter.

Before I finish with this section, I'd like to emphasize something I've touched on several times already: namely, the fact that there's a logical difference between a relation as such, on the one hand, and a picture of a relation as shown in, for example, Figs. 1.1 and 1.3, on the other. To say it one more time, the constructs in Figs. 1.1 and 1.3 aren't relations at all but, rather, pictures of relations-which I generally refer to as tables, despite the fact that table is a loaded word in SQL contexts. Of course, relations and tables do have certain points of resemblance, and in informal contexts it's usual, and usually acceptable, to say they're the same thing. But when we're trying to be precise-and right now I am trying to be a little bit precise-then we do have to recognize that the two concepts are not identical.

As an aside, I observe that, more generally, there's a logical difference between a thing of any kind and a picture of that thing. There's a famous painting by Magritte that beautifully illustrates the point I'm trying to make here. The painting is of an ordinary tobacco pipe, but underneath Magritte has written Ceçi n'est pas une pipe ... the point being, of course, that obviously the painting isn't a pipe-instead, it's a picture of a pipe.

All of that being said, I should now say too that it's actually a major advantage of the relational model that its basic abstract object, the relation, does have such a simple representation on paper; it's that simple representation on paper that makes relational systems easy to use and easy to understand, and makes it easy to reason about the way such systems behave. However, it's unfortunately also the case that that simple representation does suggest some things that aren't true (e.g., that there's a top to bottom tuple ordering).

And one further point: I've said there's a logical difference between a relation and a picture of a relation. The concept of logical difference derives from a dictum of Wittgenstein's:

## All logical differences are big differences.

This notion is an extraordinarily useful one; as a "mind tool," it's a great aid to clear and precise thinking, and it can be very helpful in pinpointing and analyzing some of the confusions that are, unfortunately, all too common in the database world. I'll be appealing to it many times in the pages ahead. Meanwhile, let me point out that we've encountered quite a few important logical differences already. Here are some of them:

[^9]SQL vs. the relational model
Model vs. implementation

Data model (first sense) vs. data model (second sense)
And we'll be meeting many more in the pages ahead.

## Some Crucial Points

At this juncture I'd like to mention some crucial points that I'll be elaborating on in later chapters (especially Chapter 3). The points in question are these:

- Every subset of a tuple is a tuple: For example, consider the tuple for supplier S1 in Fig. 1.3. That tuple has four components, corresponding to the four attributes SNO, SNAME, STATUS, and CITY. And if we remove (say) the SNAME component, what's left is indeed still a tuple: viz., a tuple with three components (a tuple of degree three).
- Every subset of a heading is a heading: For example, consider the heading of the suppliers relation in Fig. 1.3. That heading has four attributes: SNO, SNAME, STATUS, and CITY. And if we remove (say) the SNAME and STATUS attributes, what's left is still a heading, a heading of degree two.
- Every subset of a body is a body: For example, consider the body of the suppliers relation in Fig. 1.3. That body has five tuples, corresponding to the five suppliers S1, S2, S3, S4, and S5. And if we remove (say) the S1 and S3 tuples, what's left is still a body, a body of cardinality three.

Note: Perhaps I should state for the record here that throughout this book—in accordance with normal practice-I take expressions of the form " $B$ is a subset of $A$ " to include the possibility that $A$ and $B$ might be equal. Thus, for example, every tuple is a subset of itself (and so is every heading, and so is every body). When I want to exclude such a possibility, I'll talk explicitly in terms of proper subsets. For example, our usual tuple for supplier S1 is certainly a subset of itself, but it isn't a proper subset of itself. What's more, the foregoing remarks apply equally to supersets, mutatis mutandis; for example, the tuple for supplier S1 is a superset of itself, but not a proper superset of itself. ${ }^{11}$

I'd also like to say something about the crucial notion of equality-especially as that notion applies to tuples and relations specifically. In general, two values are equal if and only if they're the very same value. For example, the integer 3 is equal to the integer 3, and not to anything else-in particular, not to any other integer. In exactly the same way, two tuples are equal if and only if they're the very same tuple. With reference to Fig. 1.1, for example, the tuple for supplier S1 is equal to the tuple for supplier S1, and not to anything else-in particular, not to any other tuple. In other words, two tuples are equal if and only if (a) they involve exactly the same attributes and (b) corresponding attribute values are equal in turn.

Moreover (this might seem obvious, but it needs to be said), two tuples are duplicates of each other if and only if they're equal.

Turning now to relations: In exactly the same way, two relations are equal if and only if they're the very same relation. With reference to Fig. 1.1, for example, the suppliers relation is equal to the suppliers relation and

[^10]not to anything else-in particular, not to any other relation. In other words, two relations are equal if and only if, in turn, their headings are equal and their bodies are equal.

As I've already said, I'll be returning to these matters in Chapter 3. Here let me just add that the notion of tuple equality in particular is absolutely fundamental-just about everything in the relational model is crucially dependent on it, as we'll see.

## BASE vs. DERIVED RELATIONS

As I explained earlier, the operators of the relational algebra allow us to start with some given relations, such as the ones depicted in Fig. 1.3, and obtain further relations from those given ones (for example, by doing queries). The given relations are referred to as base relations, the others are derived relations. In order to get us started, therefore, a relational system has to provide a means for defining those base relations in the first place. In SQL, this task is performed by the CREATE TABLE statement (the SQL counterpart to a base relation being, of course, a base table, which is what CREATE TABLE creates). And base relations obviously need to be named-for example:

```
CREATE TABLE S ... ;
```

But certain derived relations, including in particular what are called views, are named too. A view (also known as a virtual relation) is a named relation whose value at any given time $t$ is the result of evaluating a certain relational expression at that time $t$. Here's an SQL example:

```
CREATE VIEW SST PARIS AS
    ( SELECT SNNO , STATUS
        FROM S
        WHERE CITY = 'Paris' ) ;
```

In principle, you can operate on views just as if they were base relations, ${ }^{12}$ but they aren't base relations. Instead, you can think of a view as being "materialized"-in effect, you can think of a base relation being constructed whose value is obtained by evaluating the specified relational expression-at the time the view in question is referenced. But I must emphasize that thinking of views being materialized in this way when they're referenced is purely conceptual; it's just a way of thinking; it's not what's really supposed to happen; and it wouldn't work for update operations in any case. How views are really supposed to work is explained in Chapter 9.

By the way, there's an important point I need to make here. You'll often hear the difference between base relations and views described like this (warning! untruths coming up!):

- Base relations really exist-that is, they're physically stored in the database.
- Views, by contrast, don't "really exist"-they merely provide different ways of looking at the base relations.

But the relational model has nothing to say as to what's physically stored!-in fact, it has nothing to say about physical storage matters at all. In particular, it categorically does not say that base relations are physically stored. The only requirement is that there must be some mapping between whatever is physically stored and those base relations, so that those base relations can somehow be obtained when they're needed (conceptually, at any rate). If the base relations can be obtained from whatever's physically stored, then everything else can be, too. For example,

[^11]we might physically store the join of suppliers and shipments, instead of storing them separately; then base relations S and SP could be obtained, conceptually, by taking appropriate projections of that join. In other words: Base relations are no more (and no less!) "physical" than views are, so far as the relational model is concerned.

The fact that the relational model says nothing about physical storage is deliberate, of course. The idea was to give implementers the freedom to implement the model in whatever way they chose-in particular, in whatever way seemed likely to yield good performance-without compromising on physical data independence. The sad fact is, however, most SQL product vendors seem not to have understood this point (or not to have risen to the challenge, at any rate); instead, they map base tables fairly directly to physical storage, ${ }^{13}$ and (as noted previously) their products therefore provide far less physical data independence than relational systems are or should be capable of. Indeed, this state of affairs is reflected in the SQL standard itself (as well as in most other SQL documentation), which typically-quite ubiquitously, in fact-talks in terms of "tables and views." Clearly, anyone who talks this way is under the impression that tables and views are different things, and probably also that "tables" always means base tables specifically, and probably also that base tables are physically stored and views aren't. But the whole point about a view is that it is a table (or, as I would prefer to say, a relation); that is, we can perform the same kinds of operations on views as we can on regular relations (at least in the relational model), because views are "regular relations." Throughout this book, therefore, I'll use the term relation to mean a relation (possibly a base relation, possibly a view, possibly a query result, and so on); and if I want to mean a base relation specifically, then I'll say "base relation." Recommendation: I suggest strongly that you adopt the same discipline for yourself. Don't fall into the common trap of thinking the term relation means a base relation specifically-or, in SQL terms, thinking the term table means a base table specifically. Likewise, don't fall into the common trap of thinking base relations (or base tables, in SQL) have to be physically stored.

## RELATIONS vs. RELVARS

Now, it's entirely possible that you already knew everything I've been telling you in this chapter so far; in fact, I rather hope you did, though I also hope that didn't mean you found the material boring. Anyway, now I come to something you might not know already. The fact is, historically there's been a lot of confusion over yet another logical difference: namely, that between relations as such, on the one hand, and relation variables on the other.

Forget about databases for a moment; consider instead the following simple programming language example.
Suppose I say in some programming language:
DECLARE N INTEGER . .. ;
Then N here is not an integer. Rather, it's a variable, whose values are integers as such-different integers at different times. We all understand that. Well, in exactly the same way, if I say in SQL-
CREATE TABLE T ... ;
-then T is not a table: It's a variable, a table variable or (as I would prefer to say, ignoring various SQL quirks such as duplicate rows and left to right column ordering) a relation variable, whose values are relations as such (different relations at different times).

Take another look at Fig. 1.3, the suppliers-and-parts database. That figure shows three relation values-namely, the relation values that happen to exist in the database at some particular time. But if we were to

[^12]
## 20 Chapter 1 / Setting the Scene

look at the database at some different time, we would probably see three different relation values appearing in their place. In other words, S, P, and SP in that database are really variables: relation variables, to be precise. For example, suppose the relation variable $S$ currently has the value-the relation value, that is-shown in Fig. 1.3, and suppose we delete the set of tuples (actually there's only one) for suppliers in Athens:

```
DELETE S WHERE CITY = 'Athens' ;
```

Here's the result:

| SNO | SNAME | STATUS | CITY |
| :---: | :--- | ---: | :--- |
| S1 | Smith | 20 | London |
| S2 | Jones | 10 | Paris |
| S3 | Blake | 30 | Paris |
| S4 | Clark | 20 | London |

Conceptually, what's happened here is that the old value of $S$ has been replaced in its entirety by a new value. Of course, the old value (with five tuples) and the new one (with four) are very similar, in a sense, but they certainly are different values. In fact, the DELETE just shown is logically equivalent to, and indeed shorthand for, the following relational assignment:

```
S := S MINUS ( S WHERE CITY = 'Athens' ) ;
```

As with all assignments, the effect here is that (a) the source expression on the right side is evaluated and then (b) the value that's the result of that evaluation is then assigned to the target variable on the left side, with the overall result already explained.

Aside: I can't show the foregoing assignment in SQL because SQL doesn't directly support relational assignment. Instead, I've shown it (as well as the original DELETE) in a more or less self-explanatory language called Tutorial D. Tutorial D is the language Hugh Darwen and I use to illustrate relational ideas in our book Databases, Types, and the Relational Model: The Third Manifesto (see Appendix G)-and I'll use it in the present book too, when I'm explaining relational concepts. ${ }^{14}$ But since my intended audience is SQL practitioners, I'll show SQL analogs as well, most of the time. Note: A BNF grammar for Tutorial D can be found in Appendix D. End of aside.

To repeat, DELETE is shorthand for a certain relational assignment-and, of course, an analogous remark applies to INSERT and UPDATE also: They too are basically just shorthand for certain relational assignments. Thus, as I mentioned in the section "A Review of the Original Model," relational assignment is the fundamental update operator in the relational model; indeed it's the only update operator we really need, logically speaking.

So there's a logical difference between relation values and relation variables. The trouble is, the database literature has historically used the same term, relation, to stand for both, and that practice has certainly led to confusion. ${ }^{15}$ In this book, therefore, I'll distinguish very carefully between the two from this point forward-I'll talk

[^13]in terms of relation values when I mean relation values and relation variables when I mean relation variables. However, I'll also abbreviate relation value, most of the time, to just relation (exactly as we abbreviate integer value most of the time to just integer). And I'll abbreviate relation variable most of the time to relvar; for example, I'll say the suppliers-and-parts database contains three relvars (more precisely, three base relvars).

As an exercise, you might like to go back over the text of this chapter so far and see exactly where I used the term relation when I really ought to have been using the term relvar instead (or as well).

## VALUES vs. VARIABLES

The logical difference between relations and relvars is actually a special case of the logical difference between values and variables in general, and I'd like to take a few moments to look at the more general case. (It's a bit of a digression, but I think it's worth taking the time here because clear thinking in this area can be such a great help, in so many ways.) Here then are some definitions:

Definition: A value is what the logicians call an "individual constant," such as the integer 3. A value has no location in time or space. However, values can be represented in memory by means of some encoding, and those representations or encodings do have location in time and space. Indeed, distinct representations of the same value can appear at any number of distinct locations in time and space-meaning, loosely, that any number of different variables (see the next definition) can have the same value, at the same time or different times. Observe in particular that, by definition, a value can't be updated; for if it could, then after such an update it wouldn't be that value any longer.

Definition: A variable is a holder for a representation of a value. A variable does have location in time and space. Also, variables, unlike values, can be updated; that is, the current value of the variable can be replaced by another value. (After all, that's what "variable" means-to be a variable is to be updatable and to be updatable is to be a variable; equivalently, to be a variable is to be assignable to, to be assignable to is to be a variable.)

Please note very carefully that it isn't just simple things like the integer 3 that are legitimate values. On the contrary, values can be arbitrarily complex-for example, a value might be a geometric point; or a polygon; or an X ray; or an XML document; or a fingerprint; or an array; or a stack; or a list; or a relation (and on and on). Analogous remarks apply to variables too, of course. I'll have more to say about such matters in the next chapter.

Now, you might think it's hard to imagine people getting confused over a distinction as obvious and fundamental as the one between values and variables. In fact, however, it's all too easy to fall into traps in this area. By way of illustration, consider the following extract from a tutorial on object databases (the italicized portions in brackets are comments by myself):

We distinguish the declared type of a variable from ... the type of the object that is the current value of the variable [so an object is a value] ... We distinguish objects from values [so an object isn't a value after all] ... A mutator [is an operator such that it's] possible to observe its effect on some object [so in fact an object is a variable].

[^14]
## 22 Chapter 1 / Setting the Scene

## CONCLUDING REMARKS

This brings us to the end of this preliminary chapter. For the most part, my aim has just been to tell you what I rather hope you knew already (and you might have felt the chapter was a little light on technical substance, therefore). Anyway, just to review briefly:

- I explained why we'd be concerned with principles, not products, and why I'd be using formal terminology such as relation, tuple, and attribute (at least in relational contexts) in place of their more "user friendly" SQL counterparts.
- I gave an overview of the original model, touching in particular on the following concepts: type (or domain), n-ary relation, tuple, attribute, candidate key (key for short), primary key, foreign key, entity integrity, referential integrity, relational assignment, and the relational algebra. (I also briefly mentioned the relational calculus.) With regard to the algebra, I mentioned the closure property and very briefly described the operators restrict, project, product, intersection, union, difference, and join.
- I discussed various properties of relations, introducing the terms heading, body, cardinality, and degree. Relations have no duplicate tuples, no top to bottom tuple ordering, and no left to right attribute ordering. I also discussed the difference between base relations (or base relvars, rather) and views. And I explained that every subset of a tuple is a tuple, every subset of a heading is a heading, and every subset of a body is a body.
- I discussed the logical differences between model and implementation, values and variables in general, and relations and relvars in particular. The model vs. implementation discussion in particular led to a discussion of physical data independence.
- I claimed that SQL and the relational model aren't the same thing. We've seen a few differences already-for example, the fact that SQL permits duplicate rows, the fact that SQL tables have a left to right column ordering, and the fact that SQL doesn't clearly distinguish between table values and table variables-and we'll see many more in the pages to come.

One last point (I didn't mention this explicitly before, but I hope it's clear from everything I did say): Overall, the relational model is declarative, not procedural, in nature; that is, it always favors declarative solutions over procedural ones, wherever such solutions are feasible. The reason is obvious: Declarative means the system does the work, procedural means the user does the work (so we're talking about productivity, among other things). That's why the relational model supports declarative queries, declarative updates, declarative view definitions, declarative integrity constraints, and on and on.

Note: After I first wrote the foregoing paragraph, I was informed that at least one well known SQL product apparently uses the term "declarative" to mean the system doesn't do the work! That is, it allows the user to state certain things declaratively (for example, the fact that a certain view has a certain key), but it doesn't enforce the constraint implied by that declaration-it simply assumes the user is going to enforce it instead. Such terminological abuses do little to help the cause of genuine understanding. Caveat lector.

## EXERCISES

1.1 (Repeated from the body of the chapter, but slightly reworded here.) If you haven't done so already, go through the chapter again and identify all of the places where I used the term relation when I should by rights have used the term relvar instead.
1.2 Who was E. F. Codd?
1.3 What's a domain?
1.4 What do you understand by the term referential integrity?
1.5 The terms heading, body, attribute, tuple, cardinality, and degree, defined in the body of the chapter for relation values, can all be interpreted in the obvious way to apply to relvars as well. Make sure you understand this remark.
1.6 Distinguish between the two meanings of the term data model.
1.7 Explain in your own words (a) physical data independence, (b) the difference between model and implementation.
1.8 In the body of the chapter, I said that tables like those in Figs. 1.1 and 1.3 weren't relations as such but, rather, pictures of relations. What are some of the specific points of difference between such pictures and the corresponding relations?
1.9 (Try this exercise without looking back at the body of the chapter.) What relvars does the suppliers-and-parts database contain? What attributes do they involve? What keys and foreign keys do they have? (The point of this exercise is that it's worth making yourself as familiar as possible with the structure, at least in general terms, of the running example. It's not so important to remember the actual data values in detail-though it wouldn't hurt if you did.)
1.10 "There's only one relational model." Explain this remark.
1.11 The following is an excerpt from a certain database textbook: "[It] is important to make a distinction between stored relations, which are tables, and virtual relations, which are views ... [We] shall use relation only where a table or a view could be used. When we want to emphasize that a relation is stored, rather than a view, we shall sometimes use the term base relation or base table." This text betrays several confusions or misconceptions regarding the relational model. Identify as many as you can.
1.12 The following is an excerpt from another database textbook: "[The relational] model ... defines simple tables for each relation and many to many relationships. Cross-reference keys link the tables together, representing the relationships between entities. Primary and secondary indexes provide rapid access to data based upon qualifications." This text is intended as a definition (!) of the relational model ... What's wrong with it?
1.13 Write CREATE TABLE statements for an SQL version of the suppliers-and-parts database.
1.14 The following is a typical SQL INSERT statement against the suppliers-and-parts database:

## 24 Chapter 1 / Setting the Scene

```
INSERT INTO SP ( SNO , PNO , QTY ) VALUES ( 'S5' , 'P6' , 250 ) ;
```

Show an equivalent relational assignment operation. Note: I realize I haven't yet explained the syntax of relational assignment in detail, so don't worry too much about giving a syntactically correct answer-just do the best you can.
1.15 (Harder.) The following is a typical SQL UPDATE statement against the suppliers-and-parts database:

```
UPDATE S SET STATUS = 25 WHERE CITY = 'Paris' ;
```

Show an equivalent relational assignment operation. (The purpose of this exercise is to get you thinking about what's involved. I haven't told you enough in this chapter to allow you to answer it fully. See the discussion of "what if" queries in Chapter 7 for a detailed explanation.)
1.16 In the body of the chapter, I said that SQL doesn't directly support relational assignment. Does it support it indirectly? If so, how? A related question: Can all relational assignments be expressed in terms of INSERT and/or DELETE and/or UPDATE? If not, why not? What are the implications?
1.17 From a practical standpoint, why do you think duplicate tuples, top to bottom tuple ordering, and left to right attribute ordering are all very bad ideas? (These questions deliberately weren't answered in the body of the chapter, and this exercise might best serve as a basis for group discussion. We'll be taking a closer look at such matters later in the book.)

## Chapter 2

## Types and Domains

A major purpose of type systems is to avoid embarrassing questions about representations, and to forbid situations in which these questions might come up.<br>-Luca Cardelli and Peter Wegner:<br>"On Understanding Types, Data Abstraction, and Polymorphism"<br>ACM Comp. Surv. 17, No. 4 (December 1985)

This chapter is related only tangentially to the main theme of the book. Types are certainly fundamental, and the ideas discussed in this chapter are certainly important (they might help to dispel certain common misconceptions, too); however, type theory as such isn't a specifically relational topic, and type-related matters don't seem-at least on the surface-to have much to do with SQL daily life, as it were. What's more, while there are certainly SQL problems in this area, there isn't much you can do about them, for the most part; I mean, there isn't much concrete advice I can offer to help with the goal of using SQL relationally (though there is some, as you'll see). So you might want to give this chapter just a "once over lightly" reading on a first pass, and come back to it after you've absorbed more of the material from later chapters.

## TYPES AND RELATIONS

Data types (types for short) are fundamental to computer science. Relational theory in particular requires a supporting type theory, because relations are defined over types; that is, every attribute of every relation is defined to be of some type (and the same is true of relvars too, of course). For example, I'm going to assume throughout this book that attribute STATUS of the suppliers relvar $S$ is defined to be of type INTEGER. Under that assumption, every relation that's a possible value for relvar S must also have a STATUS attribute of type INTEGER-which means in turn that every tuple in such a relation must also have a STATUS attribute that's of type INTEGER, which means in turn that the tuple in question must have a STATUS value that's an integer.

I'll be discussing such matters in more detail later in this chapter. For now, let me just say that-with certain important exceptions, which I'll also be discussing later-a relational attribute can be of any type whatsoever, implying among other things that such types can be arbitrarily complex. In particular, those types can be either system or user defined. In this book, however, I don't plan to say much about user defined types as such, because:

The whole point about user defined types (from the point of view of the user who is merely using them, that is, as opposed to the user who actually has the job of defining them) is that they're supposed to behave just like system defined types anyway.

- Comparatively few users will ever be faced with the job of defining a type-and type definition doesn't really involve any specifically relational considerations in any case.

From this point forward, therefore, you can take the term type to mean a system defined type specifically, unless the context demands otherwise. The relational model prescribes just one such type, BOOLEAN (the most fundamental type of all). Type BOOLEAN contains exactly two values: two truth values, to be specific, denoted by
the literals TRUE and FALSE, respectively. Of course, real systems will support a variety of other system defined types as well, and I'll assume for definiteness that types INTEGER (integers), RATIONAL (rational numbers), and CHARACTER (character strings of arbitrary length) are among those supported. Note: I'll discuss the system defined types supported by SQL in particular later in the chapter.

Aside: A rational number is a number that can be expressed as the ratio of two integers (e.g., $3 / 8,5 / 12$, $-4 / 3$ ); an irrational number is a number that can't be so expressed (e.g., $\pi, \sqrt{ } 2$ ). Rational numbers (only) have the property that, in decimal notation, the fractional part of such a number can be expressed as either (a) a finite sequence of digits followed by an infinite sequence of zeros, which can be ignored without loss (e.g., $3 / 8=0.375000 \ldots$... ) or (b) a possibly empty finite sequence of digits followed by another finite sequence of digits, the first of which is nonzero, that infinitely repeats (e.g., $5 / 12=0.41666 \ldots$..). By contrast, the fractional part of an irrational number in decimal notation consists of an infinite, nonrepeating sequence of digits (e.g., $\pi=3.14159 \ldots, \sqrt{2}=1.41421 \ldots$ ). A real number is a number that's either rational or irrational. Now, many programming languages support a numeric type they call REAL; computers being finite, however, the only real numbers computers are actually capable of representing are necessarily rational ones. Hence Tutorial D's choice of the keyword RATIONAL. End of aside.

In the interest of historical accuracy, I should now explain that when Codd first defined the relational model, he said relations were defined over domains, not types. In fact, however, domains and types are exactly the same thing. Now, you can take this claim as a position statement on my part, if you like, but I want to present a series of arguments in support of that position. I'll start with the relational model as Codd originally defined it; thus, I'll use the term domain, not type, until further notice. There are two major topics I want to discuss, one in each of the next two sections:

■ Equality comparisons and "domain check override": This part of the discussion I hope will convince you that domains really are types.

- Data value atomicity and first normal form: And this part I hope will convince you that those types can be arbitrarily complex.


## EQUALITY COMPARISONS

Despite what I said a few moments ago about ignoring user defined types, I'm going to assume in the present section, purely for the sake of the example, that the supplier number (SNO) attributes in relvars S and SP are of some user defined type-sorry, domain-which I'll assume for simplicity is called SNO as well. Likewise, I'm going to assume that the part number (PNO) attributes in relvars P and SP are also of a user defined type (or domain) with the same name, PNO. Please note that these assumptions aren't crucial to my argument; it's just that I think they make the argument a little more convincing, and perhaps easier to follow.

I'll start with the fact that, as everyone knows (?), two values can be compared for equality in the relational model only if they come from the same domain. For example, the following comparison (which might be part of the WHERE clause in some SQL query) is obviously valid:

$$
\text { SP.SNO }=\text { S.SNO } \quad / * \text { OK } \quad * /
$$

By contrast, this one obviously (?) isn't:

```
SP.PNO = S.SNO /* not OK */
```

Why not? Because part numbers and supplier numbers are different kinds of things-they're defined on different domains. So the general idea is that the $\mathrm{DBMS}^{1}$ should reject any attempt to perform any relational operation (join, union, whatever) that involves, either explicitly or implicitly, an equality comparison between values from different domains. For example, suppose some user wants to find suppliers (like supplier S 5 in the sample values of Fig. 1.3 in Chapter 1) who currently supply no parts at all. The following is an attempt to formulate this query in SQL:

```
SELECT S.SNO , S.SNAME , S.STATUS , S.CITY
FROM S
WHERE NOT EXISTS
    ( SELECT *
        FROM SP
        WHERE SP.PNO = S.SNO ) /* not OK */
```

(There's no terminating semicolon because this is an expression, not a statement. See Exercise 2.24 at the end of the chapter.)

As the comment says, this formulation is certainly not OK. The reason is that, in the last line, the user presumably meant to say WHERE SP.SNO = S.SNO, but by mistake—probably just a slip of the typing fingers-he or she said WHERE SP.PNO = S.SNO instead. And, given that we're indeed talking about a simple typo (probably), it would be a friendly act on the part of the DBMS to interrupt the user at this point, highlight the error, and perhaps ask if the user would like to correct it before proceeding.

Now, I don't know any SQL product that actually behaves in the way I've just suggested; in today's products, depending on how you've set up the database, either the query will simply fail or it'll give the wrong answer. Well ... not exactly the wrong answer, perhaps, but the right answer to the wrong question. (Does that make you feel any better?)

To repeat, therefore, the DBMS should reject a comparison like $\mathrm{SP} . \mathrm{PNO}=\mathrm{S} . \mathrm{SNO}$ if it isn't valid. However, Codd felt there should be a way in such a situation for the user to make the DBMS go ahead and do the comparison anyway, even though it's apparently not valid, on the grounds that sometimes the user will know more than the DBMS does. Now, it's hard for me to do justice to this idea, because I frankly don't think it makes sense-but let me give it a try. Suppose it's your job to design a database involving, let's say, customers and suppliers; and you therefore decide to have a domain of customer numbers and a domain of supplier numbers; and you build your database that way, and load it, and everything works just fine for a year or two. Then, one day, one of your users comes along with a query you never heard before-namely: "Are any of our customers also suppliers to us?" Observe that this is a perfectly reasonable query; observe too that it might involve a comparison between a customer number and a supplier number (a cross domain comparison) to see if they're equal. And if it does, well, certainly the system mustn't prevent you from doing that comparison; certainly the system mustn't prevent you from posing a reasonable query.

On the basis of such arguments, Codd proposed what he called "domain check override" (DCO) versions of certain of his relational operators. A DCO version of join, for example, would perform the join even if the joining attributes were defined on different domains. In SQL terms, we might imagine this proposal being realized by means of a new clause, IGNORE DOMAIN CHECKS, that could be included in an SQL query, as here:

[^15]
## 28 Chapter 2 / Types and Domains

```
SELECT ...
FROM ...
WHERE CUSTNO = SNO
IGNORE DOMAIN CHECKS
```

And this new clause would be separately authorizable-most users wouldn't be allowed to use it (perhaps only the DBA $^{2}$ would be allowed to use it).

Before analyzing the DCO idea in detail, I want to look at a simpler example. Consider the following two queries on the suppliers-and-parts database:

```
SELECT ...
FROM P , SP
WHERE P.WEIGHT = SP.QTY
```

```
SELECT ...
FROM P , SP
WHERE P.WEIGHT - SP.QTY = 0
```

Assuming, reasonably enough, that weights and quantities are defined on different domains, the query on the left is clearly invalid. But what about the one on the right? According to Codd, that one's valid! In his book The Relational Model for Database Management Version 2 (Addison-Wesley, 1990), page 47, he says that in such a situation "the DBMS [merely] checks that the basic data types are the same"; in the case at hand, those "basic data types" are all just numbers (loosely speaking), and so that check succeeds.

To me, this conclusion is unacceptable. Clearly, the expressions P.WEIGHT = SP.QTY and P.WEIGHT SP.QTY $=0$ both mean essentially the same thing. Surely, therefore, they must both be valid or both be invalid; the idea that one might be valid and the other not surely makes no sense. So it seems to me there's something strange about Codd-style domain checks in the first place, before we even get to domain check override. (In essence, in fact, Codd-style domain checks apply only in the very special case where both comparands are specified as simple attribute references. Observe that the comparison P.WEIGHT $=$ SP.QTY falls into this special category but the comparison P.WEIGHT - SP.QTY $=0$ doesn't.)

Let's look at some even simpler examples. Consider the following comparisons (each of which might appear as part of an SQL WHERE clause, for example):
S.SNO = 'X4' P.PNO = 'X4' S.SNO = P.PNO

I hope you agree it's at least plausible that the first two of these could be valid (and evaluate successfully, and possibly even give TRUE) and the third not. But if so, then I hope you also agree there's something strange going on; apparently, we can have three values $a, b$, and $c$ such that $a=c$ is true and $b=c$ is true, but as for $a=$ $b$-well, we can't even do the comparison, let alone have it come out true! So what's going on?

I return now to the fact that attributes S.SNO and P.PNO are defined on domains SNO and PNO, respectively, and my claim that domains are actually types; as previously noted, in fact, I'm assuming for the sake of the present discussion that domains SNO and PNO in particular are user defined types. Now, it's possible (even likely) that those user defined types are both physically represented in terms of the system defined type CHAR; in fact, let's assume such is indeed the case, for definiteness. However, those representations are part of the implementation, not the model-they're irrelevant to the user, and as we saw in Chapter 1 they're supposed to be hidden from the user. In particular, therefore, the operators that apply to supplier numbers and part numbers are the operators defined in connection with those types, not the operators that happen to be defined in connection with type CHAR (see the section "What's a Type?" later in this chapter). For example, we can concatenate two character strings, but we probably can't concatenate two supplier numbers (we could do this latter only if concatenation were an operator defined in connection with type SNO).

[^16]Now, when we define a type, we also have to define the operators that can be used in connection with values and variables of the type in question (again, see the section "What's a Type?"). And one operator we must define is what's called a selector operator, which allows us to select, or specify, an arbitrary value of the type in question. ${ }^{3}$ In the case of type SNO, for example, the selector (which in practice would probably also be called SNO) allows us to select the particular SNO value that has some specified CHAR representation. Here's an example:
SNO ('S1')

This expression is an invocation of the SNO selector, and it returns a certain supplier number: namely, the one represented by the character string 'S1'. Likewise, the expression
PNO ('P1')
is an invocation of the PNO selector, and it returns a certain part number: namely, the one represented by the character string 'P1'. In other words, the SNO and PNO selectors effectively work by taking a certain CHAR value and converting it to a certain SNO value and a certain PNO value, respectively.

Now let's get back to the comparison S.SNO = 'X4'. As you can see, the comparands here are of different types (types SNO and CHAR, to be specific; in fact, 'X4' is a character string literal). Since they're of different types, they certainly can't be equal (recall from the beginning of the present section that two values can be compared for equality "only if they come from the same domain"). But the system does at least know there's an operator-namely, the SNO selector-that effectively performs CHAR to SNO conversions. So it can invoke that operator, implicitly, to convert the CHAR comparand to a supplier number, thereby effectively replacing the original comparison by this one:

$$
\text { S.SNO }=\text { SNO ('X4') }
$$

Now we're comparing two supplier numbers, which is legitimate.
In the same kind of way, the system can effectively replace the comparison P.PNO = ' X 4 ' by this one:

$$
\mathrm{P} . \mathrm{PNO}=\mathrm{PNO}\left(' \mathrm{X} 4^{\prime}\right)
$$

But in the case of the comparison S.SNO = P.PNO, there's no conversion operator known to the system (at least, let's assume not) that will convert a supplier number to a part number or the other way around, and so the comparison fails on a type error: The comparands are of different types, and there's no way to make them be of the same type.

Note: Implicit type conversion as illustrated in the foregoing examples is often called coercion in the literature. In the first example, therefore, we can say the character string ' X 4 ' is coerced to type SNO; in the second it's coerced to type PNO. I'll have a little more to say about coercion in SQL in particular in the section "Type Checking and Coercion in SQL," later.

[^17]
## 30 Chapter 2 / Types and Domains

To continue with the example: Another operator we must define when we define a type like SNO or PNO is what's called, generically, a THE_ operator, which effectively converts a given SNO or PNO value to the character string (or whatever else it is) that's used to represent it. ${ }^{4}$ Assume for the sake of the example that the THE operators for types SNO and PNO are called THE_SC and THE_PC, respectively. Then, if we really did want to compare S.SNO and P.PNO for equality, the only sense I can make of that requirement is that we want to test whether the corresponding character string representations are the same, which we might do like this:

```
THE_SC (S.SNO ) = THE_PC ( P.PNO )
```

In other words: Convert the supplier number to a string, convert the part number to a string, and compare the two strings.

As I'm sure you can see, the mechanism I've been sketching, involving selectors and THE_ operators, effectively provides both (a) the domain checking we want in the first place and (b) a way of overriding that checking, when desired, in the second place. Moreover, it does all this in a clean, fully orthogonal, non ad hoc manner. By contrast, domain check override doesn't really do the job; in fact, it doesn't really make sense at all, because it confuses types and representations (as noted previously, types are a model concept, representations are an implementation concept). Note: If you're not familiar with orthogonality as an important language design principle, you can read about it in "A Note on Orthogonality" in my book Relational Database Writings 1994-1997 (Addison-Wesley, 1998).

Now, you might have realized that what I'm talking about is here is what's known in language circles as strong typing. Different writers have slightly different definitions for this term, but basically it means that (a) everything-in particular, every value and every variable-has a type, and (b) whenever we try to perform some operation, the system checks that the operands are of the right types for the operation in question (or, possibly, are coercible to those right types). Observe too that this mechanism works for all operations, not just for the equality comparisons I've been discussing; the emphasis on equality and other comparison operations in discussions of domain checking in the literature is sanctioned by historical usage but is in fact misplaced. For example, consider the following expressions:

```
P.WEIGHT * SP.QTY
P.WEIGHT + SP.QTY
```

The first of these is probably valid (it yields another weight: namely, the total weight of the pertinent shipment). The second, by contrast, is probably not valid (what could it possibly mean to add a weight and a quantity?).

I'd like to close this section by stressing the absolutely fundamental role played by the equality operator (" $=$ "). It wasn't just an accident that the discussions above happened to focus on the question of comparing two values for equality. The fact is, equality truly is central, and the relational model requires it to be supported for every type. Indeed, since a type is basically a set of values (see the section "What's a Type?"), without the "=" operator we couldn't even say what values constitute the type in question! That is, given some type $T$ and some value $v$, we couldn't say, absent that operator, whether or not $v$ was one of the values in the set of values constituting type $T$.

What's more, the relational model also specifies the semantics of the " $=$ " operator, as follows: If $v 1$ and $v 2$ are values of the same type, then $v 1=v 2$ evaluates to TRUE if $v 1$ and $v 2$ are the very same value and FALSE

[^18]otherwise. (As a matter of fact, I said exactly this in Chapter 1, as you might recall.) By contrast, if $v 1$ and $v 2$ are values of different types, then $v l=v 2$ has no meaning-it's not even a legal comparison-unless $v l$ can be coerced to the type of $v 2$ or the other way around, in which case we aren't really talking about a comparison between $v 1$ and $v 2$ as such anyway.

## DATA VALUE ATOMICITY

I hope the previous section succeeded in convincing you that domains really are types, no more and no less. Now I want to turn to the issue of data value atomicity and the related notion of first normal form (1NF for short). In Chapter 1, I said that 1NF meant that every tuple in every relation contains just a single value (of the appropriate type) in every attribute position-and it's usual to add that those "single values" are supposed to be atomic. But this latter requirement raises the obvious question: What does it mean for data to be atomic?

Well, on page 6 of the book mentioned earlier (The Relational Model for Database Management Version 2), Codd defines atomic data as data that "cannot be decomposed into smaller pieces by the DBMS (excluding certain special functions)." Even if we ignore that parenthetical exclusion, however, this definition is a trifle puzzling; at best, it's certainly not very precise. For example, what about character strings? Are character strings atomic? Well, every database product I know provides a variety of operators-LIKE, SUBSTR (substring), "|" (concatenate), and so on-that rely by definition on the fact that character strings in general can be "decomposed into smaller pieces by the DBMS." So are such strings atomic? What do you think?

Here are some other examples of values whose atomicity is at least open to question and yet we would certainly want to allow as attribute values in tuples in relations:

Bit strings
Rational numbers (which might be regarded as being decomposable into integer and fractional parts)
Dates and times (which might be regarded as being decomposable into year / month / day and hour / minute / second components, respectively)

And so on.
Now I'd like to move on to what might be considered a more startling example. Refer to Fig. 2.1 below. Relation R1 in that figure is a reduced version of the shipments relation from our running example; it shows that certain suppliers supply certain parts, and it contains one tuple for each legitimate (SNO,PNO) combination. For the sake of the example, let's agree that supplier numbers and part numbers are indeed "atomic"; then we can presumably agree that R1, at least, is in 1NF.

R1

| SNO | PNO |
| :---: | :---: |
| S2 | P1 |
| S2 | P2 |
| S3 | P2 |
| S4 | P2 |
| S4 | P4 |
| S4 | P5 |

R2

| SNO | PNO |
| :--- | :--- |
| S2 | P1, P2 |
| S3 | P2 |
| S4 | P2, P4, P5 |

R3

| SNO | PNO_SET |
| :--- | :--- |
| $=\overline{\text { S2 }}$ | $\{\mathrm{P} 1, \mathrm{P} 2\}$ |
| S3 | $\{\mathrm{P} 2\}$ |
| S4 | $\{\mathrm{P} 2, \mathrm{P} 4, \mathrm{P} 5\}$ |

Fig. 2.1: Relations R1, R2, and R3

Now suppose we replace R1 by R2, which shows that certain suppliers supply certain groups of parts (attribute PNO in R2 is what some writers would call multivalued, and values of that attribute are groups of part numbers). Then most people would surely say that R2 is not in 1 NF ; in fact, it looks like a case of "repeating groups," and repeating groups are the one thing that just about everybody agrees 1 NF is supposed to prohibit (because such groups are obviously not atomic-right?).

Well, let's agree for the sake of the argument that R2 isn't in 1NF. But suppose we now replace R2 by R3. Then I claim that $R 3$ is in $1 N F!{ }^{5}$ For consider:

- First, note that I've renamed the attribute PNO_SET, and I've shown the groups of part numbers that are PNO_SET values enclosed in braces, to emphasize the fact that each such group is indeed a single value: a set value, to be sure, but a set is still, at a certain level of abstraction, a single value.

Second (and regardless of what you might think of my first argument), the fact is that a set like $\{\mathrm{P} 2, \mathrm{P} 4, \mathrm{P} 5\}$ is no more and no less decomposable by the DBMS than a character string is. Like character strings, sets do have some inner structure; as with character strings, however, it's convenient to ignore that structure for certain purposes. In other words, if character strings are compatible with the requirements of $1 \mathrm{NF}-$ that is, if character strings are atomic-then sets must be, too.

The real point I'm getting at here is that the notion of atomicity has no absolute meaning; it just depends on what we want to do with the data. Sometimes we want to deal with an entire set of part numbers as a single thing; sometimes we want to deal with individual part numbers within that set-but then we're descending to a lower level of detail, or lower level of abstraction. The following analogy might help. In physics (which after all is where the terminology of atomicity comes from) the situation is exactly parallel: Sometimes we want to think about individual atoms as indivisible things, sometimes we want to think about the subatomic particles (i.e., the protons, neutrons, and electrons) that make up those atoms. What's more, protons and neutrons, at least, aren't really indivisible, either-they contain a variety of "subsubatomic" particles called quarks. And so on, possibly (?).

Let's return for a moment to relation R3. In Fig. 2.1, I showed PNO_SET values as general sets. But it would be more useful in practice if they were, more specifically, relations (see Fig. 2.2, where I've changed the attribute name to PNO_REL). Why would it be more useful? Because relations, not general sets, are what the relational model is all about. ${ }^{6}$ As a consequence, the full power of the relational algebra immediately becomes

[^19]available for the relations in question-they can be restricted, projected, joined, and so on. By contrast, if we were to use general sets instead of relations, then we would need to introduce new operators (set union, set intersection, and so on) for dealing with those sets ... Much better to get as much mileage as we can out of the operators we already have!


Fig. 2.2: Relation R4 (a revised version of R3)
Terminology: Attribute PNO_REL in Fig. 2.2 is a relation valued attribute (RVA). Of course, the underlying domain is relation valued too (that is, the values it's made up of are relations). I'll have more to say about RVAs in Chapter 7; here let me just note that SQL doesn't support them. (More precisely, it doesn't support what would be its analog of RVAs, table valued columns. Oddly enough, however, it does support columns whose values are arrays, and columns whose values are rows, and even columns whose values are "multisets of rows"-where a multiset, also known as a bag, is like a set except that it permits duplicates. ${ }^{7}$ Columns whose values are multisets of rows thus do look a little bit like "table valued columns"; however, they aren't table valued columns, because the values they contain can't be operated upon by means of SQL's regular table operators and thus aren't regular SQL table values, by definition.)

Now, I chose the foregoing example deliberately, for its shock value. After all, relations with RVAs do look rather like "relations" with repeating groups, and you've probably always heard that repeating groups are a "no no" in the relational world. But I could have used any number of different examples to make my point; I could have shown attributes (and therefore domains) that contained arrays; or bags (multisets); or lists; or photographs; or audio or video recordings; or X rays; or fingerprints; or XML documents; or any other kind of value, "atomic" or "nonatomic," you might care to think of. Attributes, and therefore domains, can contain anything (any values, that is).

Incidentally, you might recall that a few years ago we were hearing a great deal about so called "object/relational" systems. Well, the foregoing paragraph goes a long way toward explaining why a true

[^20]object/relational system would in fact be nothing more nor less than a true relational system-which is to say, a system that supports the relational model, with all that such support entails (after all, the whole point about an object/relational system from the user's point of view is precisely that we can have attribute values in relations that are of arbitrary complexity). Perhaps a better way to say it is: A proper object/relational system is just a relational system with proper type support (including proper user defined type support in particular)-which just means it's a proper relational system, no more and no less. And what some are pleased to call "the object/relational model" is, likewise, just the relational model, no more and no less.

## WHAT'S A TYPE?

From this point forward I'll favor the term type over the term domain. So what is a type, exactly? In essence, it's $a$ named, finite set of values-all possible values of some specific kind: for example, all possible integers, or all possible character strings, or all possible supplier numbers, or all possible XML documents, or all possible relations with a certain heading (and so on). To elaborate briefly:

- The types we're interested in are always finite because we're dealing with computers, which (as pointed out in connection with type RATIONAL earlier in the chapter) are finite by definition.

Note also that qualifier named: Types with different names are different types.

## Moreover:

- Every value is of some type-in fact, exactly one type, except possibly if type inheritance is supported, a concept that's beyond the scope of this book. Note: Since no value is of more than one type, it follows that types are disjoint (nonoverlapping), by definition. However, perhaps I need to elaborate on this point briefly. As one reviewer of this chapter said, surely types WarmBloodedAnimal and FourLeggedAnimal overlap? Indeed they do; but what I'm saying is that if types overlap, then for a variety of reasons we're getting into the realm of type inheritance-in fact, into the realm of what's called multiple inheritance. Since those reasons, and indeed the whole topic of inheritance, are independent of the context we're in, be it relational or something else, I'm not going to discuss them in this book.
- Every variable, every attribute, every operator that returns a result, and every parameter of every operator is declared to be of some type. ${ }^{8}$ And to say that, e.g., variable $V$ is declared to be of type $T$ means, precisely, that every value $v$ that can legally be assigned to $V$ is in turn of type $T$.
- Every expression denotes some value and is therefore of some type: namely, the type of the value in question, which is to say the type of the value returned by the outermost operator in the expression (where by "outermost" I mean the operator that's executed last). For example, the type of the expression

```
( a / b ) + ( x - y )
```

is the declared type of the operator " + ", whatever that happens to be.

[^21]The fact that parameters in particular are declared to be of some type touches on an issue that I've mentioned but haven't properly discussed as yet: namely, the fact that associated with every type there's a set of operators for operating on values and variables of the type in question-where to say that operator $O p$ is "associated with" type $T$ means, precisely, that operator $O p$ has a parameter of declared type $T .{ }^{9}$ For example, integers have the usual arithmetic operators; dates and times have special calendar arithmetic operators; XML documents have what are called "XPath" and "XQuery" operators; relations have the operators of the relational algebra; and every type has the operators of assignment (": $=$ ") and equality comparison ("="). Thus, any system that provides proper type support-and "proper type support" here certainly includes allowing users to define their own types-must provide a way for users to define their own operators, too, because types without operators are useless. Note: User defined operators can be defined in association with system defined types as well as user defined ones (or a mixture, of course), as you would surely expect.

Observe now that, by definition, values and variables of a given type can be operated upon only by means of the operators associated with that type. For example, in the case of the system defined type INTEGER:

The system provides an assignment operator " $:=$ " for assigning integer values to integer variables.

- It also provides a format for writing integer literals. (However, it doesn't provide any selector operators more general than simple literals, nor does it provide any THE_operators, because-as should be obvious if you think about it-such operators aren't needed for a system defined type like INTEGER.)

■ It also provides comparison operators " $="$ ", " $\neq$ ", " $<"$ ", and so on, for comparing integer values.

- It also provides arithmetic operators " + ", "*", and so on, for performing arithmetic on integer values.

■ It does not provide string operators "||" (concatenate), SUBSTR (substring), and so on, for performing string operations on integer values; in other words, string operations on integer values aren't supported.

By contrast, in the case of the user defined type SNO (still assuming it is user defined), we would certainly define the necessary selector and THE_operators, and we would also define assignment (": $=$ ") and comparison operators ("=", " $\neq "$ ", possibly " $<$ ", and so on). However, we probably wouldn’t define operators " + ", ""*", and so on, which would mean that arithmetic on supplier numbers wouldn't be supported (what could it possibly mean to add or multiply two supplier numbers?).

From everything I've said so far, then, it should be clear that defining a new type involves at least all of the following:

1. Defining a name for the type (obviously enough).
2. Defining the values that make up that type. I'll discuss this aspect in detail in Chapter 8 .
3. Defining the hidden physical representation for values of that type. As noted earlier, this is an implementation issue, not a model issue, and I won't discuss it further in this book.

[^22]4. Defining a selector operator for selecting, or specifying, values of that type.
5. Defining the operators-including in particular assignment (":="), equality comparison ("="), and THE operators-that apply to values and variables of that type (see below).
6. For those operators that return a result, defining the type of that result (again, see below).

Observe that points 4, 5, and 6 taken together imply that the system knows precisely which expressions are legal, and for those expressions that are legal it knows the type of the result as well.

By way of example, suppose we have a user defined type POINT, representing geometric points in twodimensional space. Here then is the Tutorial D definition-I could have used SQL, but operator definitions in SQL involve a number of details that I don't want to get into here-for an operator called REFLECT which, given a point P with cartesian coordinates $(x, y)$, returns the "reflected" or "inverse" point with cartesian coordinates $(-x,-y)$ :

1. OPERATOR REFLECT ( P POINT ) RETURNS POINT ;
2. RETURN POINT ( - THE_X ( P$)$, - THE_Y ( P$)$ ) ;
3. END OPERATOR ;

## Explanation:

- Line 1 shows that the operator is called REFLECT; takes a single parameter P, of type POINT; and returns a result also of type POINT (so the declared type of the operator is POINT).
- Line 2 is the operator implementation code. It consists of a single RETURN statement. The value to be returned is a point, and it's obtained by invoking the POINT selector; that invocation has two arguments, corresponding to the X and Y coordinates of the point to be returned. Each of those arguments is defined by means of a THE_ operator invocation; those invocations yield the X and Y coordinates of the point argument corresponding to parameter P , and negating those coordinates leads us to the desired result. ${ }^{10}$
- Line 3 marks the end of the definition.

Now, the discussions in this section so far have been framed in terms of user defined types, for the most part. But similar considerations apply to system defined types also, except that in this case the various definitions are furnished by the system instead of by some user. For example, if INTEGER is a system defined type, then it's the system that defines the name, defines legal integer values, defines the hidden representation, and-as we've already seen-defines a corresponding literal format, defines the corresponding operators ": $=$ ", " $=$ ", " + ", and so on (though users can define additional operators as well, of course).

There's one last point I want to make. I've mentioned selector operators several times; what I haven't said, however (at least not explicitly), is that selectors-more precisely, selector invocations-are really just a generalization of the more familiar concept of a literal. ${ }^{11}$ What I mean by this remark is that all literals are selector invocations, but not all selector invocations are literals (in fact, a selector invocation is a literal if and only if its arguments are themselves all specified as literals in turn). For example, $\operatorname{POINT}(X, Y)$ and $\operatorname{POINT}(1.0,2.5)$ are both invocations of the POINT selector, but only the second is a POINT literal. It follows that every type has (must have)

[^23]an associated format for writing literals. And for completeness I should add that every value of every type must be denotable by means of some literal.

## SCALAR vs. NONSCALAR TYPES

It's usual to think of types as being either scalar or nonscalar. Loosely, a type is scalar if it has no user visible components and nonscalar otherwise-and values, variables, attributes, operators, parameters, and expressions of some type $T$ are scalar or nonscalar according as type $T$ itself is scalar or nonscalar. For example:

- Type INTEGER is a scalar type; hence, values, variables, and so on of type INTEGER are also all scalar, meaning they have no user visible components.
- Tuple and relation types are nonscalar-the pertinent user visible components being the corresponding attributes-and hence tuple and relation values, variables, and so on are also all nonscalar.

That said, I must now emphasize that these notions are quite informal. Indeed, we've already seen that the concept of data value atomicity has no absolute meaning, and "scalarness" is just that same concept by another name. Thus, the relational model certainly doesn't rely on the scalar vs. nonscalar distinction in any formal sense. In this book, however, I do rely on it informally; I mean, I do find it intuitively useful. To be specific, I use the term scalar in connection with types that are neither tuple nor relation types, and the term nonscalar in connection with types that are either tuple or relation types. ${ }^{12}$

Aside: Another term you'll sometimes hear used to mean "scalarness" is encapsulation. Be aware, however, that this term is also used-especially in object contexts-to refer to the physical bundling, or packaging, of code and data (or operator definitions and data representation definitions, to be more precise). But to use the term in this latter sense is to mix model and implementation considerations; clearly the user shouldn't care, and shouldn't need to care, whether code and data are physically bundled together or are kept separate. End of aside.

Let's look at an example. Here's a Tutorial D definition for the base relvar S ("suppliers")-and note that, for simplicity, I now define the attributes all to be of some system defined type:

```
1. VAR S BASE
2. RELATION { SNO CHAR , SNAME CHAR , STATUS INTEGER , CITY CHAR }
3. KEY { SNO } ;
```


## Explanation:

- The keyword VAR in line 1 means this is a variable definition; S is the name of that variable, and the keyword BASE means the variable is a base relvar specifically.
- Line 2 specifies the type of this variable. The keyword RELATION shows it's a relation type; the rest of the line specifies the set of attributes that make up the corresponding heading (where, as you'll recall from

[^24]Chapter 1, an attribute is defined to be an attribute-name/type-name pair, and no two attributes in the same heading have the same attribute name). The type is, of course, a nonscalar type. No significance attaches to the order in which the attributes are specified.

Line 3 defines $\{\mathrm{SNO}\}$ to be a (candidate) key for this relvar.
In fact, the example also illustrates another point-namely, that the type
RELATION \{ SNO CHAR , SNAME CHAR , STATUS INTEGER , CITY CHAR \}
is an example of a generated type. A generated type is a type that's obtained by invoking some type generator (in the example, the type generator is, specifically, RELATION). You can think of a type generator as a special kind of operator; it's special because (a) it returns a type instead of a value, and (b) it's invoked at compile time instead of run time. For instance, most programming languages support a type generator called ARRAY, which lets users define a variety of specific array types. For present purposes, however, the only type generators we're interested in are TUPLE and RELATION. Here's an example involving the TUPLE type generator:

```
VAR STV /* tuple variable */
    TUPLE { STATUS INTEGER , SNO CHAR , CITY CHAR , SNAME CHAR } ;
```

The value of variable STV at any given time is a tuple with the same heading as that of relvar S (I've deliberately specified the attributes in a different order, just to show the order doesn't matter). ${ }^{13}$ Thus, we might imagine a code fragment that (a) extracts a one-tuple relation (perhaps the relation containing just the tuple for supplier S1) from the current value of relvar S, then (b) extracts the single tuple from that one-tuple relation, and finally (c) assigns that tuple to the variable STV. In Tutorial D:

```
STV := TUPLE FROM ( S WHERE SNO = 'S1' ) ;
```

Important: I don't want you to misunderstand me here. While a variable like STV might certainly be needed in some application program that accesses the suppliers-and-parts database, I'm not saying such a variable can appear inside the database itself. A relational database contains variables of exactly one kind-namely, relation variables (relvars); in other words, relvars are the only kind of variable allowed in a relational database. (This latter fact-i.e., that relvars are the only kind of variable allowed in a relational database-constitutes what's called The Information Principle. I'll have more to say about it in Appendix A.)

By the way, note carefully that (as the foregoing example suggests) there's a logical difference between a tuple $t$ and the relation $r$ that contains just that tuple $t$. In particular, they're of different types- $t$ is of some tuple type and $r$ is of some relation type (though the types do at least have the same heading, or in other words the same attributes).

Finally, a few miscellaneous points to close this section:

- Even though tuple and relation types do have user visible components (namely, their attributes), there's no suggestion that those components have to be physically stored as such. In fact, the physical representation of tuples and relations should be hidden from the user, just as it is for scalar values. (Recall the discussion of physical data independence in Chapter 1.)

[^25]- Like scalar types, tuple and relation types certainly need associated selector operators (and literals as a special case). I'll defer the details to the next chapter. They don't need THE_operators, however; instead, they have operators that provide access to the corresponding attributes, and those operators play a role somewhat analogous to that played by THE_ operators in connection with scalar types.
- Tuple and relation types also need assignment and equality comparison operators. I gave an example of tuple assignment earlier in the present section; I'll defer details of the other operators to the next chapter.


## SCALAR TYPES IN SQL

I turn now to SQL. SQL supports the following more or less self-explanatory system defined scalar types (it also allows users to define their own types, but as I've already said I'm more or less ignoring user defined types in this chapter):

```
BOOLEAN INTEGER CHARACTER(n)
    SMALLINT CHARACTER VARYING( }n\mathrm{ )
    BIGINT CHARACTER LARGE OBJECT(n)
    NUMERIC ( p,q) BINARY (n)
    DECIMAL ( p,q) BINARY VARYING(n)
    FLOAT(p) BINARY LARGE OBJECT(n)
```

This isn't an exhaustive list; other SQL system defined types include an "XML document" type (XML); a variety of "national character string types" (NATIONAL CHARACTER( $n$ ), etc.); and a variety of datetime types (DATE, TIME, TIMESTAMP, INTERVAL). However, I'll ignore such types, mostly, for the purposes of this book. Points arising:

- A number of defaults, abbreviations, and alternative spellings-e.g., INT for INTEGER, CHAR for CHARACTER, VARCHAR for CHARACTER VARYING, CLOB for CHARACTER LARGE OBJECT-are also supported.
- As you can see, SQL, unlike Tutorial D, requires its various character string types to have an associated length specification.
- The same goes for the various BINARY types. Note: BINARY really means bit string, or (perhaps better) byte string; the associated length specification gives the corresponding length in octets. ${ }^{14}$ Also, while BINARY LARGE OBJECT can be abbreviated to BLOB, BINARY and BINARY VARYING can't be abbreviated at all (contrary to expectations, perhaps).
- Strictly speaking, CHAR (for example) isn't really a type as such-rather, it's a type generator. By contrast, CHAR(25), for example, is a type as such, and it's obtained by invoking that type generator with the value 25 as sole argument to that invocation. What's more, analogous remarks apply to everything in the foregoing list except for type BOOLEAN and the various integer types (SMALLINT, INTEGER, BIGINT). ${ }^{15}$ For

[^26]simplicity, however, I'll overlook this point in what follows (most of the time, at any rate) and continue to refer to CHAR and the rest as if they were indeed types as such.

Literals of more or less conventional format are supported for all of these types.

- An explicit assignment operator is supported for all of these types. The syntax is:

```
SET <scalar variable ref> = <scalar exp> ;
```

Scalar assignments are also performed implicitly when various other operations (e.g., FETCH) are executed. Note: Throughout this book in formal syntax definitions like the one just shown, I use ref and exp as abbreviations for reference and expression, respectively.

- An explicit equality comparison operator is also supported for all of these types. ${ }^{16}$ The syntax is:

```
<scalar exp> = <scalar exp>
```

Equality comparisons are also performed implicitly when numerous other operations (e.g., joins and unions, grouping and duplicate elimination operations, and many others) are executed.

- Regarding type BOOLEAN in particular, I should point out that although it's included in the SQL standard, it's supported by few if any of the mainstream SQL products. Of course, boolean expressions can always appear in WHERE, ON, and HAVING clauses, even if the system doesn't support type BOOLEAN as such. In such a system, however, no table can have a column of type BOOLEAN, and no variable can be declared to be of type BOOLEAN. As a consequence, workarounds (e.g., "yes/no columns") might sometimes be needed.
- Finally, in addition to the foregoing scalar types, SQL also supports something it calls domains. However, SQL's domains aren't types at all; rather, they're just a kind of factored out "common column definition," with a number of rather strange properties that are well beyond the scope of this book. You can use them if you like, but don't make the mistake of thinking they're true relational domains (i.e., types).


## TYPE CHECKING AND COERCION IN SQL

SQL supports only a weak form of strong typing (if you see what I mean). To be specific:

- BOOLEAN values can be assigned only to BOOLEAN variables and compared only with BOOLEAN values.

[^27]- Numeric values can be assigned only to numeric variables and compared only with numeric values (where "numeric" means INTEGER, SMALLINT, BIGINT, NUMERIC, DECIMAL, or FLOAT).
- Character string values can be assigned only to character string variables and compared only with character string values (where "character string" means CHAR, VARCHAR, or CLOB).

Bit string values can be assigned only to bit string variables and compared only with bit string values (where "bit string" means BINARY, BINARY VARYING, or BLOB).

Thus, for example, an attempt to compare a number and a character string is illegal. However, an attempt to compare (say) two numbers is legal, even if those numbers are of different types-say INTEGER and FLOAT, respectively (in this example, the INTEGER value will be coerced to type FLOAT before the comparison is done). Which brings me to the question of type coercion ... It's a widely accepted principle in computing in general that coercions are best avoided, because they're error prone. In SQL in particular, one bizarre consequence of permitting coercions is that certain unions, intersections, and differences can yield a result with rows that don't appear in either operand! For example, consider the SQL tables T1 and T2 shown in Fig. 2.3 below. Let column X be of type INTEGER in table T1 but NUMERIC $(5,1)$ in table T2, and let column Y be of type $\operatorname{NUMERIC}(5,1)$ in table T1 but INTEGER in table T2. Now consider the SQL query:

```
SELECT X , Y FROM T1
UNION
SELECT X , Y FROM T2
```

The result is shown as the rightmost table in Fig. 2.3. As the figure suggests, columns $X$ and $Y$ in that result are both of type NUMERIC $(5,1)$, and all values in those columns are obtained, in effect, by coercing some INTEGER value to type NUMERIC $(5,1)$. Thus, the result consists exclusively of rows that appear in neither T1 nor T2!-a very strange kind of union, you might be forgiven for thinking. ${ }^{17}$

T1 | $X$ | $Y$ |
| :---: | :---: |
| 0 | 1.0 |
| 0 | 2.0 |

T2 | $X$ | $Y$ |
| :---: | :---: |
| 0.0 | 0 |
| 0.0 | 1 |
| 1.0 | 2 |

| $X$ | $Y$ |
| :---: | :--- |
| 0.0 | 1.0 |
| 0.0 | 2.0 |
| 0.0 | 0.0 |
| 1.0 | 2.0 |

Fig. 2.3: A very strange "union"

Strong recommendation: Do your best to avoid coercions wherever possible. (My own clear preference would be to do away with them entirely, regardless of whether we're in the SQL context or any other context.) In the SQL context in particular, I recommend that you ensure that columns with the same name are always of the same type; this discipline, along with others recommended elsewhere in this book, will go a long way toward ensuring that type conversions in general are avoided. And when they can't be avoided, I recommend doing them explicitly, using CAST or some CAST equivalent. For example (with reference to the foregoing UNION query):

[^28]```
SELECT CAST ( X AS NUMERIC (5,1) ) AS X , Y FROM T1
UNION
SELECT X , CAST ( Y AS NUMERIC(5,1) ) AS Y FROM T2
```

For completeness, however, I need to add that certain coercions are unfortunately built into the very fabric of SQL and so can't be avoided. (I realize the following remarks might not make much sense at this point in the book, but I don't want to lose them.) To be specific:

- If a table expression $t x$ is used as a row subquery, then the table $t$ denoted by $t x$ is supposed to have just one row $r$, and that table $t$ is coerced to that row $r$. Note: The term subquery occurs ubiquitously in SQL contexts. I'll explain it in detail in Chapter 12; prior to that point, you can take it to mean, albeit rather loosely, just a SELECT expression enclosed in parentheses.
- If a table expression $t x$ is used as a scalar subquery, then the table $t$ denoted by $t x$ is supposed to have just one column and just one row and hence to contain just one value $v$, and that table $t$ is doubly coerced to that value v. Note: This case occurs in connection with SQL-style aggregation in particular (see Chapter 7).
- In practice, the row expression $r x$ in the ALL or ANY comparison $r x \theta$ sq-where (a) $\theta$ is a comparison operator such as " $<$ " or " $>$ " followed by the keyword ALL or ANY and (b) $s q$ is a subquery-often consists of a simple scalar expression, in which case the scalar value denoted by that expression is effectively coerced to a row that contains just that scalar value. Note: Throughout this book, I use the term row expression to mean either a row subquery or a row selector invocation (where row selector in turn is my preferred term for what SQL calls a row value constructor-see Chapter 3); in other words, I use row expression to mean any expression that denotes a row, just as I use table expression to mean any expression that denotes a table. As for ALL or ANY comparisons, they're discussed in Chapter 11.

Finally, SQL also uses the term coercion in a very special sense in connection with character strings. The details are beyond the scope of this book.

## COLLATIONS IN SQL

SQL's rules regarding type checking and coercion, in the case of character strings in particular, are (sadly) rather more complex than I've been pretending so far, and I need to elaborate somewhat. Actually it's impossible in a book of this nature to do more than just scratch the surface of the matter, but the basic idea is this: Any given character string (a) consists of characters from one associated character set and (b) has one associated collation. A collation-also known as a collating sequence-is a rule that's associated with a specific character set and governs the comparison of strings of characters from that character set. Let $C$ be a collation for character set $S$, and let $a$ and $b$ be any two characters from $S$. Then $C$ must be such that exactly one of the comparisons $a<b, a=b$, and $a>b$ evaluates to TRUE and the other two to FALSE (under C). Note: In early versions of SQL there was just one character set, that character set had just one collation, and that collation was based on the numerical order of the binary codes used to represent the characters in that character set. But there's no intrinsic reason why collating sequences should have to depend on internal coding schemes, and there are good practical reasons why they shouldn't.

So much for the basic idea. However, there are complications. One arises from the fact that any given collation can have either PAD SPACE or NO PAD defined for it. Suppose the character strings 'AB' and 'AB' (note the trailing space in the second of these) have the same character set and the same collation. Then those two
strings are clearly distinct, and yet they're considered to "compare equal" if PAD SPACE applies.
Recommendation: Don't use PAD SPACE-always use NO PAD instead, if possible. Note, however, that the choice between PAD SPACE and NO PAD affects comparisons only-it makes no difference to assignments. ${ }^{18}$

Another complication arises from the fact that the comparison $a=b$ might evaluate to TRUE under a given collation, even if the characters $a$ and $b$ are distinct. For example, we might define a collation called CASE_INSENSITIVE in which each lowercase letter is defined to compare equal to its uppercase counterpart. As a consequence, again, strings that are clearly distinct will sometimes compare equal.

We see, therefore, that certain comparisons of the form $v l=v 2$ can give TRUE in SQL even if $v 1$ and $v 2$ are distinct (and possibly even if they're of different types, thanks to SQL's support for coercion). I'll use the term "equal but distinguishable" to refer to such pairs of values. Now, equality comparisons are performed, often implicitly, in numerous contexts-examples include MATCH, LIKE, UNIQUE, UNION, and JOIN-and the kind of equality involved in all such cases is indeed "equal even if distinguishable." For example, let collation
CASE_INSENSITIVE be as defined above, and let PAD SPACE apply to that collation. Then, if the PNO columns of tables P and SP both use that collation, and if 'P2' and 'p2 ' are PNO values in, respectively, some row of P and some row of SP, those two rows will be regarded as satisfying the foreign key constraint from SP to P, despite the lowercase ' p ' and trailing spaces in the foreign key value.

What's more, when evaluating expressions involving operators such as UNION, INTERSECT, EXCEPT, JOIN, GROUP BY, DISTINCT (and so on), the system sometimes has to decide which of several equal but distinguishable values is to be chosen as the value of some column in some result row. Unfortunately, SQL itself fails to give complete guidance in such situations. As a consequence, certain table expressions are indeterminate-the SQL term is possibly nondeterministic-in the sense that SQL doesn't fully specify how they should be evaluated; indeed, they might quite legitimately give different results on different occasions. For example, if collation CASE_INSENSITIVE applies to column $C$ in table $T$, then SELECT MAX ( $C$ ) FROM $T$ might return ' $Z Z Z$ ' on one occasion and ' $z z z$ ' on another, even if $T$ hasn't changed in the interim.

I won't give SQL's rules here for when a given expression is "possibly nondeterministic" (see Chapter 12 for further discussion). It's important to note, however, that such expressions aren't allowed in integrity constraints (see Chapter 8), because they could cause updates to succeed or fail unpredictably. Observe in particular, therefore, that this rule implies among other things that many table expressions-even simple SELECT expressions, sometimes-aren't allowed in constraints if they involve a column of some character string type! Strong
recommendation: Avoid possibly nondeterministic expressions as much as you can.

## ROW AND TABLE TYPES IN SQL

Here repeated from the section "Scalar vs. Nonscalar Types" is an example of a tuple variable definition:
VAR STV TUPLE \{ STATUS INTEGER , SNO CHAR , CITY CHAR , SNAME CHAR \} ;
The expression TUPLE $\{\ldots\}$ here is, as you'll recall, an invocation of the TUPLE type generator. SQL has a corresponding ROW type generator (though it calls it a type constructor). Here's an SQL analog of the foregoing Tutorial D example:

[^29]```
DECLARE SRV /* SQL row variable */
    ROW ( SNO VARCHAR(5) ,
        SNAME VARCHAR(25) ,
        STATUS INTEGER ,
        CITY VARCHAR(20) ) ;
```

Unlike tuples, however, rows in SQL have a left to right ordering to their components; ${ }^{19}$ in the case at hand, there are actually $24(=4 * 3 * 2 * 1)$ different row types all consisting of the same four components (!).

SQL also supports row assignment. Recall this Tutorial D tuple assignment:

```
STV := TUPLE FROM ( S WHERE SNO = 'S1' ) ;
```

Here's an SQL row assignment analog:

```
SET SRV = ( S WHERE SNO = 'S1' ) ;
```

The expression on the right side here is a row subquery-i.e., it's a table expression, syntactically speaking, but it's one that's acting as a row expression. That's why there's no explicit counterpart to Tutorial D's TUPLE FROM (see the discussion of subqueries and coercion in the section "SQL Type Checking and Coercion" a couple of pages back).

Row assignments are also involved, in effect, in SQL UPDATE statements (see Chapter 3).
Turning to tables: Interestingly, SQL doesn't really have a TABLE type generator (or type constructor, as SQL would probably call it) at all!-i.e., it has nothing directly analogous to the RELATION type generator described earlier in this chapter. However, it does have a mechanism, CREATE TABLE, for defining what by rights should be called table variables. For example, recall this definition from the section "Scalar vs. Nonscalar Types":

```
VAR S BASE
    RELATION { SNO CHAR , SNAME CHAR , STATUS INTEGER , CITY CHAR }
    KEY { SNO } ;
```

Here's an SQL analog:
CREATE TABLE S
$\begin{array}{llll}\text { ( SNO } & \text { VARCHAR (5) } & \text { NOT NULL , } \\ \text { SNAME } & \text { VARCHAR (25) } & \text { NOT NULL , } \\ \text { STATUS } & \text { INTEGER } & \text { NOT NULL , } \\ \text { CITY } & \text { VARCHAR(20) } & \text { NOT NULL , } \\ \text { UNIQUE }(S N O) ; & & \end{array}$
Note carefully, however, that there's nothing-no sequence of linguistic tokens-in this example that can logically be labeled "an invocation of the TABLE type constructor." (This fact might become more apparent when you realize that the specification UNIQUE(SNO), which defines a certain integrity constraint on suppliers, doesn't have to come after the column definitions but can appear almost anywhere-e.g., between the definitions of columns SNO and SNAME. Not to mention the NOT NULL specifications on the individual column definitions, which also define certain integrity constraints.) In fact, to the extent that the variable S can be regarded (in SQL) as having any

[^30]type at all, that type is nothing more than bag of rows, where the rows in question are of type ROW (SNO VARCHAR(5), SNAME VARCHAR(25), STATUS INTEGER, CITY VARCHAR(20)).

That said, I should say too that SQL does support something it calls "typed tables." The term isn't very appropriate, however, because if $T T$ is a "typed table" that has been defined to be "of type $T$," then $T T$ is not of type $T$, and neither are its rows! More important, I think you should avoid such tables anyway, because they're inextricably intertwined with SQL's support for pointers, and pointers are explicitly prohibited in the relational model. ${ }^{20}$ In fact, if some table has a column whose values are pointers to rows in some "target" table, then that table can't possibly represent a relation in the relational model sense. ${ }^{21}$ As I've just indicated, however, such tables are unfortunately permitted in SQL; the pointers are called reference values, and the columns that contain them are said to be of some REF type. Quite frankly, it's not clear why these features are included in SQL at all; certainly there seems to be no useful functionality that can be achieved with them that can't equally well-in fact, better-be achieved without them. Strong recommendation: Don't use them, or any features related to them.

Aside: To avoid a possible confusion, I should add that SQL actually uses the terminology of "referencing" in two quite different senses. One is as sketched above. The other, and older, sense has to do with foreign keys; a foreign key value in one row is said to "reference" the row that contains the corresponding target key value. Note, however, that foreign keys certainly aren't pointers!-there are several logical differences between the two concepts, including in particular the fact that foreign keys refer to rows, which are values, whereas pointers are addresses and therefore, by definition, refer to variables. (Recall from Chapter 1 that it's variables, not values, that "have location." Values, having no location, certainly don't have addresses.) End of aside.

## CONCLUDING REMARKS

It's a common misconception that the relational model deals only with rather simple types: numbers, strings, perhaps dates and times, and not much else. In this chapter, I've tried to show among other things that this is indeed a misconception. Rather, relations can have attributes of any type whatsoever (other than as noted in just a moment)-the relational model nowhere prescribes just what those types must be, and in fact they can be as complex as you like. In other words, the question as to what types are supported is orthogonal to the question of support for the relational model itself. Or, less precisely but more catchily: Types are orthogonal to tables.

I also remind you that the foregoing state of affairs in no way violates the requirements of first normal form-first normal form just means that every tuple in every relation contains a single value, of the appropriate type, in every attribute position. Now we know those types can be anything, we also know all relations are in first normal form by definition.

Finally, I mentioned in the introduction to this chapter that there were certain important exceptions to the rule that relational attributes can be of any type whatsoever. In fact, there are two (both of which I'll simplify just slightly for present purposes). The first is that if relation $r$ is of type $T$, then no attribute of $r$ can itself be of type $T$ (think about it!). The second is that no relation in the database can have an attribute of any pointer type. Prerelational databases were full of pointers, and access to such databases involved a lot of pointer chasing, a state of affairs that made application programming error prone and direct end user access impossible. (These aren't the

[^31]only problems with pointers, but they're among the more obvious ones.) Codd wanted to get away from such problems in his relational model, and of course he succeeded.

## EXERCISES

2.1 What's a type? What's the difference between a domain and a type?
2.2 What do you understand by the term selector? And what exactly is a literal?

### 2.3 What's a THE_ operator?

2.4 Physical representations are always hidden from the user: True or false?
2.5 This chapter has touched on several more logical differences (refer back to Chapter 1 if you need to refresh your memory regarding this important notion), including:

| argument | vs. | parameter |
| :--- | :---: | :--- |
| database | vs. | DBMS |
| foreign key | vs. | pointer |
| generated type | vs. | nongenerated type |
| relation | vs. | type |
| type | vs. | representation |
| user defined type | vs. | system defined type |
| user defined operator | vs. | system defined operator |

What exactly is the logical difference in each of these cases?
2.6 Explain in your own words the difference between the concepts scalar and nonscalar.
2.7 What do you understand by the term coercion? Why is coercion a bad idea?
2.8 Why doesn't domain check override make sense?
2.9 What's a type generator?
2.10 Define first normal form. Why do you think it's so called?
2.11 Let $X$ be an expression. What's the type of $X$ ? What's the significance of the fact that $X$ is of some type?
2.12 Using the definition of the REFLECT operator in the body of the chapter (section "What's a Type?") as a template, define a Tutorial D operator that, given an integer, returns the cube of that integer.
2.13 Let LENGTH be a user defined type, with the obvious semantics. Use Tutorial D to define an operator that, given the length of two adjacent sides of a rectangle, returns the corresponding area.
2.14 Give an example of a relation type. Distinguish between relation types, relation values, and relation variables.
2.15 Use SQL or Tutorial D or both to define relvars P and SP from the suppliers-and-parts database. If you give both SQL and Tutorial D definitions, identify as many differences between them as you can. What's the significance of the fact that relvar P (for example) is of a certain relation type?
2.16 With reference to the departments-and-employees database from Chapter 1 (see Fig. 1.1), suppose the attributes are of the following user defined types:

| DNO | $:$ | DNO |
| :--- | :--- | :--- |
| DNAME | $:$ | NAME |
| BUDGET | $:$ | MONEY |
| ENO | $:$ | ENO |
| ENAME | $:$ | NAME |
| SALARY | $:$ | MONEY |

Suppose departments also have a LOCATION attribute, of user defined type CITY (say). Which of the following scalar expressions are valid? For those that are, state the type of the result; for the others, give an expression that will achieve what appears to be the desired effect.
a. LOCATION $=$ 'London'
b. ENAME = DNAME
c. SALARY * 5
d. BUDGET + 50000
e. ENO > 'E2'
f. ENAME || DNAME
g. LOCATION || 'burg'
2.17 It's sometimes suggested that types are really variables, in a sense. For example, employee numbers might grow from three digits to four as a business expands, so we might need to update "the set of all possible employee numbers." Discuss.
2.18 A type is a set of values and the empty set is a legitimate set; thus, we might define an empty type to be a type where the set in question is empty. Can you think of any uses for such a type?
2.19 In the relational world, the equality operator " $=$ " applies to every type. By contrast, SQL doesn't require "=" to apply to every type, and it doesn't fully define the semantics in all of the cases where it does apply. What are the implications of this state of affairs?
2.20 Following on from the previous exercise, we can say that if $v l=v 2$ evaluates to TRUE in the relational world, then executing some operator $O p$ on $v l$ and executing that same operator $O p$ on $v 2$ always has exactly the
same effect, for all possible operators $O p$. But this is another precept that SQL violates. Can you think of any examples of such violation? What are the implications?
2.21 Why are pointers excluded from the relational model?
2.22 The Assignment Principle-which is very simple, but fundamental-states that after assignment of the value $v$ to the variable $V$, the comparison $V=v$ evaluates to TRUE (see Chapter 5). Yet again, however, this is a precept that SQL violates (fairly ubiquitously, in fact). Can you think of any examples of such violation? What are the implications?
2.23 Do you think that types "belong to" databases, in the same sense that relvars do?
2.24 In the first example of an SQL SELECT expression in this chapter, I pointed out that there was no terminating semicolon because the expression was an expression and not a statement. But what's the difference?
2.25 Explain as carefully as you can the logical difference between a relation with a relation valued attribute (RVA) and a "relation" with a repeating group.

### 2.26 What's a subquery?

2.27 To repeat from Exercise 2.19: In the relational world, the equality operator "=" applies to every type. But what about type BOOLEAN? And what about SQL's row and table types?

## Chapter 3

## Tuples and Relations,

## Rows and Tables

[I] have reduced several great confused Volumes into a few perspicuous Tables. -John Graunt (1662)

From the first two chapters you should have gained a pretty good understanding of what tuples and relations are, at least intuitively. Now I want to define those concepts more precisely, and I want to explore some of the consequences of those more precise definitions; also, I want to describe the analogous SQL constructs (viz., rows and tables) and offer some specific recommendations to help with our goal of using SQL relationally. Perhaps I should warn you that the formal definitions might look a little daunting-but that's not unusual with formal definitions; the concepts themselves are quite straightforward, once you've struggled through the formalism, and you should be ready to do that by now because the terminology, at least, should be quite familiar to you.

## WHAT'S A TUPLE?

Is this a tuple? -

| SNO ${ }^{\text {d }}$ CHAR | SNAME ${ }^{\text {L }}$, CHAR | STATUS ${ }^{\text {/ }}$ INTEGER | CITY ${ }^{\text {CHAR }}$ |
| :---: | :---: | :---: | :---: |
| S 1 | Smith | 20 | London |

Well, no, it isn't-it's a picture of a tuple, not a tuple as such (and note that for once I've included the type names in that picture as well as the attribute names). As we saw in Chapter 1, there's a logical difference between a thing and a picture of a thing, and that difference can be very important. For example, tuples have no left to right ordering to their attributes, and so the following is an equally good (bad?) picture of the very same tuple:

| STATUS | INTEGER | SNAME | CHAR | CITY | CHAR | SNO | CHAR |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
|  | 20 | Smith |  | London | S1 |  |  |

Thus, while I'll certainly be showing many pictures like these in the pages to follow, please keep in mind that they're only pictures, and they can sometimes suggest some things that aren't true.

With that caveat out of the way, I can now say exactly what a tuple is:
Definition: Let $T 1, T 2, \ldots, T n(n \geq 0)$ be type names, not necessarily all distinct. Associate with each $T i$ a distinct attribute name, $A i$; each of the $n$ attribute-name/type-name combinations that results is an attribute.

Associate with each attribute an attribute value, vi, of type Ti; each of the $n$ attribute/value combinations that results is a component. Then the set of all $n$ components thus defined, $t$ say, is a tuple value (or just a tuple for short) over the attributes $A 1, A 2, \ldots, A n$. The value $n$ is the degree of $t$; a tuple of degree one is unary, a tuple of degree two is binary, a tuple of degree three is ternary, ..., and more generally a tuple of degree $n$ is $n$-ary. The set of all $n$ attributes is the heading of $t$.

For example, with reference to the first of the two pictures on the previous page of the tuple for supplier S1, we have:

- Degree: 4. The heading is also said to have degree 4.
- Type names (as shown in the picture, left to right): CHAR, CHAR, INTEGER, CHAR.
- Corresponding attribute names: SNO, SNAME, STATUS, CITY.
- Corresponding attribute values: 'S1', 'Smith', 20, 'London'. Note the quotes enclosing the character string values here, incidentally; I didn't show any such quotes in the pictures, but perhaps I should have done-it would have been more correct.

Aside: Suppose for a moment, as we did in Chapter 2, that attribute SNO was of type SNO (a user defined type) instead of type CHAR. Then it would be even more incorrect to say the SNO value in the tuple we're talking about was S 1 , or even ' S 1 '; rather, it would be SNO ('S1'). A value of type SNO is a value of type SNO, not a value of type CHAR!-a difference in type is certainly a logical difference. (Recall from Chapter 2 that the expression SNO('S1') is a selector invocation-in fact, a literal-of type SNO.) End of aside.

Heading: The easiest thing to do here is show another picture:


Of course, this picture represents a set, and the order of attributes is arbitrary. Here's another picture of the same heading:

| STATUS | INTEGER | SNAME | CHAR | CITY | CHAR | SNO | CHAR |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |

Exercise: How many different pictures of this same general nature could we draw to represent this same heading? (Answer: $4!=4 * 3 * 2 * 1=24$. $)^{1}$

Now, a tuple is a value; like all values, therefore, it has a type (as we know from Chapter 2), and that type, like all types, has a name. In Tutorial D, such names take the form TUPLE $\{H\}$, where $\{H\}$ is the heading. In our example, the name is:

[^32]```
TUPLE { SNO CHAR , SNAME CHAR , STATUS INTEGER , CITY CHAR }
```

(but the order in which the attributes are specified is arbitrary).
To repeat, a tuple is a value. Like all values, therefore, it must be returned by some selector invocation (a tuple selector invocation, naturally, if the value is a tuple). Here's a tuple selector invocation for our example (Tutorial D again):

TUPLE \{ SNO 'S1' , SNAME 'Smith' , STATUS 20 , CITY 'London' \}
(where the order in which the components are specified is again arbitrary). Observe that in Tutorial D each component is specified as just an attribute-name/expression pair, where the specified expression denotes the corresponding attribute value; the attribute type is omitted because it can always be inferred from the type of the specified expression.

Here's another example (unlike the previous one, this one isn't a literal, because not all of its arguments are specified as literals in turn):

TUPLE \{ SNO SX , SNAME 'James' , STATUS TX , CITY CX \}

I'm assuming here that SX, TX, and CX are variables of types CHAR, INTEGER, and CHAR, respectively.
As these examples indicate, a tuple selector invocation in Tutorial D consists in general of the keyword TUPLE, followed by a commalist of attribute-name/expression pairs, the whole commalist enclosed in braces. Note, therefore, that the keyword TUPLE does double duty in Tutorial D-it's used in connection both with tuple selector invocations, as we've just seen, and with tuple type names as we saw earlier. An analogous remark applies to the keyword RELATION also (see the section "What's a Relation?" later in this chapter).

## Consequences of the Definitions

Now I want to highlight some important consequences of the foregoing definitions. The first is: No tuple ever contains any nulls. The reason is that, by definition, every tuple contains a value (of the appropriate type) for each of its attributes, and (as we saw in Chapter 1) nulls aren't values-despite the fact that SQL often, though not always, refers to them explicitly as null values. Recommendation: Since the phrase "null value" is a contradiction in terms, don't use it; always say just "null" instead. Note that this recommendation isn't just a matter of pedantry; rather, it's a matter of thinking straight. SQL itself manages to make numerous mistakes in its handling of nulls, and some of those mistakes can be traced directly to the fact that SQL does sometimes, but not always, think of null as a value. (Indeed, this ambivalence is reflected in the standard's very definition of the concept, which reads as follows: "null value: A special value that is used to indicate the absence of any data value." In other words: Null is a value that means there isn't a value.)

Now, if no tuple ever contains any nulls, then no relation does so either, a fortiori; so right away we have at least a formal reason for rejecting the concept of nulls-but in the next chapter I'll give some much more pragmatic reasons as well.

The next consequence-or pair of consequences, rather-is: Every subset of a tuple is a tuple and every subset of a heading is a heading. (I did mention these points in Chapter 1, but now I want to elaborate on them.) By way of example, given our usual tuple for supplier S1, what we might call "the \{SNO,CITY\} value" within that tuple is itself another tuple (of degree two):

| SNO | CHAR | CITY |
| :--- | :--- | :--- |
| S1 CHAR |  |  |

Its heading is as indicated, and its type is TUPLE \{SNO CHAR, CITY CHAR\}. Likewise, the following is a tuple also:

| SNO | CHAR |
| :--- | :--- | :--- |
| S1 |  |

This tuple is of degree one, and its type is TUPLE \{SNO CHAR\}.
Now, as I'm sure you know, the empty set-i.e., the set that contains no elements-is a subset of every set. It follows that the empty heading is a valid heading!-and hence that a tuple with an empty set of components is a valid tuple (though it's a little hard to draw pictures of such a tuple on paper, and I'm not even going to try). A tuple with an empty heading has type TUPLE $\}$; indeed, we sometimes refer to it explicitly as a 0 -tuple, in order to emphasize the fact that it has no components and is of degree zero. We also sometimes call it an empty tuple. Now, you might think such a tuple is unlikely to be of much use in practice; in fact, however, it turns out, perhaps rather surprisingly, to be of crucial importance. I'll have more to say about it in the section "TABLE_DUM and TABLE_DEE," later.

Let's get back to the original tuple for supplier S1 (i.e., the one of degree four) for a moment. $\overline{\text { Suppose we're }}$ given that tuple and we want to access the actual value of some attribute, say the SNO attribute, from that tuple. Then we have to extract that value, somehow, from the tuple that contains it. Tutorial D uses syntax of the form SNO FROM $t$ for this purpose (where $t$ is any expression that denotes a tuple with an SNO attribute). SQL uses dot qualification: $t$.SNO.

Note: It follows from the foregoing paragraph that a value $v$ and a tuple $t$ that contains just that value $v$ aren't the same thing; in particular, they're of different types. This logical difference is analogous to that described in Chapter 2, between a tuple $t$ and a relation $r$ that contains just that tuple $t$; these aren't the same thing either (they too are of different types).

Now I'd like to turn to the notion of tuple equality. (Again I mentioned this notion in Chapter 1, but now I want to elaborate on it.) Recall first from Chapter 2 that the " $=$ " comparison operator is-in fact, must be-defined for every type, and tuple types are no exception. Basically, two tuples are equal if and only if they're the very same tuple (just as, for example, two integers are equal if and only if they're the very same integer). But it's worth spelling out the semantics of tuple equality in detail, since so much in the relational model depends on it-for example, candidate keys, foreign keys, and almost all of the operators of the relational algebra are defined in terms of it. Here then is a precise definition:

Definition: Tuples $t$ and $t^{\prime}$ are equal if and only if they have the same attributes $A 1, A 2, \ldots, A n$-in other words, they're of the same type-and, for all $i(i=1,2, \ldots, n)$, the value $v$ of $A i$ in $t$ is equal to the value $v^{\prime}$ of $A i$ in $t^{\prime}$.

Also (to repeat from Chapter 1, this might seem obvious, but it needs to be said), two tuples are duplicates of each other if and only if they're equal. Thus, e.g., the tuple for supplier $S 1$ in the suppliers relation of Fig. 1.3 is equal to, and is therefore a duplicate of, itself-and it isn't equal to, or a duplicate of, anything else (any other tuple in particular).

By the way, it's an immediate consequence of the foregoing definition that all 0 -tuples are duplicates of one another. For this reason, we're within our rights if we talk in terms of the 0 -tuple instead of "a" 0 -tuple, and indeed we usually do. Note, moreover, that we can validly say that the 0 -tuple is a subset of every tuple (just as we can say the empty set is a subset of every set).

So the comparison operator " $=$ ", and therefore the comparison operator " $\neq$ " also, do both necessarily apply to tuples. However, the operators " $<$ " and " $>$ " do not apply. The reason is that tuples are fundamentally sets (sets of components, to be specific), and such operators make no sense for sets.

In closing this section, let me draw your attention to Exercise 3.16 at the end of the chapter (also the discussion of that exercise in Appendix F), which I strongly recommend you devote some thought to. Later chapters in the book will appeal to some of the points raised by that exercise.

## ROWS IN SQL

SQL supports rows, not tuples; in particular, it supports row types, a row type constructor, and row value constructors, which are analogous, somewhat, to Tutorial D's tuple types, TUPLE type generator, and tuple selectors, respectively. (Row types and row type constructors, though not row value constructors, were also discussed in Chapter 2.) But these analogies are loose at best, because, crucially, rows, unlike tuples, have a left to right ordering to their components. For example, the expressions ROW (1,2) and ROW(2,1)-both of which are legitimate row value constructor invocations in SQL—represent two different SQL rows. Note: The keyword ROW in an SQL row value constructor invocation is optional; in practice, it's almost always omitted.

Thanks to that left to right ordering, row components ("fields") in SQL can be, and indeed are, identified by ordinal position instead of by name. For example, consider the following row value constructor invocation (actually it's a row literal, though SQL doesn't use that term):

$$
\text { ( 'S1' , 'Smith' , } 20 \text {, 'London' ) }
$$

This row clearly has (among other things) a component with the value 'Smith'; logically speaking, however, we can't say that component is "the SNAME component," we can only say it's the second component.

I should add that rows in SQL always contain at least one component; SQL has no analog of the 0-tuple of the relational model (there's no "0-row").

As discussed in Chapter 2-recall the example involving the SQL row variable SRV—SQL also supports a row assignment operation. ${ }^{2}$ In particular, such assignments are involved (in effect) in SQL UPDATE statements. For example, the following UPDATE statement-

```
UPDATE S
SET STATUS = 20, CITY = 'London'
WHERE CITY = 'Paris' ;
```

-is defined to be logically equivalent to this one (note the row assignment in the second line):

```
UPDATE S
SET (STATUS , CITY ) = (20 , 'London' )
WHERE CITY = 'Paris' ;
```

[^33]
## 54 Chapter 3 / Tuples and Relations, Rows and Tables

As for comparison operations, most boolean expressions in SQL, including (believe it or not) simple "scalar" comparisons in particular, are actually defined in terms of rows rather than scalars. Here's an example of a SELECT expression in which the WHERE clause contains an explicit row comparison:

```
SELECT SNO
FROM S
WHERE ( STATUS , CITY ) = ( 20 , 'London' )
```

This SELECT expression is logically equivalent to the following one:

```
SELECT SNO
FROM S
WHERE STATUS = 20 AND CITY = 'London'
```

As another example, the expression

```
SELECT SNO
FROM S
WHERE ( STATUS , CITY ) <> ( 20 , 'London' )
```

is logically equivalent to:

```
SELECT SNO
FROM S
WHERE STATUS <> 20 OR CITY <> 'London'
```

Note carefully in the expanded form of this example that the two individual comparisons in the WHERE clause are connected by OR, not AND.

Moreover, since row components have a left to right ordering, SQL is also able to support "<" and " $>$ " as row comparison operators. Here's an example:

```
SELECT SNO
FROM S
WHERE ( STATUS , CITY ) > ( 20 , 'London' )
```

This expression is logically equivalent to:

```
SELECT SNO
FROM S
WHERE STATUS > 20 OR ( STATUS = 20 AND CITY > 'London' )
```

In practice, however, the vast majority of row comparisons involve rows of degree one, as here:

```
SELECT SNO
FROM S
WHERE ( STATUS ) = ( 20 )
```

Now, all of the comparand expressions in the examples so far have been, specifically, row value constructor invocations. But now I need to explain that SQL has a syntax rule to the effect that if such an invocation consists of a single scalar expression enclosed in parentheses, then the parentheses can optionally be dropped, as here:

```
SELECT SNO
FROM
    S
WHERE STATUS = 20
```

The "row comparison" in the WHERE clause in this example is thus effectively a scalar comparison (STATUS and 20 are both scalar expressions). Strictly speaking, however, there's no such thing as a scalar comparison in SQL; the expression STATUS $=20$ is still technically a row comparison (and the "scalar" comparands are effectively coerced to rows), so far as SQL is concerned.

Recommendation: Unless the rows being compared are of degree one (and are thus effectively scalars), don't use the comparison operators " $<$ ", " $<="$ ", " $>$ ", and " $>=$ "; they rely on left to right column ordering, they have no direct counterpart in the relational model, and in any case they're seriously error prone. (It's relevant to note in this connection that when this functionality was first proposed for SQL, the standardizers had great difficulty in defining the semantics properly; in fact, it took them several iterations before they got it right.)

## WHAT'S A RELATION?

I'll use our usual suppliers relation as a basis for examples in this section. Here's a picture:

| SNO | CHAR | SNAME | CHAR | STATUS | INTEGER |
| :--- | :--- | :--- | :--- | :--- | :--- |
| S1 | CITY | CHAR |  |  |  |
| S2 | Smith |  | 20 | London |  |
| S3 |  | Jones |  | 10 | Paris |
| S4 | Blake |  | 30 | Paris |  |
| S5 | Clark |  | 20 | London |  |

## And here's a definition:

Definition: Let $\{H\}$ be a tuple heading and let $t 1, t 2, \ldots, t m(m \geq 0)$ be distinct tuples, all with heading $\{H\} .{ }^{3}$ Then the combination, $r$ say, of $\{H\}$ and the set of tuples $\{t 1, t 2, \ldots, t m\}$ is a relation value (or just a relation for short) over the attributes $A 1, A 2, \ldots, A n$, where $A 1, A 2, \ldots, A n$ are all of the attributes in $\{H\}$. The heading of $r$ is $\{H\} ; r$ has the same attributes (and hence the same attribute names and types) and the same degree as that heading does. The set of tuples $\{t 1, t 2, \ldots, t m\}$ is the body of $r$. The value $m$ is the cardinality of $r$.

I'll leave it as an exercise for you to interpret the suppliers relation in terms of the foregoing definition. However, I will at least explain why we call such things relations. Basically, each tuple in a relation represents an $n$-ary relationship, in the ordinary natural language sense of that term, interrelating a set of $n$ values (one such value for each tuple attribute); the full set of tuples in a given relation represents the full set of such relationships that happen to exist at some given time; and, mathematically speaking, that set of tuples is a relation. Thus, the

[^34]
## 56 Chapter 3 / Tuples and Relations, Rows and Tables

explanation often heard, to the effect that the relational model is so called because it lets us "relate one table to another," though accurate in a kind of secondary sense, really misses the basic point. The relational model is so called because it deals with certain abstractions that we can think of informally as "tables" but are known in mathematics, formally, as relations.

Now, a relation, like a tuple, is itself a value and has a type, and that type has a name. In Tutorial D, such names take the form RELATION $\{H\}$, where $\{H\}$ is the heading-for example:

```
RELATION { SNO CHAR , SNAME CHAR , STATUS INTEGER , CITY CHAR }
```

(The order in which the attributes are specified is arbitrary.) Also, every relation value is denoted by some relation selector invocation-for example:

```
RELATION
    { TUPLE { SNO 'S1' , SNAME 'Smith' , STATUS 20 , CITY 'London' }
        TUPLE { SNO 'S2' , SNAME 'Jones' , STATUS 10 , CITY 'Paris' } ,
        TUPLE { SNO 'S3' , SNAME 'Blake' , STATUS 30 , CITY 'Paris' } ,
        TUPLE { SNO 'S4' , SNAME 'Clark' , STATUS 20 , CITY 'London' } ,
        TUPLE { SNO 'S5' , SNAME 'Adams' , STATUS 30 , CITY 'Athens' } }
```

The order in which the tuples are specified is arbitrary. Here's another example (unlike the previous one, this one isn't a literal):

```
RELATION { tx1 , tx2 , tx3 }
```

I'm assuming here that $t x 1, t x 2$, and $t x 3$ are tuple expressions and are all of the same tuple type. As these examples suggest, a relation selector invocation in Tutorial D consists in general ${ }^{4}$ of the keyword RELATION, followed by a commalist of tuple expressions enclosed in braces (and those tuple expressions must all be of the same tuple type).

## Consequences of the Definitions

Most of the properties of relations I talked about in Chapter 1 are direct consequences of the definitions discussed above, but there are some points I didn't call out explicitly before, and I want to elaborate on some of the others. The first two I want to mention are as follows:

- Relations never contain duplicate tuples-because the body of a relation is a set (a set of tuples) and sets in mathematics don't contain duplicate elements.
- Relations never contain nulls-because the body of a relation is a set of tuples, and we've already seen that tuples in turn never contain nulls.

[^35]But these two points are so significant, and there's so much I need to say about them, that I'll defer detailed treatment to the next chapter. In the next few sections, I'll address a series of possibly less weighty issues (?) arising from the definitions.

## RELATIONS AND THEIR BODIES

The first point I want to discuss is this: Every subset of a body is a body-or, loosely, every subset of a relation is a relation. (Once again I mentioned this fact in Chapter 1, but now I want to say a little more about it.) In particular, since the empty set is a subset of every set, a relation can have a body that consists of an empty set of tuples (and we call such a relation an empty relation). For example, suppose there are no shipments right now. Then relvar SP will have as its current value the empty shipments relation, which we might draw like this (and now I revert to the convention by which we omit the type names from a heading in informal contexts; throughout the rest of the book, in fact, I'll feel free to regard headings as either including or excluding type names-whichever best suits my purpose at the time):


Note that, given any particular relation type, there's exactly one empty relation of that type-but empty relations of different types aren't the same thing, precisely because they're of different types. For example, the empty suppliers relation isn't equal to the empty parts relation (their bodies are equal but their headings aren't). Consider now the relation depicted here:

| SNO | PNO | QTY |
| :---: | :---: | :---: |
| S1 | P1 | 300 |

This relation contains just one tuple (equivalently, it's of cardinality one). If we want to access the single tuple it contains, then we'll have to extract it somehow from its containing relation. Tutorial $\mathbf{D}$ uses syntax of the form TUPLE FROM $r x$ for this purpose, where $r x$ is any expression that denotes a relation of cardinality one-for example, it might be the expression RELATION \{TUPLE \{SNO ‘S1', PNO ‘P1', QTY 300\}\}, which is in fact a relation selector invocation (actually it's a literal). SQL, by contrast, uses coercion: If (a) $t x$ is a table expression that's being used as a row subquery (meaning it appears where a row expression is expected), then (b) the table $t$ denoted by $t x$ is supposed to contain just one row $r$, and (c) that table $t$ is coerced to that row $r$. Here's an example (it's the row assignment example from the section "Row and Table Types in SQL" in Chapter 2):

```
SET SRV = ( S WHERE SNO = 'S1' ) ;
```

We also need to be able to test whether a given tuple $t$ appears in a given relation $r$. In Tutorial D:

```
t\inr
```

This expression returns TRUE if $t$ appears in $r$ and FALSE otherwise. The symbol " $\in$ " denotes the set membership operator, the expression $t \in r$ can be read as " $t$ [is] in $r$ " or " $t$ appears in $r$." In fact, as you've probably realized, " $\epsilon$ "
is essentially SQL's IN-except that the left operand of SQL's IN is usually a scalar, not a row, which means there's some coercion going on once again (i.e., the scalar is coerced to the row that contains it). ${ }^{5}$ Here's an example ("Get suppliers who supply at least one part"):

```
SELECT SNO , SNAME , STATUS , CITY
FROM S
WHERE SNO IN /* "SNO" coerced to "ROW(SNO)" */
    ( SELECT SNO
```

Note: As I'm sure you know, SQL also supports NOT IN. The Tutorial D analog is " $\neq$ "; in other words, the Tutorial D expression " $t \notin r$ " means tuple $t$ isn't in relation $r$.

## RELATIONS ARE $\boldsymbol{n}$-DIMENSIONAL

I've stressed the point several times that, while a relation can be pictured as a table, it isn't a table. (To say it yet again, a picture of a thing isn't the same as the thing.) Of course, it can be very convenient to think of a relation as a table; after all, tables are user friendly; indeed, as noted in Chapter 1, it's the fact that we can think of relations, informally, as tables-sometimes more explicitly as "flat" or "two-dimensional" tables-that makes relational systems intuitively easy to understand and use, and makes it intuitively easy to reason about the way such systems behave. In other words, it's a very nice property of the relational model that its basic data structure, the relation, has such an intuitively attractive pictorial representation.

Unfortunately, however, many people seem to have been blinded by that attractive pictorial representation into thinking that relations as such are "flat" or "two-dimensional." But they're not. Rather, if relation $r$ has $n$ attributes, then each tuple in $r$ represents a point in a certain $n$-dimensional space (and the relation overall represents a set of such points). For example, each of the five tuples appearing in our usual suppliers relation represents a certain point in a certain 4-dimensional space (the four dimensions corresponding, of course, to the four attributes of that relation), and the relation overall can thus be said to be 4 -dimensional. Thus, relations are $n$-dimensional, not two-dimensional. ${ }^{6}$ As I've written elsewhere (in quite a few places, in fact): Let's all vow never to say "flat relations" ever again.

## RELATIONAL COMPARISONS

Like tuple types, relation types are no exception to the rule that the "=" comparison operator must be defined for every type; that is, given two relations $r 1$ and $r 2$ of the same relation type $T$, we must at least be able to test whether they're equal. Other comparisons might be useful, too; for example, we might want to test whether $r 1$ includes $r 2$ (meaning every tuple in $r 2$ is also in $r 1$ ), or whether $r 1$ properly includes $r 2$ (meaning every tuple in $r 2$ is also in $r 1$ but $r l$ contains at least one tuple that isn't in $r 2$ ). Here's an example, expressed in Tutorial D as usual, of an equality comparison on relations:

```
S { CITY } = P { CITY }
```

[^36]The left comparand here is the projection of suppliers on $\{$ CITY $\},{ }^{7}$ the right comparand is the projection of parts on $\{$ CITY $\}$, and the comparison returns TRUE if these two projections are equal, FALSE otherwise. In other words, the comparison (which is a boolean expression) means: "The set of supplier cities is equal to the set of part cities" (and it evaluates to either TRUE or FALSE, of course).

Here's another example:
S \{ SNO \} $\supset \operatorname{SP}$ \{ SNO \}

The symbol "כ" here means "properly includes" (or, equivalently, "is a proper superset of"). The meaning of this expression (considerably paraphrased) is: "Some suppliers supply no parts at all" (which again necessarily evaluates to either TRUE or FALSE).

Other useful relational comparison operators include " $\supseteq$ " ("includes"), " $\subseteq$ " ("is included in"), and " $\subset$ " ("is properly included in"). Note: Of these operators, the " $\subseteq$ " operator in particular is usually referred to, a trifle arbitrarily, as "the" relational inclusion operator.

One extremely common requirement is to be able to perform an "=" comparison between some given relation $r$ and an empty relation of the same type-in other words, a test to see whether $r$ is empty. So it's convenient to define a shorthand:

```
IS_EMPTY ( r )
```

This expression is defined to return TRUE if relation $r$ is empty and FALSE otherwise. I'll be relying on it heavily in chapters to come (especially Chapter 8). The inverse operator is useful too:

```
IS_NOT_EMPTY ( r )
```

This expression is logically equivalent to NOT (IS_EMPTY $(r)$ ).

## TABLE_DUM AND TABLE_DEE

Recall from the discussion of tuples earlier in this chapter that the empty set is a subset of every set, and hence that there's such a thing as the empty tuple (also called the 0-tuple), and of course that tuple has an empty heading. For exactly the same reason, a relation too might have an empty heading-a heading is a set of attributes, and there's no reason why that set shouldn't be empty. Such a relation is of type RELATION $\}$, and its degree is zero.

Let $r$ be a relation of degree zero, then. How many such relations are there? The answer is: Just two. First, $r$ might be empty (meaning it contains no tuples)-remember there's always exactly one empty relation of any given type. Second, if $r$ isn't empty, then the tuples it contains must all be 0 -tuples. But there's only one 0 -tuple!-equivalently, all 0 -tuples are duplicates of one another-and so $r$ can't possibly contain more than one of them. So there are indeed just two relations with no attributes: one with just one tuple, and one with no tuples at all. For fairly obvious reasons, I'm not going to try drawing pictures of these relations (in fact, this is the one place where the idea of thinking of relations as tables breaks down completely).

Now, you might well be thinking: So what? Why on earth would I ever want a relation that has no attributes at all? Even if they're mathematically respectable (which they are), surely they're of no practical significance? In fact, however, it turns out they're of very great practical significance indeed: so much so, that we have pet names for them-we call them TABLE_DUM and TABLE_DEE, or DUM and DEE for short (DUM is the empty one, DEE is

[^37]the one with one tuple). And what makes them so significant is their meanings, which are FALSE (or no) for DUM and TRUE (or yes) for DEE. They have the most fundamental meanings of all. Note: I'll be discussing the whole notion of relations and their meaning in much more detail in Chapters 5 and 6 .

By the way, a good way to remember which is which is this: DEE and yes both have an "E"; DUM and no don't.

Now, I haven't covered enough in this book yet to show concrete examples of DUM and DEE in action, as it were, but we'll see plenty of examples of their use in the pages ahead. Here I'll just mention one point that should make at least intuitive sense at this early juncture: These two relations (especially TABLE_DEE) play a role in the relational algebra that's analogous to the role played by zero in conventional arithmetic. And we all know how important zero is; in fact, it's hard to imagine an arithmetic without zero (the ancient Romans tried, but it didn't get them very far). Well, it should be equally hard to imagine a relational algebra without TABLE_DEE. Which brings us to SQL ... SQL, since it has no counterpart to the 0 -tuple, clearly (but unfortunately) has no counterpart to TABLE_DUM or TABLE_DEE either. ${ }^{8}$

## TABLES IN SQL

Note: Throughout this section, by the term table I mean a table value specifically-an SQL table value, that is-and not a table variable (which is what CREATE TABLE and CREATE VIEW create). I'll discuss table variables in Chapter 5.

Now, I explained in Chapter 2 that SQL doesn't really have anything analogous to the concept of a relation type at all; instead, an SQL table is just a collection of rows (a bag of rows, in general, not necessarily a set) that are of a certain row type. It follows that SQL doesn't really have anything analogous to the RELATION type generator, either-though as we know from Chapter 2 it does support other type generators, including ROW, ARRAY, and MULTISET. It does, however, have something called a table value constructor that's analogous, somewhat, to a relation selector. Here's an example:

```
VALUES ( 1 , 2 ), ( 2 , 1 ), ( 1 , 1 ), ( 1 , 2 )
```

This expression (actually it's a table literal, though SQL doesn't use this term) evaluates to a table with four-not three!-rows and two columns. What's more, those columns have no names. As I've already explained, the columns of an SQL table are ordered, left to right; as a consequence, those columns can be, and sometimes have to be, identified by ordinal position instead of name.

By way of another example, consider the following table value constructor invocation:


Note that, in order for this expression to be regarded as a fair approximation to its relational counterpart (i.e., a relation literal denoting the relation that's the current value of relvar S as shown in Fig. 1.3), we must:

[^38]1. Ensure, for each column of the table specified by the VALUES expression, that all of the values are of the pertinent type. (In particular, if some given ordinal position in any of the specified rows corresponds to attribute $A$ of the intended relational counterpart, then we must ensure that the same ordinal position in all of those rows corresponds to that same attribute $A$.)
2. Ensure that we don't specify the same row twice.

Note: As you know, in the relational model a heading is a set of attributes. In SQL, by contrast, because columns have a left to right ordering, it would be more correct to regard a heading as a sequence, not a set, of attributes (or columns, rather). If the recommendations of this book are followed, however, this logical difference can mostly (?) be ignored.

What about table assignment and comparison operators? Well, table assignment is a big topic, and I'll defer the details to Chapter 5. As for table comparisons, SQL has no direct support—not even for equality! ${ }^{9}$-but workarounds are available. For example, here's an SQL counterpart to the Tutorial D comparison S $\{$ CITY $\}=$ P \{CITY $\}$ :

```
NOT EXISTS ( SELECT CITY FROM S
    EXCEPT
    SELECT CITY FROM P )
AND
NOT EXISTS ( SELECT CITY FROM P
    EXCEPT
    SELECT CITY FROM S )
```

And here's a counterpart to the Tutorial D comparison $\mathrm{S}\{\mathrm{SNO}\} \supset \mathrm{SP}\{\mathrm{SNO}\}:$

```
EXISTS ( SELECT SNO FROM S
    EXCEPT
    SELECT SNO FROM SP )
AND
NOT EXISTS ( SELECT SNO FROM SP
        EXCEPT
        SELECT SNO FROM S )
```

[^39]
## COLUMN NAMING IN SQL

In the relational model, (a) every attribute of every relation has a name (i.e., anonymous attributes are prohibited), and (b) such names are unique within the relevant relation (i.e., duplicate attribute names are prohibited). In SQL, analogous rules are enforced sometimes, but not always. To be specific, they're enforced for the tables that happen to be the current values of table variables-defined via CREATE TABLE or CREATE VIEW-but not for the tables that result from evaluation of some table expression. ${ }^{10}$ Strong recommendation: Use AS clauses whenever necessary (and possible) to give proper column names to columns that otherwise (a) wouldn't have a name at all or (b) would have a name that wasn't unique. Here are some examples:

```
SELECT DISTINCT SNAME , 'Supplier' AS TAG
FROM S
SELECT DISTINCT SNAME , 2 * STATUS AS DOUBLE_STATUS
FROM S
SELECT MAX ( WEIGHT ) AS MBW
FROM P
WHERE COLOR = 'Blue'
CREATE VIEW SDS AS
    ( SELECT DISTINCT SNAME , 2 * STATUS AS DOUBLE_STATUS
        FROM S ) ;
SELECT DISTINCT S.CITY AS SCITY , P.CITY AS PCITY
FROM S , SP , P
WHERE S.SNO = SP.SNO
AND SP.PNO = P.PNO
SELECT TEMP.*
FROM ( SELECT * FROM S JOIN P ON S.CITY > P.CITY ) AS TEMP
    ( SNO , SNAME , STATUS , SCITY
        PNO , PNAME , COLOR , WEIGHT , PCITY )
```

Of course, the foregoing recommendation can safely be ignored if there's no subsequent need to reference the otherwise anonymous or nonuniquely named columns. For example, the third of the foregoing examples could safely be abbreviated in some circumstances (in a WHERE or HAVING clause, perhaps) to just:

```
SELECT MAX ( WEIGHT )
FROM
WHERE COLOR = 'Blue'
```

Perhaps more important, note that the recommendation unfortunately can't be followed at all in the case of tables specified by means of VALUES expressions. However, workarounds are possible. For example, the following is legal:

[^40]```
SELECT TEMP.*
FROM ( VALUES ( 'S1' , 'Smith' , 20 , 'London' )
    ( 'S2' , 'Jones' , 10 , 'Paris' ) ,
    ( 'S3' , 'Blake' , 30 , 'Paris' ) ,
    ( 'S4' , 'Clark' , 20 , 'London' ) ,
    ( 'S5', 'Adams' , 30 , 'Athens' ) )
    AS TEMP ( SNO , SNAME , STATUS , CITY )
```

Explanation: I've enclosed the VALUES expression in parentheses (thereby making it a subquery), attached an AS clause, and specified column names as well as a "correlation name" within that AS clause (see Chapter 12).

Important note: The operators of the relational algebra rely on proper attribute naming in a variety of ways. For example, as we'll see in Chapter 6, the relational UNION operator requires its operands to have the same heading (and hence the same attribute names), and the result then has the same heading as well. One advantage of this scheme is precisely that it avoids the complexities caused (in SQL) by reliance on ordinal position! In order to use SQL relationally, therefore, you should apply the same discipline to the SQL analogs of those relational operators. Strong recommendation: As a prerequisite to enforcing such a discipline, if two columns in SQL represent "the same kind of information," give them the same name wherever possible. (That's why, for example, the two supplier number columns in our running example, the suppliers-and-parts database, are both called SNO and not, say, SNO in one table and SNR in the other.) Conversely, if two columns represent different kinds of information, it's usually a good idea to give them different names.

The only case where it's impossible to follow the foregoing recommendation is when two columns in the same table both represent the same kind of information. For example, consider an SQL table EMP with columns representing employee number and manager number, respectively, where manager number is itself another employee number. These two columns will have to have different names, say ENO and MNO, respectively. As a consequence, some column renaming will sometimes have to be done, as in this example (note the specification "ENO AS MNO" in the third line):

```
( SELECT ENO , MNO FROM EMP ) AS TEMP1
    NATURAL JOIN
( SELECT ENO AS MNO , ... FROM EMP ) AS TEMP2
/* where "..." is EMP columns other than ENO and MNO */
```

Such renaming will also have to be done, if you want to use SQL relationally, if columns simply haven't been named appropriately in the first place (e.g., if you're confronted with a database that's been defined by somebody else-doubtless a common state of affairs in practice). A strategy you might want to consider in such circumstances is the following:

■ For each table $T$ in the database, define a view $V$ that's identical to table $T$ except possibly for some column renaming.

Make sure all views so defined abide by the column naming discipline described above.
Operate in terms of those views instead of the underlying tables.
Unfortunately, it's impossible to ignore the fact 100 percent that columns have an ordinal position in SQL. (Of course, it's precisely because of this fact that SQL is able to get away with its anonymous columns and duplicate column names.) Note in particular that columns still have an ordinal position in SQL even when they don't need to (i.e., when they're all properly named anyway); this observation applies to columns in base tables and views in particular. Strong recommendation: Never write SQL code that relies on such ordinal positioning. Examples of where SQL attaches significance to such positioning include (but probably aren't limited to):

- SELECT * (see Chapter 12)
- The FROM clause, if more than one table is specified
- Explicit JOIN operations (see Chapter 6)
- UNION, INTERSECT, and EXCEPT operations, if CORRESPONDING isn't specified (see Chapter 6)
- In the column name commalist, if specified, following the definition of a range variable (see Chapter 12)
- In the column name commalist, if specified, in CREATE VIEW (see Chapter 9)
- INSERT, if no column name commalist is specified (see Chapter 5)
- VALUES expressions
- Row assignments and comparisons
- ALL and ANY comparisons, if the comparands are of degree greater than one (see Chapter 11)


## CONCLUDING REMARKS

In this chapter I've given precise definitions for the fundamental concepts tuple and relation. As I said earlier, those definitions can be a little daunting at first, but I hope you were able to make sense of them after having read the first two chapters. I also discussed tuple and relation types, selectors, and comparisons, as well as a number of important consequences of the definitions; in particular, I briefly described the important relations TABLE_DUM and TABLE_DEE. And I discussed the SQL counterparts of all of these notions, where such counterparts exist. In closing, I'd like to stress the importance of the recommendations, in the section immediately preceding this one, regarding column naming in SQL. Later chapters will rely heavily on those recommendations.

## EXERCISES

3.1 Define as precisely as you can the terms attribute, body, cardinality, degree, heading, relation, relation type, and tuple.
3.2 State as precisely as you can what it means for (a) two tuples to be equal; (b) two relations to be equal.
3.3 Write Tutorial D tuple selector invocations for a typical tuple from (a) the parts relvar, (b) the shipments relvar. Also show SQL's counterparts, if any, to those selector invocations.
3.4 Write a typical Tutorial D relation selector invocation. Also show SQL's counterpart, if any, to that selector invocation.

## 3.5 (This is essentially a repeat of Exercise 1.8 from Chapter 1, but you should be able to give a more comprehensive answer now.) There are many differences between a relation and a table. List as many as you can.

3.6 The attributes of a tuple can be of any type whatsoever (well, almost; can you think of any exceptions?). Give an example of (a) a tuple with a tuple valued attribute, (b) a tuple with a relation valued attribute (RVA).
3.7 Give an example of a relation with (a) one RVA, (b) two RVAs. Also give two more relations that represent the same information as those relations but don't involve RVAs. Also give an example of a relation with an RVA such that there's no relation that represents precisely the same information but has no RVA.
3.8 Explain the relations TABLE_DUM and TABLE_DEE in your own words. Why exactly doesn't SQL support them?
3.9 As we saw in the body of the chapter, TABLE_DEE means TRUE and TABLE_DUM means FALSE. Do these facts mean we could dispense with the usual BOOLEAN data type? Also, DEE and DUM are relations, not relvars. Do you think it would ever make sense to define a relvar of degree zero?
3.10 What's the logical difference if any-as opposed to the obvious syntactic difference-between the following two SQL expressions?

```
VALUES ( 1 , 2 ), ( 2 , 1 ), ( 1 , 1 ), ( 1 , 2 )
VALUES ( ( 1 , 2 ), ( 2 , 1 ), ( 1 , 1 ), ( 1 , 2 ) )
```

3.11 What exactly does the following SQL expression mean?

```
SELECT SNO
FROM S
WHERE ( NOT ( ( STATUS , SNO ) <= ( 20, 'S4' ) ) ) IS NOT FALSE
```

3.12 Explain in your own words what it means to say that relations are $n$-dimensional.
3.13 List as many situations as you can think of in which SQL regards left to right column ordering as significant.
3.14 Give an SQL analog for the Tutorial D expression IS_NOT_EMPTY( $r$ ).
3.15 I said in the body of the chapter that a relation selector invocation in Tutorial D consists of the keyword RELATION, followed by a commalist of tuple expressions enclosed in braces (and those tuple expressions must all be of the same tuple type)-and I implied, though I didn't say as much explicitly, that the type of the relation denoted by the overall expression was RELATION $\{H\}$, where TUPLE $\{H\}$ was the common type of all of the specified tuple expressions. But what if the set of specified tuple expressions is empty?-in other words, what if the relation being selected is empty? How can its type be determined?

Following on from the foregoing, how can we specify an empty table in SQL?
3.16 A tuple is a set (a set of components); so do you think it might make sense to define versions of the usual set operators (union, intersection, etc.) that apply to tuples?
3.17 State in your own words, as carefully as you can, the discipline described in the body of the chapter regarding SQL column names.
3.18 The column naming discipline referred to in the previous exercise relies on the use of AS clauses. But such clauses can appear in SQL in several different contexts; moreover, the syntax sometimes takes the form " $X$ AS <something>" and sometimes " $<$ something $>$ AS $X$ " (if you see what I mean); and the keyword is sometimes optional and sometimes mandatory. ${ }^{11}$ List all of the contexts in which AS can appear, showing which are of the form " $X$ AS ..." and which "... AS $X$ ", and in which cases the keyword is optional.

[^41]
## Chapter 4

## No Duplicates, No Nulls

I haven't even mentioned yet the way the silly notions<br>Discussed so far interreact and lead us into oceans<br>Of complication and despond and general distress. Are two nulls equal (duplicates)? I fear, both NO and YES.

-Anon.: Where Bugs Go

In the previous chapter, I said the following (approximately):

- Relations never contain duplicate tuples, because the body of a relation is a set (a set of tuples) and sets in mathematics don't contain duplicate elements.
- Relations never contain nulls, because the body of a relation is a set of tuples, and tuples in turn never contain nulls.

I also suggested that since there was so much to be said about these topics, it was better to devote a separate chapter to them. This is that chapter. Note: By definition, the topics in question are SQL topics, not relational ones; in what follows, therefore, I'll use the terminology of SQL rather than that of the relational model (for the most part, at any rate).

## WHAT'S WRONG WITH DUPLICATES?

There are numerous practical arguments in support of the position that duplicate rows ("duplicates" for short) should be prohibited. Here I want to emphasize just one-but I think it's a powerful one. ${ }^{1}$ However, it does rely on certain notions I haven't discussed yet in this book, so I need to make a couple of preliminary assumptions:

1. I assume you know that relational DBMSs include a component called the optimizer, ${ }^{2}$ whose job is to try to figure out the best way to implement user queries and the like (where "best" basically means best performing).

[^42]2. I assume you also know that one of the things optimizers do is what's sometimes called query rewrite. Query rewrite is the process of transforming some relational expression expl (representing some user query, say) into another such expression $\exp 2$, such that $\exp 1$ and $\exp 2$ are guaranteed to produce the same result when evaluated but $\exp 2$ performs better than $\exp 1$ (at least, we hope so). Note: Be aware, however, that the term query rewrite is also used in certain commercial products with a different (typically more limited) meaning.

Now I can present my argument. The fundamental point I want to make is that certain expression transformations, and hence certain optimizations, that would be valid if SQL were truly relational aren't valid in the presence of duplicates. By way of example, consider the (nonrelational) database shown in Fig. 4.1. Note right away that the tables in that database have no keys (which is why there's no double underlining in the figure). And by the way: If you're thinking the database is unrealistic-and especially if you're thinking you're not going to be convinced by the arguments that follow, therefore-please see the further remarks on this example at the beginning of the next section.

P | PNO | PNAME |
| :--- | :--- |
| P1 | Screw |
| P1 | Screw |
| P1 | Screw |
| P2 | Screw |

SP | SNO | PNO |
| :---: | :---: |
| S1 | P1 |
| S1 | P1 |
| S1 | P2 |

Fig. 4.1: A nonrelational database, with duplicates
Before going any further, perhaps I should ask the question: What does it mean to have three (P1,Screw) rows in table P and not two, or four, or seventeen? It must mean something, for if it means nothing, then why are the duplicates there in the first place? As I once heard Ted Codd say: If something is true, saying it twice doesn't make it any more true. ${ }^{3}$

So I have to assume there's some meaning attached to the duplication, even though that meaning, whatever it is, is hardly very explicit. ${ }^{4}$ Given that duplicates do have some meaning, therefore, there are presumably going to be business decisions made on the basis of the fact that, for example, there are three ( $\mathrm{P} 1, \mathrm{Screw}$ ) rows in table P and not two or four or seventeen. For if not, then (to repeat) why are the duplicates there in the first place?

Now consider the following query on the database of Fig. 4.1: "Get part numbers for parts that either are screws or are supplied by supplier S1, or both." Here are some candidate SQL formulations for this query, together with the result produced in each case:

[^43]```
1. SELECT P.PNO
    FROM P
    WHERE P.PNAME = 'Screw'
    OR P.PNO IN
        ( SELECT SP.PNO
        FROM SP
        WHERE SP.SNO = 'S1' )
    Result: P1 * 3, P2 * 1.
2. SELECT SP.PNO
    FROM SP
    WHERE SP.SNO = 'S1'
    OR SP.PNO IN
        ( SELECT P.PNO
        FROM P
        WHERE P.PNAME = 'Screw' )
    Result: P1 * 2, P2 * 1.
3. SELECT P.PNO
    FROM P , SP
    WHERE ( SP.SNO = 'S1' AND
        SP.PNO = P.PNO )
    OR P.PNAME = 'Screw'
    Result: P1 * 9, P2 * 3.
4. SELECT SP.PNO
    FROM P , SP
    WHERE ( SP.SNO = 'S1' AND
            SP.PNO = P.PNO )
    OR P.PNAME = 'Screw'
    Result: P1 * 8, P2 * 4.
5. SELECT P.PNO
    FROM P
    WHERE P.PNAME = 'Screw'
    UNION ALL
    SELECT SP.PNO
    FROM SP
    WHERE SP.SNO = 'S1'
    Result: P1 * 5, P2 *2.
6. SELECT DISTINCT P.PNO
    FROM P
    WHERE P.PNAME = 'Screw'
    UNION ALL
    SELECT SP.PNO
    FROM SP
    WHERE SP.SNO = 'S1'
    Result: P1 * 3, P2 * 2.
```

```
7. SELECT P.PNO
    FROM P
    WHERE P.PNAME = 'Screw'
    UNION ALL
    SELECT DISTINCT SP.PNO
    FROM SP
    WHERE SP.SNO = 'S1'
```

    Result: P1 * 4, P2 * 2 .
    8. SELECT DISTINCT P.PNO
FROM P
WHERE P.PNAME = 'Screw'
OR P.PNO IN
( SELECT SP.PNO
FROM SP
WHERE SP.SNO = 'S1' )
Result: P1 * 1, P2 * 1 .
9. SELECT DISTINCT SP.PNO
FROM SP
WHERE SP.SNO = 'S1'
OR SP.PNO IN
( SELECT P.PNO
FROM P
WHERE P.PNAME = 'Screw' )
Result: $\mathrm{P} 1 * 1, \mathrm{P} 2 * 1$.
10. SELECT P.PNO
FROM P
GROUP BY P.PNO, P.PNAME
HAVING P.PNAME = 'Screw'
OR P.PNO IN
( SELECT SP.PNO
FROM SP
WHERE SP.SNO = 'S1' )
Result: P 1 * $1, \mathrm{P} 2$ * 1 .
11. SELECT P.PNO
FROM P , SP
GROUP BY P.PNO , P.PNAME , SP.SNO , SP.PNO
HAVING ( SP.SNO = 'S1' AND
SP.PNO = P.PNO )
OR P.PNAME = 'Screw'

Result: P1 * 2, P2 * 2 .
12. SELECT P.PNO

FROM P
WHERE P.PNAME = 'Screw'
UNION
SELECT SP.PNO
FROM SP
WHERE SP.SNO = 'S1'
Result: P1 * $1, \mathrm{P} 2$ * 1.

Aside: Actually, certain of the foregoing formulations-which?-are a little suspect, because they effectively assume that every screw is supplied by at least one supplier. But this fact makes no material difference to the argument that follows. End of aside.

The first point to notice, then, is that the twelve different formulations produce nine different results: different, that is, with respect to their degree of duplication. (By the way, I make no claim that the twelve different formulations and the nine different results are the only ones possible; indeed, they aren't, in general.) Thus, if the user really cares about duplicates, then he or she needs to be extremely careful in formulating the query in such a way as to obtain exactly the desired result.

Furthermore, analogous remarks apply to the system itself: Because different formulations can produce different results, the optimizer too has to be extremely careful in its task of expression transformation. For example, the optimizer isn't free to transform, say, formulation 1 into formulation 12 or the other way around, even if it would like to. In other words, duplicate rows act as a significant optimization inhibitor. Here are some implications of this fact:

- The optimizer code itself is harder to write, harder to maintain, and probably more buggy-all of which combine to make the product more expensive and less reliable, as well as later in delivery to the marketplace, than it might be.
- System performance is likely to be worse than it might be.
- Users are going to have to get involved in performance issues. To be more specific, they're going to have to spend time and effort in figuring out how to formulate a given query in order to get the best performance-a state of affairs that (as noted in Chapter 1) the relational model was expressly intended to avoid.

The fact that duplicates serve as an optimization inhibitor is particularly frustrating in view of the fact that, in most cases, users probably don't care how many duplicates appear in the result. In other words:

- Different formulations produce different results.
- However, the differences are probably irrelevant from the user's point of view.
- But the optimizer is unaware of this latter fact and is therefore prevented, unnecessarily, from performing the transformations it might like to perform.

On the basis of examples like the foregoing, I'm tempted to say you should always ensure that query results contain no duplicates-for example, by always specifying DISTINCT in your SQL queries-and thus simply forget
about the whole problem (and if you follow this advice, there can be no good reason for having duplicates in the first place!). However, I'll have more to say about this suggestion in the section immediately following.

## DUPLICATES: FURTHER ISSUES

There's much, much more that could be said regarding duplicates and what's wrong with them, but I'll limit myself here to just three further points. First of all, you might reasonably object that in practice base tables, at least, never do include duplicates, and the foregoing example thus intuitively fails. True enough (probably); but the trouble is, SQL can generate duplicates in query results. Indeed, different formulations of the same query can produce results with different degrees of duplication, even if the input tables themselves have no duplicates at all. For example, here are two possible formulations of the query "Get supplier numbers for suppliers who supply at least one part" on our usual suppliers-and-parts database (and note here that the input tables certainly don't contain any duplicates):

```
SELECT SNO
FROM S
WHERE SNO IN
    ( SELECT SNO
        FROM SP )
```

SELECT SNO
FROM $S$ NATURAL JOIN SP

At least one of these expressions-which?-will produce a result with duplicates, in general. (Exercise: Given our usual sample data values, what results do the two expressions produce?) So if you don't want to think of the tables in Fig. 4.1 as base tables specifically, fine: Just take them to be the output from previous queries, and the rest of the analysis goes through unchanged.

Second, there's another at least psychological argument against duplicates that I think is quite persuasive (thanks to Jonathan Gennick for this one): If, in accordance with the $n$-dimensional perspective on relations introduced in Chapter 3, you think of a table as a plot of points in some $n$-dimensional space, then duplicate rows clearly don't add anything-they simply amount to plotting the same point twice.

My last point is this. Suppose table $T$ does permit duplicates. Then we can't tell the difference between "genuine" duplicates in $T$ and duplicates that arise from errors in data entry on $T$ ! For example, suppose the person responsible for data entry unintentionally enters the very same row twice-e.g., by inadvertently hitting the return key twice (easily done, by the way). Then there's no straightforward way to delete the "second" row without deleting the "first" as well. Note that we presumably do want to delete that "second" row, since it shouldn't have been entered in the first place.

## AVOIDING DUPLICATES IN SQL

The relational model prohibits duplicates; to use SQL relationally, therefore, steps must be taken to prevent them from occurring. Now, if every base table has at least one key (see Chapter 5), then duplicates will never occur in base tables as such. As already mentioned, however, certain SQL expressions can still yield result tables with duplicates. Here are some of the cases in which such tables can be produced:

## SELECT ALL

## UNION ALL

VALUES (i.e., table value constructor invocations)

Regarding VALUES, see Chapter 3. Regarding ALL, note first that this keyword (and its alternative, DISTINCT) can be specified:

In a SELECT clause, immediately following the SELECT keyword

- In a union, intersection, or difference, immediately following the applicable keyword (UNION, INTERSECT, and EXCEPT, respectively)
- Inside the parentheses in an invocation of a "set function" such as SUM, immediately preceding the argument expression

Note: DISTINCT is the default for UNION, INTERSECT, and EXCEPT; ALL is the default in the other cases. Now, the "set function" case is special; you must specify ALL, at least implicitly, if you want the function to take duplicate values into account, which sometimes you do (see Chapter 7). But the other cases have to do with elimination of duplicate rows, which must always be done, at least in principle, if you want to use SQL relationally. Thus, the obvious recommendations in those cases are: Always specify DISTINCT; preferably do so explicitly; and never specify ALL. Then you can just forget about duplicate rows entirely.

In practice, however, matters aren't quite that simple. Why not? Well, I don't think I can do better here than repeat the essence of what I wrote in this book's predecessor (Database in Depth, O'Reilly Media Inc., 2005):

At this point in the original draft, I added that if you find the discipline of always specifying DISTINCT annoying, don't complain to me-complain to the SQL vendors instead. But my reviewers reacted with almost unanimous horror to my suggestion that you should always specify DISTINCT. One wrote: "Those who really know SQL well will be shocked at the thought of coding SELECT DISTINCT by default." Well, I'd like to suggest, politely, that (a) those who are "shocked at the thought" probably know the implementations well, not SQL, and (b) their shock is probably due to their recognition that those implementations do such a poor job of optimizing away unnecessary DISTINCTs. ${ }^{5}$ If I write SELECT DISTINCT SNO FROM S ..., that DISTINCT can safely be ignored. If I write either EXISTS (SELECT DISTINCT ...) or IN (SELECT DISTINCT ...), those DISTINCTs can safely be ignored. If I write SELECT DISTINCT SNO FROM SP ... GROUP BY SNO, that DISTINCT can safely be ignored. If I write SELECT DISTINCT ... UNION SELECT DISTINCT ..., those DISTINCTs can safely be ignored. And so on. Why should I, as a user, have to devote time and effort to figuring out whether some DISTINCT is going to be a performance hit and whether it's logically safe to omit it?-and to remembering all of the details of SQL's inconsistent rules for when duplicates are automatically eliminated and when they're not?

Well, I could go on. However, I decided-against my own better judgment, but in the interest of maintaining good relations (with my reviewers, I mean)-not to follow my own advice elsewhere in this book but only to request duplicate elimination explicitly when it seemed to be logically necessary to do so. It wasn't always easy to decide when that was, either. But at least now I can add my voice to those complaining to the vendors, I suppose.

So the recommendation (sadly) boils down to this: First, make sure you know when SQL eliminates duplicates without you asking it to. Second, in those cases where you do have to ask, make sure you know whether

[^44]it matters if you don't. Third, in those cases where it matters, specify DISTINCT (but, as Hugh Darwen once said, be annoyed about it). And never specify ALL!

## WHAT'S WRONG WITH NULLS?

The opening paragraph from the section "What's Wrong with Duplicates?" applies equally well here, with just one tiny text substitution, so I'll basically just repeat it: There are numerous practical arguments in support of the position that nulls should be prohibited. Here I want to emphasize just one-but I think it's a powerful one. But it does rely on certain notions I haven't discussed yet in this book, so I need to make a couple of preliminary assumptions:

1. I assume you know that any comparison in which at least one of the comparands is null evaluates to the UNKNOWN truth value instead of TRUE or FALSE. The justification for this state of affairs is the intended interpretation of null as value unknown: If the value of $A$ is unknown, then it's also unknown whether, for example, $A>B$, regardless of the value of $B$ (even-perhaps especially-if the value of $B$ is unknown as well). Note: That same state of affairs is also the source of the term three-valued logic (3VL). That is, the notion of nulls, as understood in SQL, inevitably leads to a logic in which there are three truth values instead of the usual two. (The relational model, by contrast, is based on conventional two-valued logic, 2VL.)
2. I assume you're also familiar with the 3 VL truth tables for the familiar logical operators-also known as connectives - NOT, AND, and OR ( $\mathrm{T}=\mathrm{TRUE}, \mathrm{F}=\mathrm{FALSE}, \mathrm{U}=\mathrm{UNKNOWN}$ ):

| $p$ | NOT $p$ |
| :---: | :---: |
| $T$ | $F$ |
| $U$ | $U$ |
| $F$ | $T$ |


| $p$ | $q$ | $p$ | AND |
| :---: | :---: | :---: | :---: |
|  | $q$ |  |  |
| $T$ | $T$ | $T$ |  |
| $T$ | $U$ | $U$ |  |
| $T$ | $F$ | F |  |
| U | T | U |  |
| U | U | U |  |
| U | F | F |  |
| F | T | F |  |
| F | U | F |  |
| F | F | F |  |


| $p$ | $q$ | $p$ | OR |
| :---: | :---: | :---: | :---: |
|  | $q$ |  |  |
| $T$ | $T$ | $T$ |  |
| $T$ | $U$ | $T$ |  |
| $T$ | $F$ | $T$ |  |
| $U$ | $T$ | $T$ |  |
| $U$ | $U$ | $U$ |  |
| $U$ | $F$ | $U$ |  |
| $F$ | $T$ | $T$ |  |
| $F$ | $U$ | $U$ |  |
| $F$ | $F$ | $F$ |  |

Observe in particular that NOT returns UNKNOWN if its input is UNKNOWN; AND returns UNKNOWN if one input is UNKNOWN and the other is either UNKNOWN or TRUE; and OR returns UNKNOWN if one input is UNKNOWN and the other is either UNKNOWN or FALSE.

Now I can present my argument. The fundamental point I want to make is that certain boolean expressions-and therefore certain queries in particular-can produce results that are correct according to threevalued logic but not correct in the real world. By way of example, consider the (nonrelational) database shown in Fig. 4.2, in which "the CITY is null" for part P1. Note carefully that the shading in that figure, in the place where the CITY value for part P1 ought to be, stands for nothing at all; conceptually, there's nothing at all-not even a string of blanks or an empty string-in that position (which means the "tuple" for part P1 isn't really a tuple, a point I'll come back to near the end of this section).

| SNO | CITY |
| :---: | :--- |
| S1 | London |

P

| PNO | CITY |
| :--- | :--- |
| P1 | MA/A/A/A. |

Fig. 4.2: A nonrelational database, with a null
Consider now the following (admittedly rather contrived) query on the database of Fig. 4.2: "Get (SNO,PNO) pairs where either the supplier and part cities are different or the part city isn't Paris (or both)." Here's an SQL formulation of this query:

```
SELECT S.SNO , P.PNO
FROM S , P
WHERE S.CITY <> P.CITY
OR P.CITY <> 'Paris'
```

Now I want to focus on the boolean expression in the WHERE clause:

```
( S.CITY <> P.CITY ) OR ( P.CITY <> 'Paris' )
```

(I've added some parentheses for clarity.) For the only data we have, this expression evaluates to UNKNOWN OR UNKNOWN, which reduces to just UNKNOWN. Now, queries in SQL retrieve data for which the expression in the WHERE clause evaluates to TRUE, not to FALSE and not to UNKNOWN; ${ }^{6}$ in the example, therefore, nothing is retrieved at all.

But part P1 does have some corresponding city in the real world;" in other words, "the null CITY" for part P1 does stand for some real value, say $c$. Now, either $c$ is Paris or it isn't. If it is, then the expression

```
( S.CITY <> P.CITY ) OR ( P.CITY <> 'Paris' )
```

becomes (for the only data we have)
( 'London' <> 'Paris' ) OR ( 'Paris' <> 'Paris' )
which evaluates to TRUE, because the first term evaluates to TRUE. Alternatively, if $c$ isn't Paris, then the expression becomes (again, for the only data we have)
( 'London' <> c ) OR ( c <> 'Paris' )
which also evaluates to TRUE, because the second term evaluates to TRUE. Thus, the boolean expression is always true in the real world, and the query should therefore return the pair (S1,P1), regardless of what real value the null

[^45]stands for. In other words, the result that's correct according to the logic (meaning, specifically, 3VL) and the result that's correct in the real world are different!

By way of another example, consider the following query on that same table P from Fig. 4.2 (I didn't lead with this example because it's even more contrived than the previous one, but in some ways it makes the point with even more force):

```
SELECT PNO
FROM P
WHERE CITY = CITY
```

The real world answer here is surely the set of part numbers currently appearing in $P$ (in other words, the set containing just part number P1, given the sample data shown in Fig. 4.2). SQL, however, will return no part numbers at all.

To sum up: If you have any nulls in your database, then you're getting wrong answers to certain of your queries. What's more, you have no way of knowing, of course, just which queries you're getting wrong answers to and which not; all results become suspect. You can never trust the answers you get from a database with nulls. In my opinion, this state of affairs is a complete showstopper.

Aside: To all of the above, I can't resist adding that even though SQL does support 3VL, and even though it does support the keyword UNKNOWN, that keyword does not-unlike the keywords TRUE and FALSE-denote a value of type BOOLEAN. (This is just one of the numerous flaws in SQL's 3VL support; there are many, many others, but most of them are beyond the scope of this book.) To elaborate briefly: As with 2VL, the SQL type BOOLEAN contains just two values, TRUE and FALSE; "the third truth value" is represented, quite incorrectly, by null! Here are some consequences of this fact:

- Assigning UNKNOWN to a variable B of type BOOLEAN actually sets B to null.
- After such an assignment, the comparison $\mathrm{B}=\mathrm{UNKNOWN}$ doesn't give TRUE-instead, it gives null (meaning, to spell the point out, that SQL apparently believes, or claims, that it's unknown whether B is UNKNOWN). Note, incidentally, that this state of affairs constitutes a violation of The Assignment Principle (see Exercise 2.22 in Chapter 2, also Chapter 5).
- In fact, the comparison $\mathrm{B}=\mathrm{UNKNOWN}$ always gives null (meaning UNKNOWN), regardless of the value of B, because it's logically equivalent to the comparison " $B=$ NULL" (not meant to be valid SQL syntax).

To understand the seriousness of such flaws, you might care to meditate on the analogy of a numeric type that uses null instead of zero to represent zero. End of aside.

As with the business of duplicates earlier, there's a lot more that could be said on the whole issue of nulls, but I just want to close with a brief look at the formal argument against them. Recall that, by definition, a null isn't a value. It follows that:

- A "type" that contains a null inn't a type (because types contain values).
- A "tuple" that contains a null isn't a tuple (because tuples contain values).
- A "relation" that contains a null isn't a relation (because relations contain tuples, and tuples don't contain nulls).
- In fact, nulls (like duplicates) violate one of the most fundamental relational principles of all-viz., The Information Principle. Once again, see Appendix A for further discussion of that principle.

The net of all this is that if nulls are present, then we're certainly not talking about the relational model (I don't know what we are talking about, but it's not the relational model); the entire edifice crumbles, and all bets are off.

## AVOIDING NULLS IN SQL

The relational model prohibits nulls; to use SQL relationally, therefore, steps must be taken to prevent them from occurring. First of all, a NOT NULL constraint should be specified, explicitly or implicitly, for every column in every base table (see Chapter 5); then nulls will never occur in base tables as such. Unfortunately, however, certain SQL expressions can still yield result tables containing nulls. Here are some of the situations in which nulls can be produced:

- The SQL "set functions" such as SUM all return null if their argument is empty (except for COUNT and $\operatorname{COUNT}(*)$, which correctly return zero in such a situation).
- If a scalar subquery evaluates to an empty table, that empty table is coerced to a null.
- If a row subquery evaluates to an empty table, that empty table is coerced to a row of all nulls. Note: A row of all nulls and a null row aren't the same thing at all, logically speaking (another logical difference here, in fact)-yet SQL does think they're the same thing, at least some of the time. But it would take us much too far afield to get into the detailed implications of this state of affairs here.
- Outer joins and "union joins" are expressly designed to produce nulls in their result. ${ }^{8}$
- If the ELSE clause is omitted from a CASE expression, an ELSE clause of the form ELSE NULL is assumed.

■ The expression $\operatorname{NULLIF}(x, y)$ returns null if $x=y$ evaluates to TRUE.
■ The "referential triggered actions" ON DELETE SET NULL and ON UPDATE SET NULL can both generate nulls (obviously enough).

## Strong recommendations:

■ Do specify NOT NULL, explicitly or implicitly, for every column in every base table.

[^46]- Don't use the keyword NULL in any other context whatsoever (i.e., anywhere other than a NOT NULL constraint or logical equivalent).
- Don't use the keyword UNKNOWN in any context whatsoever.

■ Don't omit the ELSE clause from a CASE expression unless you're certain it would never have been reached anyway.

Don't use NULLIF.

- Don't use outer join, and don't use the keywords OUTER, FULL, LEFT, and RIGHT (except possibly as suggested in the section "A Remark on Outer Join" below).
- Don't use union join.
- Don't specify either PARTIAL or FULL on MATCH (they have meaning only when nulls are present). For similar reasons, don't use the MATCH option on foreign key constraints, and don't use IS DISTINCT FROM. (In the absence of nulls, the expression $a$ IS DISTINCT FROM $b$ is logically equivalent to the expression $a<>b$.)
- Don't use IS TRUE, IS NOT TRUE, IS FALSE, or IS NOT FALSE. The reason is that, if $b x$ is a boolean expression, then the following logical equivalences fail to be valid only if nulls are present:

| $b x$ | IS TRUE | $\equiv b x$ |
| :--- | :--- | :--- |
| $b x$ | IS NOT TRUE | $\equiv$ |
| NOT $b x$ |  |  |
| $b x$ | IS FALSE | $\equiv$ NOT bx |
| $b x$ | IS NOT FALSE | $\equiv b x$ |

In other words, IS TRUE and the rest are distractions at best, in the absence of nulls.

- Finally, do use COALESCE on every scalar expression that might "evaluate to null" without it. (Apologies for the quotation marks, but the fact is that the phrase "evaluates to null" is a solecism.)

In case you're not familiar with COALESCE, let me elaborate briefly on the last of these recommendations. Essentially, COALESCE is an operator that lets us replace a null by some nonnull value "as soon as it appears" (i.e., before it has a chance to do any significant damage). Here's the definition: Let $a, b, \ldots, c$ be scalar expressions. Then the expression COALESCE $(a, b, \ldots, c)$ returns null if its arguments are all null, or the value of its first nonnull argument otherwise. Of course, to use it "sensibly," you do need to make sure at least one of $a, b, \ldots, c$ is nonnull! Here's a fairly realistic example:

```
SELECT S.SNO , ( SELECT COALESCE ( SUM ( ALL SP.QTY ) , 0 )
    FROM SP
    WHERE SP.SNO = S.SNO ) AS TOTQ
FROM S
```

In this example, if the SUM invocation "evaluates to null"-which it will do in particular for any supplier that doesn't have any matching shipments-then the COALESCE invocation will replace that null by a zero. (Incidentally, this example also illustrates a situation in which use of ALL instead of DISTINCT isn't just
acceptable but is logically required, though it might be implicit. See Chapter 7.) Given our usual sample data, therefore, the query produces the following result:

| SNO | TOTQ |
| ---: | ---: |
| S1 | 1300 |
| S2 | 700 |
| S3 | 200 |
| S4 | 900 |
| S5 | 0 |

## A REMARK ON OUTER JOIN

Outer join is expressly designed to produce nulls in its result and should therefore be avoided, in general.
Relationally speaking, it's a kind of shotgun marriage: It forces tables into a kind of union-yes, I do mean union, not join-even when the tables in question fail to conform to the usual requirements for union (see Chapter 6). It does this, in effect, by padding one or both of the tables with nulls before doing the union, thereby making them conform to those usual requirements after all. But there's no reason why that padding shouldn't be done with proper values instead of nulls, as in this example:

```
SELECT SNO , PNO FROM SP
UNION
SELECT SNO , 'nil' AS PNO FROM S
WHERE SNO NOT IN ( SELECT SNO FROM SP )
```

Result (note the line for supplier S5 in particular):

| SNO | PNO |
| :---: | :---: |
| S1 | P1 |
| S1 | P2 |
| S1 | P3 |
| S1 | P4 |
| S1 | P5 |
| S1 | P6 |
| S2 | P1 |
| S2 | P2 |
| S3 | P2 |
| S4 | P2 |
| S4 | P4 |
| S4 | P5 |
| S5 | nil |

Alternatively, the same result could be obtained by using the explicit SQL outer join operator in conjunction with COALESCE, as here:

```
SELECT SNO, COALESCE ( PNO , 'nil') AS PNO
FROM S NATURAL LEFT OUTER JOIN SP
```


## CONCLUDING REMARKS

There are a few final remarks I want to make regarding nulls and 3VL specifically. Nulls and 3VL are supposed to be a solution to the "missing information" problem-but I believe I've shown that, to the extent they can be considered a "solution" at all, they're a disastrously bad one. Before I leave the topic, however, I'd like to raise, and respond to, an argument that's often heard in this connection. That argument goes something like this:

All of those examples you give where nulls lead to wrong answers are very artificial. Real world queries aren't like that! More generally, most of your criticisms seem very academic and theoretical-I bet you can't show any real practical situations where nulls have given rise to the kinds of problems you worry about, and I bet you can't prove such practical situations do occur.

Needless to say, I have several responses to this argument. The first is: How do we know nulls haven't caused real practical problems, anyway? It seems to me that if some serious real world situation-an oil spill, a collapsed bridge, a wrong medical diagnosis-were found to be due to nulls, there might be valid reasons (nontechnical ones, I mean) why the information would never get out. We've all heard stories of embarrassing failures caused by software glitches of other kinds, even in the absence of nulls; in my opinion, nulls can only serve to make such failures more likely.

Second, suppose someone-me, for example-were to go around claiming that some software product or application contained a serious logical error due to nulls. Can you imagine the lawsuits?

Third and most important, I think those of us who criticize nulls don't need to be defensive, anyway; I think we should stand the counterarguments on their head, as it were. After all, it's undeniable that nulls can lead to errors in certain cases. So it's not up to us to prove those "certain cases" might include practical, real world situations; rather, it's up to those who want to defend nulls to prove they don't. And I venture to suggest that in practice it would be quite difficult, and very likely impossible, to prove any such thing.

Of course, if nulls are prohibited, then missing information will have to be handled by some other means. Unfortunately, those other means are a little too complex, in general, to be discussed in detail here. The SQL mechanism of (nonnull) default values can be used in simple cases; but for a more comprehensive approach to the problem-including in particular an explanation of how you can still get "don't know" answers when you want them, even from a database without nulls-I refer you to Appendix C.

## EXERCISES

4.1 "Duplicates in databases are a good idea in because duplicates occur naturally in the real world. For example, all pennies are duplicates of one another." How would you respond to this argument?
4.2 Let $r$ be a relation and let $b x$ and $b y$ be boolean expressions. Then there's a law (used in relational systems to help with optimization, among other things) that states that ( $r$ WHERE $b x$ ) UNION ( $r$ WHERE $b y$ ) $\equiv r$ WHERE $b x$ OR by. If $r$ isn't a relation but an SQL table with duplicates, does this law still apply?
4.3 Let $a, b$, and $c$ be sets. Then the distributive law of intersection over union (also used in relational systems to help with optimization among other things) states that $a$ INTERSECT ( $b$ UNION $c$ ) $\equiv(a$ INTERSECT $b$ ) UNION $(a$ $\operatorname{INTERSECT} c$ ). If $a, b$, and $c$ are bags instead of sets, does this law still apply?

### 4.4 Part of the SQL standard's explanation of the FROM clause (as in a SELECT - FROM - WHERE

 expression) reads as follows:[The] result of the $<$ from clause $>$ is the ... cartesian product of the tables identified by [the specifications in that $<$ from clause>]. The ... cartesian product, $C P$, is the multiset of all rows $r$ such that $r$ is the concatenation of a row from each of the identified tables ...

Note, therefore, that $C P$ isn't well defined!-the fact that the standard goes on to say that "The cardinality of $C P$ is the product of the cardinalities of the identified tables" notwithstanding. For example, consider the tables T1 and T2 shown here:


Observe now that all of the following fit the above definition for "the" cartesian product $C P$ of T1 and T2 (that is, any of them could be "the" multiset referred to):
CP1 $\left.\begin{array}{c|c|}\hline \mathrm{C} 1 & \mathrm{C} 2 \\ \hline 0 & 1 \\ 0 & 1 \\ 0 & 1 \\ 0 & 2 \\ \hline\end{array}\right] \quad \mathrm{CP} 2\left[\begin{array}{c|c|}\hline \mathrm{C} 1 & \mathrm{C} 2 \\ \hline 0 & 1 \\ 0 & 1 \\ 0 & 2 \\ 0 & 2 \\ \hline\end{array}\right]$

Can you fix up the wording of the standard appropriately?

### 4.5 Consider the following SQL cursor definition:

DECLARE X CURSOR FOR SELECT SNO , QTY FROM SP ;
Note that (a) cursor X permits updates, (b) the table visible through cursor X permits duplicates, but (c) the underlying table SP doesn't (permit duplicates, that is). Now suppose the operation DELETE ... WHERE CURRENT OF X is executed. Then there's no way, in general, of saying which specific row of table SP is deleted by that operation. How would you fix this problem?
4.6 Please write out one googol times: There's no such thing as a duplicate. Note: A googol is one followed by 100 zeros (i.e., 10 to the hundredth power). A googolplex is one followed by a googol zeros (i.e., 10 to the "googolth" power).
4.7 Do you think nulls occur naturally in the real world?
4.8 There's a logical difference between null and the third truth value: True or false? (Perhaps I should ask: True, false, or unknown?)
4.9 In the body of the chapter, I gave truth tables for one monadic 3 VL connective (NOT) and two dyadic 3 VL connectives (AND and OR), but there are many other connectives as well (see Exercise 4.10 below). Another useful monadic connective is MAYBE, ${ }^{9}$ with truth table as follows:

| $P$ | MAYBE $p$ |
| :---: | :---: |
| $T$ | F |
| U | T |
| F | F |

Does SQL support this connective?
4.10 Following on from the previous exercise, how many distinct connectives are there altogether in 2VL? What about 3VL? What do you conclude from your answers to these questions?
4.11 A logic is truth functionally complete if it supports, directly or indirectly, all possible connectives. Truth functional completeness is an extremely important property; a logic that didn't satisfy it would be like an arithmetic that had no support for certain operations, say " + ". Is classical 2VL truth functionally complete? Is SQL's 3VL truth functionally complete?
4.12 Let $b x$ be a boolean expression. Then $b x$ OR NOT $b x$ is also a boolean expression, and in 2VL it's guaranteed to evaluate to TRUE (it's an example of what logicians call a tautology). Is it a tautology in 3VL? If not, is there an analogous tautology in 3VL?
4.13 With $b x$ as in the previous exercise, $b x$ AND NOT $b x$ is also a boolean expression, and in 2VL it's guaranteed to evaluate to FALSE (it's an example of what logicians call a contradiction). Is it a contradiction in 3 VL ? If not, is there an analogous contradiction in 3VL?
4.14 In 2VL, $r$ JOIN $r$ is equal to $r$ and INTERSECT and TIMES are both special cases of JOIN (see Chapter 6). Are these observations still valid in 3VL?
4.15 The following is a legitimate SQL row value constructor invocation: ROW (1,NULL). Is the row it denotes null or nonnull?
4.16 Let $b x$ be an SQL boolean expression. Then NOT ( $b x$ ) and ( $b x$ ) IS NOT TRUE are both SQL boolean expressions. Are they equivalent?
4.17 Let $x$ be an SQL expression. Then $x$ IS NOT NULL and NOT ( $x$ IS NULL) are both SQL boolean expressions. Are they equivalent?
4.18 Let DEPT and EMP be SQL tables; let DNO be a column in both; let ENO be a column in EMP; and consider the expression DEPT.DNO = EMP.DNO AND EMP.DNO = 'D1' (this expression might be part of the WHERE clause in some query, for example). Now, a "good" optimizer might very well transform this expression into DEPT.DNO = EMP.DNO AND EMP.DNO = 'D1' AND DEPT.DNO = 'D1', on the grounds that $a=b$ and $b=$

[^47]$c$ together imply that $a=c$ (see Exercise 6.13 in Chapter 6). But is this transformation valid? If not, why not? And what are the implications?
4.19 Suppose the suppliers-and-parts database permits nulls; in particular, suppose columns SP.SNO and SP.PNO permit nulls. ${ }^{10}$ Here then is a query on that database, expressed for reasons beyond the scope of this chapter not in SQL but in a kind of pidgin form of relational calculus (see Chapter 10):

S WHERE NOT EXISTS SP ( SP.SNO $=$ S.SNO AND SP.PNO = 'P2' )

What does this query mean? And is the following formulation equivalent?

```
S WHERE NOT ( S.SNO IN ( SP.SNO WHERE SP.PNO = 'P2' ) )
```

4.20 Let $k l$ and $k 2$ be values of the same type. In SQL, then, what exactly do the following statements mean?
a. $k l$ and $k 2$ are "the same" for the purposes of a comparison in, e.g., a WHERE clause.
b. $k 1$ and $k 2$ are "the same" for the purposes of key uniqueness.
c. $k 1$ and $k 2$ are "the same" for the purposes of duplicate elimination.
4.21 In the body of the chapter, I said UNION ALL can generate duplicates. But what about INTERSECT ALL and EXCEPT ALL?
4.22 Are the recommendations "Always specify DISTINCT" and "Never specify ALL" duplicates of each other?
4.23 If TABLE_DEE corresponds to TRUE (or yes) and TABLE_DUM to FALSE (or no), then what corresponds to UNKNOWN ( or maybe)?
4.24 The following quotes are taken from the SQL standard: ${ }^{11}$
"The data type boolean comprises the distinct truth values True and False. Unless prohibited by a NOT NULL constraint, the boolean data type also supports the truth value Unknown as the null value. This [standard] does not make a distinction between the null value of the boolean data type and the truth value Unknown ... [They] may be used interchangeably to mean exactly the same thing."

- "All boolean values and SQL truth values are comparable ... The value True is greater than the value False, and any comparison involving the null value or an Unknown truth value will return an Unknown result."

[^48]Do you have any comments on these quotes? In particular, which if any of the following do you think are legal SQL expressions? And what do they return, if they're legal?
a. TRUE OR FALSE
b. TRUE OR UNKNOWN
c. TRUE OR NULL
d. TRUE > FALSE
e. TRUE > UNKNOWN
f. TRUE > NULL
4.25 In his book Using the New DB2 (Morgan Kaufmann, 1996), in a section titled "A Brief History of SQL," Don Chamberlin-who is widely acknowledged to be "the father of SQL"-has the following to say (I'm quoting the text more or less verbatim, except that I've added some italics):

During the early development of SQL ... some decisions were made that were ultimately to generate a great deal [of] controversy ... Chief among these were the decisions to support null values [sic] and to permit duplicate rows ... I will [briefly examine] the reasons for these decisions ... My purpose here is historical rather than persuasive ... I recognize that nulls and duplicates are religious topics, and I do not expect anyone to have a conversion experience after reading this chapter.

Do you agree with Chamberlin that nulls and duplicates are "religious topics"?

## Chapter 5

## Base Relvars, Base Tables

Said a young mathematician named Gene<br>"I always say what I mean-<br>Or mean what I say-<br>It's the same, anyway-<br>Or-at least-well, you know what I mean."

-Anon.: Where Bugs Go

By now you should be very familiar with the idea that relation values (relations for short) vs. relation variables (relvars for short) is one of the great logical differences. Now it's time to take a closer look at that difference; more specifically, it's time to take a closer look at issues that are relevant to relvars in particular, as opposed to relations. Caveat: Unfortunately, you might find the SQL portions of the discussion that follows a little confusing, because SQL doesn't clearly distinguish between the two concepts-as you know, it uses the same term table to mean sometimes a table value, sometimes a table variable. For example, the keyword TABLE in CREATE TABLE clearly refers to a table variable; but when we say, e.g., that table S has five rows, the phrase "table S " clearly refers to a table value (namely, the current value of the table variable called S ). Be on your guard for potential confusion in this area.

Let me also remind you of a few further points:

- First of all, a relvar is a variable whose permitted values are relations, and it's specifically relvars, not relations, that are the target for INSERT, DELETE, and UPDATE operations (more generally, for relational assignment operations-recall that INSERT, DELETE, and UPDATE are all just shorthand for certain relational assignments).
- Next, if $R$ is a relvar and $r$ is a relation to be assigned to $R$, then $R$ and $r$ must be of the same (relation) type.
- Last, the terms heading, body, attribute, tuple, cardinality, and degree, formally defined in Chapter 3 for relations, can all be interpreted in the obvious way to apply to relvars as well (see Exercise 1.5 in Chapter 1).

The present chapter deals with base relvars (base tables, in SQL). In fact, it won't hurt too much if you assume throughout this book until further notice that all relvars are base relvars and all tables are base tables, barring explicit statements to the contrary; Chapter 9 discusses the special considerations, such as they are, that apply to virtual relvars or views. The topics I'll be covering in the present chapter form something of a mixed bag, but generally speaking they fall into the following broad categories:

## - Updating (relational assignment)

- Candidate and foreign keys

Predicates

As a basis for examples, I'll use the following definitions for the suppliers-and-parts database (Tutorial D on the left and SQL on the right, a pattern I'll follow in most of my examples in this chapter and throughout the rest of the book):

```
VAR S BASE RELATION
    { SNO CHAR ,
        SNAME CHAR ,
        STATUS INTEGER ,
        CITY CHAR }
    KEY { SNO } ;
VAR P BASE RELATION
    { PNO CHAR ,
        PNAME CHAR ,
        COLOR CHAR ,
        WEIGHT RATIONAL ,
        CITY CHAR }
    KEY { PNO } ;
VAR SP BASE RELATION
    { SNO CHAR ,
        PNO CHAR,
        QTY INTEGER }
    KEY { SNO , PNO }
    FOREIGN KEY { SNO }
            REFERENCES S
    FOREIGN KEY { PNO }
        REFERENCES P ;
```

CREATE TABLE S
( SNO VARCHAR (5) NOT NULL,
$\begin{array}{lll}\text { SNO } & \text { VARCHAR (5) } & \text { NOT NULL, } \\ \text { SNAME } & \text { VARCHAR(25) } & \text { NOT NULL, }\end{array}$
STATUS INTEGER NOT NULL
CITY VARCHAR(20) NOT NULL,
UNIQUE ( SNO ) ) ;
CREATE TABLE P
( PNO VARCHAR (6) NOT NULL ,
PNAME VARCHAR(25) NOT NULL,
COLOR CHAR(10) NOT NULL,
WEIGHT NUMERIC $(5,1)$ NOT NULL,
CITY VARCHAR(20) NOT NULL,
UNIQUE ( PNO ) ) ;
CREATE TABLE SP
( SNO VARCHAR(5) NOT NULL,
PNO VARCHAR(6) NOT NULL,
QTY INTEGER NOT NULL ,
UNIQUE ( SNO , PNO ) ,
FOREIGN KEY ( SNO )
REFERENCES S ( SNO ) ,
FOREIGN KEY ( PNO )
REFERENCES P ( PNO ) ) ;

## UPDATING IS SET LEVEL

The first point I want to stress is that, regardless of what syntax we use to express it, relational assignment is a set level operation. (In fact, all operations in the relational model are set level, meaning they take entire relations or relvars as operands, not just individual tuples.) Thus, INSERT inserts a set of tuples into the target relvar; DELETE deletes a set of tuples from the target relvar; and UPDATE updates a set of tuples in the target relvar. Now, it's true that we often talk in terms of (for example) updating some individual tuple as such, but you need to understand that:
a. Such talk really means the set of tuples we're updating just happens to have cardinality one.
b. What's more, updating a set of tuples of cardinality one sometimes isn't possible anyway.

For example, suppose relvar S is subject to the integrity constraint (see Chapter 8) that suppliers S1 and S4 are always in the same city. Then any "single tuple UPDATE" that tries to change the city for just one of those two suppliers will necessarily fail. Instead, we must change them both at the same time, perhaps like this:

```
UPDATE S
WHERE SNO = 'S1'
    OR SNO = 'S4':
        { CITY := 'New York' } ;
```

UPDATE S
SET CITY = 'New York'
WHERE $\quad$ SNO $=$ 'S1'
OR $\quad$ SNO $=$ 'S4' ;

What's being updated in this example is a set of two tuples.

One consequence of the foregoing is that there's nothing in the relational model corresponding to SQL's "positioned updates" (i.e., UPDATE or DELETE "WHERE CURRENT OF cursor"), because those operations are tuple level (or row level, rather), not set level, by definition. They do happen to work, most of the time, in today's SQL products, but that's because those products aren't very good at supporting integrity constraints. If they were to improve in that regard, those "positioned updates" might not work any more; that is, applications that succeed today might fail tomorrow—not a very desirable state of affairs, it seems to me. Recommendation: Don't do SQL updates through a cursor, unless you can be absolutely certain that problems like the one in the example will never arise. (I say this in full knowledge of the fact that many SQL updates are done through a cursor at the time of writing.) Note: For another argument against updating through cursors, see Exercise 4.5 in Chapter 4.

Now I need to 'fess up to something. The fact is, to talk as I've been doing of "updating a tuple"-or set of tuples, rather-is very imprecise (not to say sloppy) anyway. Recall the definitions of value and variable from Chapter 1. If $V$ is subject to update, then $V$ must be a variable, by definition-but tuples (like relations) are values and can't be updated, again by definition. What we really mean when we talk of updating tuple $t 1$ to $t 2$ (say), within some relvar $R$, is that we're replacing tuple $t 1$ in $R$ by another tuple $t 2$. And that kind of talk is still sloppy!-what we really mean is that we're replacing the relation $r l$ that's the original value of $R$ by another relation $r 2$. And what exactly is relation $r 2$ here? Well, let $s 1$ and $s 2$ be relations containing just tuple $t 1$ and tuple $t 2$, respectively; then $r 2$ is ( $r 1$ MINUS $s 1$ ) UNION $s 2$. In other words, "updating tuple $t 1$ to $t 2$ in relvar $R$ " can be thought of as, first, deleting $t l$ and then inserting $t 2$-if despite everything I've been saying you'll let me talk in terms of deleting and inserting individual tuples in this loose fashion.

In the same kind of way, it doesn't really make sense to talk in terms of "updating attribute $A$ within tuple $t "$-or within relation $r$, or even within relvar $R$. Of course, we do it anyway, because it's convenient (it saves a lot of circumlocution); I mean, we say things like "update the city for supplier S1 from London to New York"; but it's like that business of user friendly terminology I discussed in Chapter 1-it's OK to talk this way only if we all understand that such talk is only an approximation to the truth, and indeed that it tends to obscure the essence of what's really going on.

## Triggered Actions

The fact that updating is set level implies among other things that "referential triggered actions" such as ON DELETE CASCADE (see the section "More on Foreign Keys" later in this chapter)-more generally, triggered actions of all kinds - mustn't be done until all of the explicitly requested updating has been done. In other words, a set level update must not be treated as a sequence of individual tuple level updates (or row level updates, in SQL). SQL, however, unfortunately does treat set level updates as a sequence of row level ones, at least in its support for "row level triggers" if nowhere else. Recommendation: Try to avoid operations that are inherently row level. Of course, this recommendation doesn't prohibit set level operations in which the set just happens to be of cardinality one, as in the following example:

```
UPDATE S WHERE SNO = 'S1' :
    { CITY := 'New York' } ;
```

| UPDATE | S |
| :--- | :--- |
| SET | CITY = 'New York' |
| WHERE | SNO = 'S1'; |

## Constraint Checking

The fact that updating is set level has another implication too: namely, that integrity constraint checking also mustn't be done until all of the updating (including triggered actions, if any) has been done. (The constraint discussed earlier, involving a change to the city for suppliers S1 and S4, illustrates this point very clearly. See Chapter 8 for further discussion.) Again, therefore, a set level update mustn't be treated as a sequence of individual tuple level updates (or row level updates, in SQL). Now, I believe the SQL standard does conform to this requirement-or
maybe not; its row level triggers might be a little suspect in this regard (see the subsection immediately preceding this one). In any case, even if the standard does conform, that's not to say all commercial products do; ${ }^{1}$ thus, you should still be on your lookout for violations in this connection.

## A Final Remark

The net of the discussions in this section overall is that update operations-in fact, all operations-in the relational model are always semantically atomic; that is, either they execute in their entirety, or they have no effect at all (except possibly for returning a status code or equivalent). Thus, although we do sometimes describe some set level operation, informally, as if it were shorthand for a sequence of tuple level operations, it's important to understand that such descriptions are (as I said before) strictly incorrect and only approximations to the truth.

## RELATIONAL ASSIGNMENT

Relational assignment in general works by assigning a relation value, denoted by some relational expression, to a relation variable, denoted by a relvar reference (where a relvar reference is basically just the pertinent relvar name). Here's a Tutorial D example:

```
S := S WHERE NOT ( CITY = 'Athens' ) ;
```

Now, it's easy to see that this particular assignment is logically equivalent to the following DELETE statement:

```
DELETE S WHERE CITY = 'Athens' ;
```

More generally, the Tutorial D DELETE statement

```
DELETE R WHERE bx ;
```

(where $R$ is a relvar name and $b x$ is a boolean expression) is shorthand for, and hence logically equivalent to, the following relational assignment:

```
R := R WHERE NOT ( bx ) ;
```

Alternatively, we might say it's shorthand for this one (either way, it comes to the same thing):

```
R := R MINUS ( R WHERE bx ) ;
```

Turning to INSERT, the Tutorial D INSERT statement

```
INSERT R rx ;
```

(where $R$ is again a relvar name and $r x$ is a relational expression-typically but not necessarily a relation selector invocation) is shorthand for:

[^49]```
R := R UNION rx ;
```

For example, the INSERT statement-

```
INSERT SP RELATION { TUPLE { SNO 'S5' , PNO 'P6' , QTY 700 } } ;
```

-effectively inserts a single tuple into the shipments relvar SP.
Finally, the Tutorial D UPDATE statement also corresponds to a certain relational assignment. However, the details are a little more complicated in this case than they are for INSERT and DELETE, and I'll defer them to Chapter 7.

## D_INSERT and I_DELETE

I've said the INSERT statement

```
INSERT R rx ;
```

is shorthand for:

```
R := R UNION rx ;
```

Observe now, however, that this definition implies that an attempt to insert "a tuple that already exists" (i.e., an INSERT in which the relations denoted by $R$ and $r x$ aren't disjoint) will succeed. (It won't insert a duplicate tuple, of course-it just won't have any effect.) For that reason, Tutorial D additionally supports an operator called D_INSERT ("disjoint INSERT"), with syntax as follows:

```
D_INSERT R rx ;
```

This statement is shorthand for:

$$
R:=R \text { D_UNION } r x \text {; }
$$

D_UNION here stands for disjoint union. Disjoint union is just like regular union, except that its operand relations are required to have no tuples in common (see Chapter 6). It follows that an attempt to use D_INSERT to insert a tuple that already exists will fail.

What about DELETE? Well, observe first that the syntax presented above-
DELETE $R$ WHERE bx ;
-is actually just a special case (though it's far and away the commonest case in practice). The more general form parallels the syntax of INSERT:

```
DELETE R rx ;
```

Here $R$ is a relvar name and $r x$ is a relational expression (possibly but not necessarily a relation selector invocation). ${ }^{2}$ This more general form of DELETE is defined to be shorthand for:

```
R := R MINUS rx ;
```

For example, the DELETE statement-

```
DELETE SP RELATION { TUPLE { SNO 'S1' , PNO 'P1' , QTY 300 } } ;
```

-effectively deletes a single tuple from the shipments relvar SP.
It should be clear, however, that the foregoing definition implies that an attempt to delete "a tuple that doesn't exist" (i.e., a DELETE in which the relation denoted by $r x$ isn't wholly included in the relation denoted by $R$ ) will succeed. For that reason, Tutorial D additionally supports an operator called I_DELETE ("included DELETE"), with syntax as follows:

```
I_DELETE R rx ;
```

This statement is shorthand for:

$$
R:=R \text { I_MINUS } r x \text {; }
$$

I_MINUS here stands for included minus; the expression $r 1$ I_MINUS $r 2$ is defined to be the same as $r 1$ MINUS $r 2$ (see Chapter 6), except that every tuple appearing in $r 2$ must also appear in $r 1$-in other words, $r 2$ must be included in $r 1$. It follows that an attempt to use I_DELETE to delete a tuple that doesn't exist will fail.

Note: Now that I've introduced D_INSERT and I_DELETE, please understand that discussions elsewhere in this book that refer to INSERT and DELETE operations in Tutorial D should be taken for simplicity as applying to D_INSERT and I_DELETE operations as well, where the sense demands it.

## Table Assignment in SQL

SQL has nothing directly comparable to Tutorial D's D_INSERT and I_DELETE. Apart from this difference, however, SQL's support for INSERT, DELETE, and UPDATE operations resembles that of Tutorial D fairly closely and there's little more to be said, except for a few points regarding INSERT specifically:

- First, the source for an SQL INSERT operation is specified by means of a table expression (typically but not necessarily a VALUES expression-see Chapter 3). Contrary to popular opinion, therefore, INSERT in SQL really does insert a table, not a row, though that table (the source table) might and often will contain just one row, or even no rows at all.
- Second, INSERT in SQL is defined in terms of neither UNION nor D_UNION, but rather in terms of SQL's "UNION ALL" operator (see Chapter 6). As a consequence, an attempt to insert a row that already exists will fail if the target table is subject to a key constraint but will succeed (and will insert a duplicate row) otherwise.

[^50]Third, INSERT in SQL supports an option according to which the target table specification can be followed by a parenthesized column name commalist, identifying the columns into which values are to be inserted; the $i$ th target column corresponds to the $i$ th column of the source table. Omitting this option is equivalent to specifying all of the columns of the target table, in the left to right order in which they appear within that table. Recommendation: Never omit this option. For example, the INSERT statement

```
INSERT INTO SP ( PNO , SNO , QTY ) VALUES ( 'P6' , 'S5' , 700 ) ;
```

is preferable to this one-

```
INSERT INTO SP VALUES ( 'S5' , 'P6' , 700 ) ;
```

-because this second formulation relies on the left to right ordering of columns in table SP and the first one doesn't. ${ }^{3}$ Here's another example (incidentally, this one makes it clear that INSERT really does insert a table and not a row):

```
INSERT INTO SP ( SNO , PNO , QTY ) VALUES ( 'S3' , 'P1' , 500 ) ,
    ( 'S2' , 'P5' , 400 ) ;
```

As for relational assignment: Unfortunately SQL doesn't have a direct counterpart to this operator. The closest it can get to the generic assignment

```
R := rx ;
```

is the following sequence of statements:

```
DELETE FROM T ;
INSERT INTO T ( ... ) tx ;
```

( $T$ and $t x$ here are the SQL analogs of $R$ and $r x$, respectively.) Note in particular that (as noted in the answer to Exercise 1.16 in Appendix $F$ ) this sequence of statements could fail where its relational counterpart, the relational assignment, would succeed-for example, if table $T$ is subject to the constraint that it mustn't be empty.

## The Assignment Principle

I'd like to close this section by drawing your attention to a principle that, though it's really quite simple, has far reaching consequences: The Assignment Principle, which states that after assignment of value $v$ to variable $V$, the comparison $v=V$ must evaluate to TRUE. Note: The Assignment Principle is a fundamental principle, not just for the relational model, but for computing in general. It applies to relational assignment in particular, of course, but (to repeat) it's actually relevant to assignments of all kinds. In fact, as I'm sure you realize, it's more or less the definition of the assignment operation. I'll have more to say about it in Chapter 8, when I discuss what's called multiple assignment.

[^51]
## MORE ON CANDIDATE KEYS

I explained the basic idea of candidate keys in Chapter 1, but now I want to make the concept more precise. Here first is a definition:

Definition: Let $K$ be a subset of the heading of relvar $R$. Then $K$ is a candidate key (or just key for short) for $R$ if and only if it possesses both of the following properties:

1. Uniqueness: No valid value for $R$ contains two distinct tuples with the same value for $K$.
2. Irreducibility: No proper subset of $K$ has the uniqueness property.

If $K$ consists of $n$ attributes, then $n$ is the degree of $K$.
Now, the uniqueness property is self-explanatory, but I need to say a little more about the irreducibility property. Consider relvar S and the set of attributes-let's call it SC-\{SNO,CITY\}, which is certainly a subset of the heading of $S$ that has the uniqueness property (no relation that's a valid value for relvar $S$ ever has two distinct tuples with the same SC value). But it doesn't have the irreducibility property, because we could discard the CITY attribute and what's left, the singleton set $\{\mathrm{SNO}\}$, would still have the uniqueness property. So we don’t regard SC as a key, because it's "too big." By contrast, $\{\mathrm{SNO}\}$ is irreducible, and it's a key.

Why do we want keys to be irreducible? One important reason is that if we were to specify a "key" that wasn't irreducible, the DBMS wouldn't be able to enforce the proper uniqueness constraint. For example, suppose we told the DBMS (lying!) that SC was a key for relvar S. Then the DBMS couldn't enforce the constraint that supplier numbers are "globally" unique; instead, it could enforce only the weaker constraint that supplier numbers are "locally" unique, in the sense that they're unique within the pertinent city. So this is one reason-not the only one-why we require keys not to contain any attributes that aren't needed for unique identification purposes.
Recommendation: In SQL, never lie to the system by defining as a key some column combination that you know isn't irreducible. (By the way, you might think this recommendation rather obvious, but I've certainly seen it violated in practice; in fact, I've even seen such a violation explicitly recommended, by writers who really ought to know better.)

Now, all of the relvars we've seen so far have had just one key. Here by contrast are several self-explanatory examples (in Tutorial D only, for simplicity) of relvars with two or more. Note the overlapping nature of the keys in the second and third examples. Note: I assume the availability of certain user defined types in these definitions.

```
VAR TAX BRACKET BASE RELATION
    { LOW MONEY , HIGH MONEY , PERCENTAGE INTEGER }
    KEY { LOW }
    KEY { HIGH }
    KEY { PERCENTAGE } ;
VAR ROSTER BASE RELATION
    { DAY DAY OF WEEK , TIME TIME OF DAY , GATE GATE , PILOT NAME }
    KEY { DAY', \overline{TIME , GATE }}
    KEY { DAY , TIME , PILOT } ;
```

```
VAR MARRIAGE BASE RELATION
    { SPOUSE_A NAME , SPOUSE_B NAME , DATE_OF_MARRIAGE DATE }
    /* assūme no polygamy ānd no persons-ma\overline{rrying */}
    /* each other more than once ... */
    KEY { SPOUSE_A , DATE_OF_MARRIAGE }
    KEY { DATE_O\overline{F}}\mathrm{ MARRIAGE ,- SPOUSE_B }
    KEY { SPOUSE_\overline{B}, SPOUSE_A } ;
```

By the way, you might have noticed a tiny sleight of hand here. A key is supposed to be a set of attributes, and an attribute is supposed to be an attribute-name/type-name pair; yet the Tutorial D KEY syntax specifies just attribute names, not attribute-name/type-name pairs. The syntax works, however, because attribute names are unique within the pertinent heading, and the corresponding type names are thus specified implicitly. In fact, analogous remarks apply at various points in the Tutorial D language, and I won't bother to repeat them every time, letting this one paragraph do duty for all.

I'll close this section with a few miscellaneous points. First, note that the key concept applies to relvars, not relations. Why? Because to say something is a key is to say a certain integrity constraint is in effect-a certain uniqueness constraint, to be specific-and integrity constraints apply to variables, not values. ${ }^{4}$ (By definition, integrity constraints constrain updates, and updates apply to variables, not values. See Chapter 8 for further discussion.)

Second, in the case of base relvars in particular, it's usual, as noted in Chapter 1, to single out one key as the primary key (and any other keys for the relvar in question are then sometimes said to be alternate keys). But whether some key is chosen as primary, and if so which one, are essentially psychological issues, beyond the purview of the relational model as such. As a matter of good practice, most base relvars probably should have a primary key-but, to repeat, this rule, if it is a rule, really isn't a relational issue as such. Certainly it isn't inviolable.

Third, if $R$ is a relvar, then $R$ certainly does have, and in fact must have, at least one key. The reason is that every possible value of $R$ is a relation and therefore contains no duplicate tuples, by definition; at the very least, therefore, the combination of all of the attributes of $R$-i.e., the entire heading-certainly has the uniqueness property. Thus, either that combination also has the irreducibility property, or there's some proper subset of that combination that does. Either way, there's certainly something that's both unique and irreducible. Note: These remarks don't necessarily apply to SQL tables-SQL tables allow duplicate rows and so might have no key at all.
Strong recommendation: In SQL, for base tables at any rate, use UNIQUE and/or PRIMARY KEY specifications to ensure that every such table does have at least one key.

Fourth, note that key values are tuples (rows, in SQL), not scalars. In the case of relvar S, for example, with its sole key $\{\mathrm{SNO}\}$, the value of that key for some specific tuple-say that for supplier S1-is:

```
TUPLE { SNO 'S1' }
```

(a subtuple of the pertinent tuple-recall that every subset of a tuple is a tuple in turn). Of course, in practice we would usually say, informally, that the key value in this example is just S 1 -or ' S 1 ', rather-but it really isn't. And so now it should be clear just how keys, like so much else in the relational model, rely crucially on the concept of tuple equality. To spell the point out: In order to enforce some key uniqueness constraint, we need to be able to tell whether two key values are equal, and that's precisely a matter of testing two tuples for equality-even when, as in the case of relvar S , the tuples in question are of degree one and "look like" simple scalar values.

[^52]Fifth, let $S K$ be a subset of the heading of relvar $R$ that possesses the uniqueness property but not necessarily the irreducibility property. Then $S K$ is a superkey for $R$ (and a superkey that isn't a key is called a proper superkey). For example, $\{\mathrm{SNO}\}$ and $\{\mathrm{SNO}, \mathrm{CITY}\}$ are both superkeys-and the latter is a proper superkey-for relvar S . Note that the heading of any relvar $R$ is always a superkey for $R$, by definition.

My final point has to do with the notion of functional dependency. ${ }^{5}$ I don't want to get into a lot of detail regarding that concept here-I'll come back to it in Chapter 8 -but you're probably familiar with it anyway. All I want to do here is call your attention to the following. Let $S K$ be a superkey (possibly a key) for relvar $R$, and let $X$ be any subset of the heading of $R$. Then the functional dependency (FD)

$$
S K \rightarrow X
$$

holds in $R$, necessarily. To elaborate briefly: In general, the functional dependency $S K \rightarrow X$ means that whenever two tuples of $R$ have the same value for $S K$, they also have the same value for $X$. But if two tuples have the same value for $S K$, where $S K$ is a superkey, then by definition they must be the very same tuple!-and so they must have the same value for $X$. In other words, loosely: We always have functional dependency arrows "out of superkeys" (and therefore out of keys in particular) to everything else in the relvar.

## MORE ON FOREIGN KEYS

I remind you from Chapter 1 that, loosely speaking, a foreign key is a set of attributes in one relvar whose values are supposed to correspond to values of some candidate key-the target key-in some other relvar (or possibly in the same relvar). In the suppliers-and-parts database, for example, $\{\mathrm{SNO}\}$ and $\{\mathrm{PNO}\}$ are foreign keys in SP whose values are required to match, respectively, values of the candidate key $\{\mathrm{SNO}\}$ in S and values of the candidate key $\{\mathrm{PNO}\}$ in P . (By required to match here, I mean that if, e.g., relvar SP contains a tuple with SNO value S1, then relvar S must also contain a tuple with SNO value S1-for otherwise SP would show some shipment as being supplied by a nonexistent supplier, and the database wouldn't be "a faithful model of reality.")

Here now is a more precise definition:
Definition: Let $R 1$ and $R 2$ be relvars, not necessarily distinct, and let $K$ be a key for $R 1$. Let $F K$ be a subset of the heading of $R 2$ such that there exists a possibly empty sequence of attribute renamings on $R 1$ that maps $K$ into $K^{\prime}$ (say), where $K^{\prime}$ and $F K$ contain exactly the same attributes (i.e., are of the same type). Further, let $R 2$ and $R 1$ be subject to the constraint that, at all times, every tuple $t 2$ in $R 2$ has an $F K$ value that's the $K^{\prime}$ value for some (necessarily unique) tuple $t l$ in $R 1$ at the time in question. Then $F K$ is a foreign key (with the same degree as $K$ ); $K$ (not $K^{\prime}$ ) is the corresponding target key; the associated constraint is a referential constraint; and R2 and R1 are the referencing relvar and the corresponding referenced relvar (or target relvar), respectively, for that constraint.

As an aside, I note that the relational model as originally formulated required foreign keys to correspond not just to some key, but very specifically to the primary key, of the referenced relvar. Since we don't insist on primary keys, however, we certainly can't insist that foreign keys correspond to primary keys specifically, and we don't (and SQL agrees with this position).

In the suppliers-and-parts database, to repeat, $\{\mathrm{SNO}\}$ and $\{\mathrm{PNO}\}$ are foreign keys in SP , referencing the sole candidate key-which we can therefore regard, harmlessly, as the primary key, if we want to-in S and P , respectively. Here now is a more complicated example:

[^53]```
VAR EMP BASE RELATION
    { ENO CHAR ,
        MNO CHAR ,
        ..... }
    KEY { ENO }
    FOREIGN KEY { MNO }
        REFERENCES EMP { ENO }
            RENAME { ENO AS MNO } ;
```

```
CREATE TABLE EMP
```

CREATE TABLE EMP

```
CREATE TABLE EMP
```

CREATE TABLE EMP
( ENO VARCHAR(6) NOT NULL ,
( ENO VARCHAR(6) NOT NULL ,
( ENO VARCHAR(6) NOT NULL ,
( ENO VARCHAR(6) NOT NULL ,
MNO VARCHAR(6) NOT NULL ,
MNO VARCHAR(6) NOT NULL ,
MNO VARCHAR(6) NOT NULL ,
MNO VARCHAR(6) NOT NULL ,
UNIQUE ( ENO ) ,
UNIQUE ( ENO ) ,
UNIQUE ( ENO ) ,
UNIQUE ( ENO ) ,
FOREIGN KEY ( MNO )
FOREIGN KEY ( MNO )
FOREIGN KEY ( MNO )
FOREIGN KEY ( MNO )
REFERENCES EMP ( ENO ) ) ;

```
                        REFERENCES EMP ( ENO ) ) ;
```

                        REFERENCES EMP ( ENO ) ) ;
    ```
                        REFERENCES EMP ( ENO ) ) ;
```

As you can see, there's a significant difference between the Tutorial D and SQL FOREIGN KEY specifications in this example. I'll explain the Tutorial D one first. Attribute MNO denotes the employee number of the manager of the employee identified by ENO; for example, the EMP tuple for employee E3 might include an MNO value of E2, which constitutes a reference to the EMP tuple for employee E2. So the referencing relvar ( $R 2$ in the definition) and the referenced relvar ( $R 1$ in the definition) are one and the same in this example. More to the point, foreign key values, like candidate key values, are tuples; so we have to do some renaming in the foreign key specification, in order for the tuple equality comparison to be at least syntactically valid. (What tuple equality comparison? Answer: The one that's implicit in the process of checking the foreign key constraint-recall that tuples must certainly be of the same type if they're to be tested for equality, and "same type" means they must have the same attributes and thus certainly the same attribute names.) That's why, in the Tutorial D specification, the target is specified not just as EMP but rather as EMP $\{E N O\}$ RENAME \{ENO AS MNO \}. Note: The RENAME operator is described in detail in the next chapter; for now, I'll just assume it's self-explanatory.

Turning now to SQL: In SQL the key $K$ in the referenced table $T 1$ and the corresponding foreign key $F K$ in the referencing table $T 2$ are sequences, not sets, of columns. (In other words, key and foreign key values in SQL are rows, not tuples, and left to right column ordering is significant once again.) Let those columns, in sequence as defined within the FOREIGN KEY specification in the definition of table $T 2$, be $B 1, B 2, \ldots, B n$ (for $F K$ ) and $A 1, A 2$, ..., $A n$ (for $K$ ), thus: ${ }^{6}$

```
FOREIGN KEY ( B1 , B2 , ..., Bn )
    REFERENCES T1 (A1 , A2 , ..., An )
```

Then columns Bi and $A i(1 \leq i \leq n)$ must be of the same type-no coercions here-but they don't have to have the same name. That's why the SQL specification

FOREIGN KEY ( MNO ) REFERENCES EMP ( ENO )
is sufficient as it stands, without any need for renaming.
Recommendation: Despite this last point, ensure that foreign key columns do have the same name in SQL as the corresponding key columns wherever possible (see the discussion of column naming in Chapter 3). However, there are certain situations-two of them, to be precise-in which this recommendation can't be followed 100 percent:

When some table $T$ has a foreign key corresponding to some key of $T$ itself (as in the EMP example)

[^54]When some table $T 2$ has two distinct foreign keys both corresponding to the same key $K$ in table $T 1$
Even here, however, you should at least try to follow the recommendation in spirit, as it were. For example, you might want to ensure in the second case that one of the foreign keys has the same column names as $K$, even though the other one doesn't (and can't). See Exercise 5.16 at the end of the chapter, and the answer to that exercise in Appendix F, for further discussion.

## Referential Actions

As you probably know, SQL supports not just foreign keys as such but also certain associated referential actions, such as CASCADE. Such actions can be specified as part of either an ON DELETE clause or an ON UPDATE clause. For example, the CREATE TABLE statement for shipments might include the following:

FOREIGN KEY ( SNO ) REFERENCES $S$ ( SNO ) ON DELETE CASCADE
Given this specification, an attempt to delete a specific supplier will cascade to delete all shipments for that supplier as well.

Now, referential actions might well be useful in practice, but they aren't part of the relational model as such. But that's not necessarily a problem! The relational model is the foundation of the database field, but it's only the foundation. In other words, there's no reason why additional features shouldn't be built on top of, or alongside, that foundation-just so long as those additions don't violate any of the prescriptions of the model (and are in the spirit of the model and can be shown to be useful, I suppose I should add). To elaborate:

- Type theory: Type theory provides the most obvious example of such an "additional feature." We saw in Chapter 2 that "types are orthogonal to tables," but we also saw that full and proper type support in relational systems-including support for user defined types, and perhaps even support for type inheritance-is highly desirable, to say the least. (In my own opinion, in fact, a system without such support scarcely deserves the label "relational." See Appendix A for further discussion.)

Triggered procedures: Strictly speaking, a triggered procedure is an action (the triggered action) to be performed if a specified event (the triggering event) occurs-but the term is often used loosely to include the triggering event as well. Referential triggered actions such as ON DELETE CASCADE are just a pragmatically important example of this more general construct, in which the action is DELETE (actually the "procedure" in this particular case is specified declaratively), and the triggering event is ON DELETE. ${ }^{7}$ No triggered procedures are prescribed by the relational model, but they aren't necessarily proscribed either-though they would be if they led to a violation of either the model's set level nature or The Assignment Principle, both of which they're likely to do in practice. Note: The combination of a triggering event and the corresponding triggered action is often known just as a trigger. Recommendation: As discussed earlier, avoid use of SQL's row level triggers, and don't use triggers of any kind in such a way as to violate The Assignment Principle.

- Recovery and concurrency: By way of a third example, the relational model has almost nothing to say about recovery and concurrency controls, but this fact obviously doesn't mean that relational systems shouldn't provide such controls. (Actually it could be argued that the relational model does say something about such

[^55]matters implicitly, because it does rely on the DBMS to implement updates properly and not to lose data-but it doesn't prescribe anything specific.)

One final remark to close this section: I've discussed foreign keys because they're of considerable pragmatic importance, also because they're part of the model as originally defined. But I'd like to stress the point that they're not truly fundamental-they're really just shorthand for certain integrity constraints that are commonly required in practice, as we'll see in Chapter 8. (In fact, much the same could be said for candidate keys as well, but in that case the practical benefits of providing a shorthand are overwhelming.)

## RELVARS AND PREDICATES

Now we come to what in many ways is the most important part of this chapter. The essence of it is this: There's another way to think about relvars. I mean, most people think of relvars as if they were just files in the traditional computing sense-rather abstract files, perhaps (disciplined might be a better word than abstract), but files nonetheless. But there's a different way to look at them, a way that I believe can lead to a much deeper understanding of what's really going on. It goes like this.

Consider the suppliers relvar S. Like all relvars, that relvar is supposed to represent some portion of the real world. In fact, I can be more precise: The heading of that relvar represents a certain predicate, meaning it's a kind of generic statement about some portion of the real world (it's generic because it's parameterized, as I'll explain in a moment). The predicate in question looks like this:

Supplier SNO is under contract, is named SNAME, has status STATUS, and is located in city CITY.
This predicate is the intended interpretation-in other words, the meaning, also called the intension (note the spelling)-for relvar S.

In general, you can think of a predicate as a truth valued function. Like all functions, it has a set of parameters; it returns a result when it's invoked; and (because it's truth valued) that result is either TRUE or FALSE. In the case of the predicate just shown, for example, the parameters are SNO, SNAME, STATUS, and CITY (corresponding of course to the attributes of the relvar), and they stand for values of the applicable types (CHAR, CHAR, INTEGER, and CHAR, respectively). When we invoke the function-when we instantiate the predicate, as the logicians say-we substitute arguments for the parameters. Suppose we substitute the arguments S1, Smith, 20, and London, respectively. Then we obtain the following statement:

Supplier S1 is under contract, is named Smith, has status 20, and is located in city London.
This statement is in fact a proposition, which in logic is something that's unequivocally either true or false. Here are a couple of examples:

1. Edward Abbey wrote The Monkey Wrench Gang.

## 2. William Shakespeare wrote The Monkey Wrench Gang.

The first of these is true and the second false. Don't fall into the common trap of thinking that propositions must always be true! However, the ones I'm talking about at the moment are supposed to be true ones specifically, as I now explain:

- First of all, every relvar has an associated predicate, called the relvar predicate for the relvar in question. (The predicate shown above is thus the relvar predicate for relvar S.)

■ Let relvar $R$ have predicate $P$. Then every tuple $t$ appearing in $R$ at some given time can be regarded as representing a certain proposition $p$, derived by invoking (or instantiating) $P$ at that time with the attribute values from $t$ as arguments.

- And (very important!) we assume by convention that each proposition $p$ that's obtained in this manner evaluates to TRUE.

Given our usual sample value for relvar S , for example, we assume the following propositions all evaluate to TRUE at this time:

Supplier S1 is under contract, is named Smith, has status 20, and is located in city London.
Supplier S2 is under contract, is named Jones, has status 10, and is located in city Paris.
Supplier S3 is under contract, is named Blake, has status 30, and is located in city Paris.
And so on. What's more, we go further: If at some given time $t$ a certain tuple plausibly could appear in some relvar but doesn't, then we assume the corresponding proposition is false at that time $t$. For example, the tuple

```
TUPLE { SNO 'S6' , SNAME 'Lopez' , STATUS 30 , CITY 'Madrid' }
```

is-let's agree-a plausible supplier tuple but doesn't appear in relvar $S$ at this time, and so we're entitled to assume it's not the case that the following proposition is true at this time:

Supplier S6 is under contract, is named Lopez, has status 30, and is located in city Madrid.
To sum up: A given relvar $R$ contains, at any given time, all and only the tuples that represent true propositions (true instantiations of the relvar predicate for $R$ ) at the time in question-or, at least, that's what we always assume in practice. In other words, in practice we adopt what's called The Closed World Assumption (see Appendixes A and C for more on this topic).

More terminology: Again, let $P$ be the relvar predicate, or intension, for relvar $R$, and let the value of $R$ at some given time be relation $r$. Then $r$-or the body of $r$, to be more precise-constitutes the extension of $P$ at that time. Note, therefore, that the extension for a given relvar varies over time, but the intension does not.

Two final points regarding terminology:

- You're probably familiar with the term predicate already, since SQL uses it extensively to refer to what this book calls a boolean expression (i.e., SQL talks about "comparison predicates," "IN predicates," "EXISTS predicates," and so on). Now, this usage on SQL's part isn't exactly incorrect, but it does usurp a very general term-one that's extremely important in relational contexts-and give it a rather specialized meaning, which is why I prefer not to follow that usage myself.
- Talking of usurping general terms and giving them specialized meanings, there's another potential confusion in this area. It has to do with the term statement. As you might have realized, logic uses this term in a sense that's very close to its natural language meaning. By contrast, programming languages give it a different and rather specialized meaning: They use it to mean a construct that causes some action to occur, such as
defining or updating a variable or changing the flow of control. And I'm afraid this book uses the term in both senses, relying on context to make it clear which meaning is intended. Caveat lector.


## RELATIONS vs. TYPES

Chapter 2 discussed types and relations, among other things. However, I wasn't in a position in that chapter to explain the most important logical difference between those two concepts-but now I am, and I will.

I've shown that the database at any given time can be thought of as a collection of true propositions: for example, the proposition Supplier S1 is under contract, is named Smith, has status 20, and is located in city London. More specifically, I've shown that the argument values appearing in such a proposition (S1, Smith, 20, and London, in the example) are, precisely, the attribute values from the corresponding tuple, where each such attribute value is a value of the associated type. It follows that:

## Types are sets of things we can talk about; relations are (true) statements we make about those things.

In other words, types give us our vocabulary-the things we can talk about-and relations give us the ability to say things about the things we can talk about. For example, if we limit our attention to suppliers only, for simplicity, we see that:

- The things we can talk about are character strings and integers-and nothing else. (In a real database, of course, our vocabulary will usually be much more extensive than this, especially if any user defined types are involved.)

The things we can say are things of the form "The supplier with the supplier number denoted by the specified character string is under contract; has the name denoted by another specified character string; has the status denoted by the specified integer; and is located in the city denoted by yet another specified character string"-and nothing else. (Nothing else, that is, except for things logically implied by things we can say explicitly. For example, given the things we already know we can say explicitly about supplier S1, we can also say things like Supplier S1 is under contract, is named Smith, has status 20, and is located in some city-where the city is left unspecified. (And if you're thinking that what I've just said is very reminiscent of, and probably has some deep connection to, relational projection ... well, you'd be absolutely right. See the section "What Do Relational Expressions Mean?" in Chapter 6 for further discussion.)

The foregoing state of affairs has at least three important corollaries. To be specific, in order to "represent some portion of the real world" (as I put it in the previous section):

1. Types and relations are both necessary-without types, we would have nothing to talk about; without relations, we couldn't say anything.
2. Types and relations are sufficient, as well as necessary-we don't need anything else, logically speaking. (Well, we do need relvars, in order to reflect the fact that the real world changes over time, but we don't need them to represent the situation at any given time. $)^{8}$

[^56]3. Types and relations aren't the same thing. Beware of anyone who tries to pretend they are! In fact, pretending a type is just a special kind of relation is precisely what certain products try to do (though it goes without saying that they don't usually talk in such terms)-and I hope it's clear that any product that's founded on such a logical error is doomed to eventual failure. (As a matter of fact, at least one of the products I have in mind here already has failed.) The products in question aren't relational products, though; typically, they're products that support "objects" in the object oriented sense, or products that try somehow to marry such objects and SQL tables. Further details of such products are beyond the scope of this book.

Here's a slightly more formal perspective on what I've been saying. As we've seen, a database can be thought of as a collection of true propositions. In fact, a database, together with the operators that apply to the propositions represented in that database (or sets of such propositions, rather), is a logical system. And by "logical system" here, I mean a formal system-like euclidean geometry, for example-that has axioms ("given truths") and rules of inference by which we can prove theorems ("derived truths") from those axioms. Indeed, it was Codd's very great insight, when he invented the relational model back in 1969, that a database (despite the name) isn't really just a collection of data; rather, it's a collection of facts, or in other words true propositions. Those propositions-the given ones, that is to say, which are the ones represented by the tuples in the base relvars-are the axioms of the logical system under discussion. And the inference rules are essentially the rules by which new propositions can be derived from the given ones; in other words, they're the rules that tell us how to apply the operators of the relational algebra. ${ }^{9}$ Thus, when the system evaluates some relational expression (in particular, when it responds to some query), it's really deriving new truths from given ones; in effect, it's proving theorems!

Once we understand the foregoing, we can see that the whole apparatus of formal logic becomes available for use in attacking "the database problem." In other words, questions such as

What should the database look like to the user?
What should integrity constraints look like?
What should the query language look like?
How can we best implement queries?
More generally, how can we best evaluate database expressions?
How should results be presented to the user?
How should we design the database in the first place?
(and others like them) all become, in effect, questions in logic that are susceptible to logical treatment and can be given logical answers.

It goes without saying that the relational model supports the foregoing perception very directly-which is why, in my opinion, that model is rock solid, and "right," and will endure. It's also why, again in my opinion, other data models are simply not in the same ballpark. Indeed, I seriously question whether those other data models deserve to be called models at all, in the same sense that the relational model does. Certainly most of them are ad

[^57]hoc to a degree, instead of being firmly founded, as the relational model is, on set theory and predicate logic. I'll expand on these issues in Appendix A.

## EXERCISES

5.1 It's sometimes suggested that a relvar is really just a traditional computer file, with tuples instead of records and attributes instead of fields. Discuss.
5.2 Explain in your own words why remarks like (for example) "This UPDATE operation updates the status for suppliers in London" aren't very precise. Give a replacement for that remark that's as precise as you can make it.

### 5.3 Why are SQL's "positioned update" operations a bad idea?

5.4 In Tutorial D, INSERT and D_INSERT are defined in terms of UNION and D_UNION, respectively, and DELETE and I_DELETE are defined in terms of MINUS and I_MINUS, respectively. In SQL, by contrast, INSERT is defined in terms of UNION ALL, and there's nothing analogous to D_INSERT. There's also nothing in SQL analogous to I_DELETE; but what about the regular SQL DELETE operator? How do you think that's defined?
5.5 Let the SQL base table SS have the same columns as table S. Consider the following SQL INSERT statements:

```
INSERT INTO SS ( SNO , SNAME , STATUS , CITY )
    ( SELECT SNO , SNAME , STATUS , CITY
        FROM S
        WHERE SNO = `S6' ) ;
INSERT INTO SS ( SNO , SNAME , STATUS , CITY ) VALUES
    ( SELECT SNO , SNAME , STATUS , CITY
        FROM S
        WHERE SNO = 'S6' ) ;
```

Are these statements logically equivalent? If not, what's the difference between them? Note: Thinking about Tutorial D analogs of the two statements might help you answer this question.
5.6 (This is essentially a repeat of Exercise 2.22 from Chapter 2, but you should be able to give a more comprehensive answer now.) State The Assignment Principle. Can you think of any situations in which SQL violates that principle? Can you identify any negative consequences of such violations?
5.7 Give definitions for SQL base tables corresponding to the TAX_BRACKET, ROSTER, and MARRIAGE relvars in the section "More on Candidate Keys."
5.8 Why doesn't it make sense to say a relation has a key?
5.9 In the body of the chapter, I gave one reason why key irreducibility is a good idea. Can you think of any others?

### 5.10 "Key values are not scalars but tuples." Explain this remark.

5.11 Let relvar $R$ be of degree $n$. What's the maximum number of keys $R$ can have?
5.12 What's the difference between a key and a superkey? And given that the superkey concept makes sense, do you think it would make sense to define any kind of subkey concept?
5.13 Relvar EMP from the section "More on Foreign Keys" is an example of what's sometimes called a selfreferencing relvar. Invent some sample data for that relvar. Do such relvars lead inevitably to a requirement for null support? (Answer: No, they don't, but they do serve to show how seductive the nulls idea can be.) What can be done in the example if nulls are prohibited?
5.14 Why doesn't SQL have anything analogous to Tutorial D's renaming option in its foreign key specifications?
5.15 Can you think of a situation in which two relvars $R 1$ and $R 2$ might each have a foreign key referencing the other? What are the implications of such a situation?
5.16 The well known bill of materials application involves a relvar-PP, say-showing which parts ("major" parts) contain which parts ("minor" parts) as immediate components, and showing also the corresponding quantities (e.g., "part P1 contains part P2 in quantity 4"). Of course, immediate components are themselves parts, and they can have further immediate components of their own. Give appropriate base relvar (Tutorial D) and base table (SQL) definitions. What referential actions do you think might make sense in this example?
5.17 Investigate any SQL product available to you. What referential actions does that product support? Which ones do you think are useful? Can you think of any others the product doesn't support but might be useful?
5.18 Define the terms proposition and predicate. Give examples.
5.19 State the predicates for relvars P and SP from the suppliers-and-parts database.
5.20 What do you understand by the terms intension and extension?
5.21 Let $D B$ be any database you happen to be familiar with and let $R$ be any relvar in $D B$. What's the predicate for $R$ ? Note: The point of this exercise is to get you to apply some of the ideas discussed in the body of this chapter to your own data, in an attempt to get you thinking about data in general in such terms. Obviously the exercise has no unique right answer.

### 5.22 Explain The Closed World Assumption in your own terms. Could there be such a thing as The Open World Assumption?

5.23 A key is a set of attributes and the empty set is a legitimate set; thus, we could define an empty key to be a key where the pertinent set of attributes is empty. What are the implications? Can you think of any uses for such a key?
5.24 A predicate has a set of parameters and the empty set is a legitimate set; thus, a predicate could have an empty set of parameters. What are the implications?
5.25 What's the predicate for a relvar of degree zero? (Does this question even make sense? Justify your answer.)
5.26 Every relvar has some relation as its value. Is the converse true?-that is, is every relation a value of some relvar?
5.27 In Chapter 1 I said I'd be indicating primary key attributes, in tabular pictures of relations, by double underlining. At that point, however, I hadn't discussed the logical difference between relations and relvars; and in this chapter we've seen that keys in general apply to relvars, not relations. Yet I've shown numerous tabular pictures in previous chapters that represent relations as such (I mean, relations that aren't just a sample value for some relvar), and I've certainly been using the double underlining convention in those pictures. So what can we say about that convention now?

## Chapter 6

## SQLand Relational Algebra I:

## The Original Operators

Join the union!
-Susan B. Anthony (1869)

This is the first of two chapters on the operators of the relational algebra; it discusses the original operators (i.e., the ones briefly described in Chapter 1) in depth, and it also examines certain ancillary but important issues-e.g., the significance of proper attribute (or column) naming once again. It also explains the implications of such matters for our overall goal of using SQL relationally.

## SOME PRELIMINARIES

Let me begin by reviewing a few points from Chapter 1. First, recall that each algebraic operator takes at least one relation as input and produces another relation as output. Second, recall too that the fact that the output is the same kind of thing as the input(s)-they're all relations-constitutes the closure property of the algebra, and it's that property that lets us write nested relational expressions. Third, I gave outline descriptions in Chapter 1 of what I called "the original operators" (restrict, project, product, intersect, union, difference, and join); now I'm in a position to define those operators, and others, much more carefully. Before I can do that, however, I need to make a few more general points:

- The operators of the algebra are generic: They apply, in effect, to all possible relations. For example, we don't need one specific join operator to join departments and employees and another, different, join operator to join suppliers and shipments. (Incidentally, do you think an analogous remark applies to object systems?).

■ The operators are also read-only: They "read" their operands and they return a result, but they don't update anything. In other words, they operate on relations, not relvars.

- Of course, the previous point doesn't mean that relational expressions can't refer to relvars. For example, if $R 1$ and $R 2$ are relvar names, then $R 1$ UNION $R 2$ is certainly a valid relational expression in Tutorial D (so long as the relvars denoted by those names are of the same type, that is). In that expression, however, RI and $R 2$ don't denote those relvars as such; rather, they denote the relations that happen to be the current values of those relvars at that time. In other words, we can certainly use a relvar name to denote a relation
operand-and such a relvar reference in itself thus constitutes a valid relational expression ${ }^{1}$-but in principle we could equally well denote the very same operand by means of an appropriate relation literal instead. ${ }^{2}$

An analogy might help clarify this latter point. Suppose N is a variable of type INTEGER, and at time $t$ it has the value 3 . Then $\mathrm{N}+2$ is certainly a valid expression, but at time $t$ it means exactly the same thing as $3+2$, no more and no less.

■ Finally, given that the operators of the algebra are indeed all read-only, it follows that INSERT, DELETE, and UPDATE (and relational assignment), though they're certainly relational operators, aren't relational algebra operators as such-though, regrettably, you'll often come across statements to the contrary in the literature.

I also need to say something here about the design of Tutorial D, because its support for the algebra in particular is significantly different from that of SQL. The overriding point is that, in operations like UNION or JOIN that need some correspondence to be established between operand attributes, Tutorial D does so by requiring the attributes in question to be, formally, the very same attribute (i.e., to have the same name and same type). For example, here's a Tutorial D expression for the join of parts and suppliers on cities:

```
P JOIN S
```

The join operation here is performed, by definition, on the basis of part and supplier cities, CITY being the sole attribute that P and S have in common (i.e., the sole common attribute).

To repeat, Tutorial D establishes the correspondence between operand attributes, when such a correspondence is required, by insisting that the attributes in question in fact be the very same attribute. And it applies this same technique uniformly and consistently across the board, in all pertinent contexts. By contrast, SQL uses different techniques in different contexts. Sometimes it uses ordinal position (we've already seen an example of this case in connection with foreign keys, as discussed in the previous chapter). Sometimes it uses explicit specification. Sometimes it requires the attributes in question (or columns, rather) to have the same name-and then the correspondence is sometimes established explicitly, sometimes implicitly. And regardless of whether it requires the columns in question to have the same name, sometimes it requires those columns to be of the same type, and sometimes it doesn't. In order to illustrate some but not all of these possibilities, let's consider the P JOIN S example again. Here's one possible formulation of that join in SQL:

```
SELECT P.PNO , P.PNAME , P.COLOR , P.WEIGHT , P.CITY
                                    /* or S.CITY */ ,
    S.SNO , S.SNAME , S.STATUS
FROM P , S
WHERE P.CITY = S.CITY
```

In this formulation, the required column correspondence is specified explicitly in the WHERE clause. As you probably know, however, examples like this one can in fact be formulated in several different ways in SQL. Here are three more formulations for the case at hand (as you can see, the second and third are a little closer to the spirit of Tutorial D): ${ }^{3}$

[^58]```
SELECT P.PNO , P.PNAME , P.COLOR , P.WEIGHT , P.CITY
    S.SNO , S.SNAME , S.STATUS
FROM P JOIN S
ON P.CITY = S.CITY
SELECT P.PNO , P.PNAME , P.COLOR , P.WEIGHT , CITY ,
    S.SNO , S.SNAME , S.STATUS
FROM P JOIN S
USING ( CITY )
SELECT P.PNO , P.PNAME , P.COLOR , P.WEIGHT , CITY ,
    S.SNO , S.SNAME , S.STATUS
FROM P NATURAL JOIN S
```

Observe now that:

- In the first of these three formulations, the column correspondence is again specified explicitly, but this time by means of an ON clause instead of a WHERE clause.
- In the second formulation, the correspondence is based on common column names, but it's still specified explicitly, by means of the USING clause.
- In the third formulation, the correspondence is again based on common column names, but this time it's implicit.

Now I'd like to go back to the SQL formulation I gave first of all, partly because it was the only one supported in SQL as originally defined and partly, and more importantly, because it allows me to make a number of additional points concerning differences between SQL and Tutorial D:

- SQL permits, and sometimes requires, dot qualified names. Tutorial D doesn't. Note: I'll have more to say about SQL's dot qualified names in Chapter 12.
- Tutorial D sometimes needs to rename attributes in order to avoid what would otherwise be naming clashes or mismatches. SQL usually doesn't (though it does support an analog of the RENAME operator that Tutorial D uses for the purpose, as we'll see in the next section).
- Partly as a consequence of the previous point, Tutorial D has no need for SQL's "correlation name" concept; in effect, it replaces that concept by the idea that attributes sometimes need to be renamed, as previously explained. Note: I'll be discussing SQL's correlation names in detail in Chapter 12.
- As well as either explicitly or implicitly supporting certain features of the relational algebra, SQL also explicitly supports certain features of the relational calculus (correlation names are a case in point, and EXISTS is another). Tutorial $\mathbf{D}$ doesn't. One consequence of this difference is that SQL is a highly redundant language, in that it typically provides numerous different ways of formulating the same query, a fact that can have serious negative implications for both the user and the optimizer. (I once wrote a paper on this topic called "Fifty Ways to Quote Your Query"-see Appendix G-in which I showed that even a query
as simple as "Get names of suppliers who supply part P2" can be expressed in well over 50 different ways in SQL.)

SQL requires most queries to conform to its SELECT - FROM - WHERE template. Tutorial D has no analogous requirement. Note: I'll have more to say on this particular issue in the next chapter.

In what follows, I'll show examples in both Tutorial D and SQL.

## MORE ON CLOSURE

To say it again, the result of every relational operation is a relation. Conversely, any operator that produces a result that isn't a relation is, by definition, not a relational operator. ${ }^{4}$ For example, any operator that produces an ordered result isn't a relational operator (see the discussion of ORDER BY in the next chapter). And in SQL in particular, the same is true of any operator that produces a result with duplicate rows, or left to right column ordering, or nulls, or anonymous columns, or duplicate column names. Closure is crucial! As I've already said, closure is what makes it possible to write nested expressions in the relational model, and (as we'll see later) it's also important in expression transformation, and hence in optimization. Strong recommendation: Don't use any operation that violates closure if you want the result to be amenable to further relational processing.

Now, when I say the result of every algebraic operation is another relation, I hope it's clear that I'm talking from a conceptual point of view; I don't mean the system always has to materialize those results in their entirety. For example, consider the following expression (a restriction of a join-Tutorial $\mathbf{D}$ on the left and SQL on the right as usual, and I've deliberately shown all name qualifications explicitly in the SQL version): ${ }^{5}$

```
( P JOIN S )
WHERE PNAME > SNAME
\begin{tabular}{ll} 
SELECT & P.*, S.SNO, S.SNAME , S.STATUS \\
FROM & P,, S \\
WHERE & P.CITY \(=\) S.CITY \\
AND & P.PNAME \(>\) S.SNAME
\end{tabular}
```

Clearly, as soon as any given tuple of the join is formed, the system can test that tuple right away against the restriction condition PNAME $>$ SNAME (P.PNAME $>\mathrm{S}$. SNAME in the SQL version) to see if it belongs in the final output, discarding it if not. Thus, the intermediate result that's the output from the join might never have to exist as a fully materialized relation in its own right at all. In practice, in fact, the system tries very hard not to materialize intermediate results in their entirety, for obvious performance reasons. (As an aside, I remark that the process by which tuples of an intermediate result are produced and passed on to another operation one at a time instead of en bloc is sometimes referred to as pipelining.)

The foregoing example raises another point, however. Consider the boolean expression PNAME $>$ SNAME in the Tutorial D version. That expression applies, conceptually, to the result of P JOIN S, and the attribute names PNAME and SNAME in that expression therefore refer to attributes of that result-not to the attributes of the same names in relvars P and S . But how do we know that result has any such attributes? What is the heading of that result? More generally, how do we know what the heading is for the result of any algebraic operation? Clearly,

[^59]what we need is a set of rules-to be specific, relation type inference rules-such that if we know the headings (and therefore the types) of the input relations for an operation, we can infer the heading (and therefore the type) of the output relation from that operation. And the relational model does include such a set of rules. In the case of join, for example, those rules say the output from P JOIN S is of this type:

```
RELATION { PNO CHAR , PNAME CHAR , COLOR CHAR , WEIGHT RATIONAL ,
    CITY CHAR , SNO CHAR , SNAME CHAR , STATUS INTEGER }
```

In fact, for join, the heading of the output is the union of the headings of the inputs (where by union I mean the regular set theory union, not the special relational union I'll be discussing later in this chapter). In other words, the output has all of the attributes of the inputs, except that common attributes-just CITY in the example-appear once, not twice, in that output. Of course, those attributes don't have any left to right order, so I could equally well say the type of the result of P JOIN S is (for example):

```
RELATION { SNO CHAR , PNO CHAR , SNAME CHAR , WEIGHT RATIONAL ,
    CITY CHAR , STATUS INTEGER , PNAME CHAR , COLOR CHAR }
```

Note that type inference rules of some kind are definitely needed in order to support the closure property fully-closure says every result is a relation, and relations have a heading as well as a body; thus, every result must have a proper relational heading as well as a proper relational body.

Now, the RENAME operator mentioned in the previous section is needed in large part because of the foregoing type inference rules; it allows us to perform, e.g., a join, even when the relations involved don't meet the attribute naming requirements for that operation (speaking a trifle loosely). Here's the definition:

Definition: Let $r$ be a relation and let $A$ be an attribute of $r$. Then the (attribute) renaming $r$ RENAME $\{A$ AS $B\}$ is a relation with (a) heading identical to that of $r$ except that attribute $A$ in that heading is renamed $B$ and (b) body identical to that of $r$ (except that references to $A$ in that body are replaced by references to $B$, a nicety that can be ignored for present purposes). Note: I assume for simplicity that relation $r$ doesn't already have an attribute named $B$.

For example:

```
S RENAME { CITY AS SCITY }
\begin{tabular}{ll} 
SELECT & SNO, SNAME , STATUS , \\
S.CITY AS SCITY \\
FROM & S
\end{tabular}
```

Given our usual sample values, the result looks like this (it's identical to our usual suppliers relation, except that the city attribute is called SCITY):

| SNO | SNAME | STATUS | SCITY |
| :---: | :---: | ---: | :--- |
| S1 | Smith | 20 | London |
| S2 | Jones | 10 | Paris |
| S3 | Blake | 30 | Paris |
| S4 | Clark | 20 | London |
| S5 | Adams | 30 | Athens |

Note: I won't usually bother to show results explicitly in this chapter unless I think the particular operator I'm talking about might be unfamiliar to you, as in the case at hand.

Important: The foregoing example does not change relvar S in the database! RENAME isn't like SQL's ALTER TABLE; the RENAME invocation is only an expression (just as, for example, P JOIN S or $\mathrm{N}+2$ are only expressions), and like any expression it simply denotes a value. What's more, since it is an expression, not a statement or "command," it can be nested inside other expressions. We'll see plenty of examples of such nesting later.

So how does SQL handle this business of result type inference? The answer is: Not very well. First of all, as we saw in Chapter 3, it doesn't really have a notion of "relation type" (or table type, rather) anyway. Second, it can produce results with columns that effectively have no name at all (for example, consider SELECT PNO, 2 * WEIGHT FROM P). Third, it can also produce results with duplicate column names (for example, consider SELECT DISTINCT P.CITY, S.CITY FROM P, S). Strong recommendation: Follow the column naming discipline from Chapter 3 wherever necessary to ensure that SQL conforms as far as possible to the relational rules described in this chapter. Just to remind you, that discipline involved using AS specifications to give proper column names to columns that otherwise (a) wouldn't have a name at all or (b) would have a name that wasn't unique. My SQL examples in this chapter and the next (indeed, throughout the rest of this book) will all abide by this discipline.

I haven't finished with the example from the beginning of this section. Here it is again:

```
( P JOIN S ) SELECT P.* , S.SNO , S.SNAME , S.STATUS
WHERE PNAME > SNAME
\begin{tabular}{ll} 
SELECT P.* & , S.SNO, S.SNAME , S.STATUS \\
FROM & P, S \\
WHERE & P.CITY \(=\) S.CITY \\
AND & P.PNAME \(>\) S.SNAME
\end{tabular}
```

As you can see, the counterpart in the SQL version to Tutorial D's PNAME > SNAME is P.PNAME > S.SNAME (note the "P." and "S." qualifiers)-which is curious when you come to think about it, because that expression is supposed to apply to the result of the FROM clause (see the section "Evaluating SQL Expressions," later), and tables P and S certainly aren't part of that result! Indeed, it's quite difficult to explain how references to the names P and S in the WHERE and SELECT clauses (and possibly elsewhere in the overall expression) can make any sense at all in terms of the result of the FROM clause. The SQL standard does explain it, but the machinations it has to go through in order to do so are much more complicated than Tutorial D's type inference rules-so much so that I won't even try to explain them here, but will simply rely on the fact that they can be explained if necessary. I justify this omission by appealing to the fact that you're supposed to be familiar with SQL already. It's tempting to ask, though, whether you had ever thought about this issue before ... but I won't.

Now I can go on to describe some other algebraic operators. Please note that I'm not trying to be exhaustive in this chapter (or the next); I won't be covering "all known operators," and I won't even describe all of the operators I do cover in full generality. In most cases, in fact, I'll just give a careful but somewhat informal definition and show some simple examples.

## RESTRICTION

Definition: Let $r$ be a relation and let $b x$ be a boolean expression in which every attribute reference identifies some attribute of $r$ and there aren't any relvar references. Then $b x$ is a restriction condition, and the restriction of $r$ according to $b x, r$ WHERE $b x$, is a relation with (a) heading the same as that of $r$ and (b) body consisting of all tuples of $r$ for which $b x$ evaluates to TRUE.

For example:
P WHERE WEIGHT < 17.5
SELECT *
FROM P
WHERE WEIGHT < 17.5

Let $r$ be a relation. Then the restriction $r$ WHERE TRUE (or, more generally, any expression of the form $r$ WHERE $b x$ where $b x$ is a boolean expression such as $1=1$ that's identically TRUE) ${ }^{6}$ just returns $r$. Such a restriction is known as an identity restriction.

Note: Tutorial D does support expressions of the form $r$ WHERE $b x$, of course, but those expressions aren't limited to being simple restrictions as defined above, because the boolean expression $b x$ isn't limited to being a restriction condition but can be more general. Similar remarks apply to SQL also. Examples are given in later chapters.

As an aside, I remark that restrict is sometimes called select; I prefer not to use this term, however, because of the potential confusion with SQL's SELECT operator. SQL's SELECT operator-meaning, more precisely, the SELECT clause portion of a SELECT expression-isn't restriction at all but is, rather, a kind of loose combination of UNGROUP, EXTEND, RENAME, and "project" ("project" in quotes because it doesn't eliminate duplicates unless explicitly asked to do so). Note: UNGROUP and EXTEND are described in the next chapter.

## PROJECTION

Definition: Let $r$ be a relation and let $A, B, \ldots, C$ be attributes of $r$. Then the projection of $r$ on (or over) those attributes, $r\{A, B, \ldots, C\}$, is a relation with (a) heading $\{A, B, \ldots, C\}$ and (b) body the set of all tuples $x$ such that there exists some tuple $t$ in $r$ with $A$ value equal to the $A$ value in $x, B$ value equal to the $B$ value in $x, \ldots$, and $C$ value equal to the $C$ value in $x$.

For example:

```
P { COLOR , CITY } SELECT DISTINCT COLOR , CITY
FROM P
```

To repeat, the result is a relation; thus, "duplicates are eliminated," to use the common phrase, and that DISTINCT in the SQL formulation is really needed, therefore. ${ }^{7}$ The result heading has attributes (or columns) COLOR and CITY-in that left to right order, in SQL.

Let $r$ be a relation. Then:
The projection $r\{H\}$, where $\{H\}$ is all of the attributes-in other words, the heading-of $r$, just returns $r$. Such a projection is known as an identity projection.

- The projection $r\}$-in other words, the projection of $r$ on no attributes at all-returns TABLE_DEE if $r$ is nonempty, TABLE_DUM otherwise. Such a projection is sometimes called a nullary projection; however, the term nullary is best avoided because of the potential confusion with SQL-style nulls. (Just to remind you, TABLE_DEE is the unique relation with no attributes and just one tuple-the 0 -tuple, of course-and TABLE_DUM is the unique relation with no attributes and no tuples at all. The fact that projecting $r$ on no attributes always yields one of these two relations is a direct consequence of the fact that every tuple has the same value for the empty set of attributes: namely, the 0 -tuple. See the answer to Exercise 3.16 in Appendix F if you need to refresh your memory regarding this point.)

[^60]
## 112 Chapter 6 / SQL and Relational Algebra I

Tutorial D also allows a projection to be expressed in terms of the attributes to be removed instead of the ones to be kept. Thus, for example, the Tutorial D expressions

```
P { COLOR , CITY } and P { ALL BUT PNO , PNAME , WEIGHT }
```

are equivalent. This feature can save a lot of writing (think of projecting a relation of degree 100 on 99 of its attributes). ${ }^{8}$ Analogous remarks apply, where they make sense, to all of the operators in Tutorial D.

In concrete syntax, it turns out to be convenient to assign high precedence to the projection operator. In Tutorial D, for example, we take the expression

```
P JOIN S { CITY }
```

to mean

```
P JOIN ( S { CITY } )
```

and not

```
( P JOIN S ) { CITY }
```

Exercise: Show the difference between these two interpretations, given our usual sample data.

## JOIN

Before I get to the join operator as such, it's helpful to introduce the concept of "joinability." Relations $r 1$ and $r 2$ are joinable if and only if attributes with the same name are of the same type (meaning they are in fact the very same attribute)-equivalently, if and only if the set theory union of the headings of $r 1$ and $r 2$ is itself a legal heading. Note that this concept applies not just to join as such but to various other operations as well, as we'll see in the next chapter. Anyway, armed with this notion, I can now define the join operation (note how the definition appeals to the fact that tuples are sets and hence can be operated upon by set theory operators such as union):

Definition: Let relations $r 1$ and $r 2$ be joinable. Then their natural join (or just join for short), $r 1$ JOIN $r 2$, is a relation with (a) heading the set theory union of the headings of $r 1$ and $r 2$ and (b) body the set of all tuples $t$ such that $t$ is the set theory union of a tuple from $r l$ and a tuple from $r 2$.

The following example is repeated from the section "Some Preliminaries," except that now I've dropped the explicit name qualifiers in the SQL version where they aren't needed:

P JOIN S

```
SELECT PNO , PNAME , COLOR , WEIGHT ,
    P.CITY /* or S.CITY */ ,
    SNO , SNAME , STATUS
FROM P , S
WHERE P.CITY = S.CITY
```

[^61]I remind you, however, that SQL also allows this join to be expressed in an alternative style that's a little closer to that of Tutorial D (and this time I deliberately replace that long commalist of column references in the SELECT clause by a simple "*"):

```
SELECT *
FROM P NATURAL JOIN S
```

The result heading, given this latter formulation, has attributes-or columns, rather-CITY, PNO, PNAME, COLOR, WEIGHT, SNO, SNAME, and STATUS (in that order in SQL, though not of course in the Tutorial D analog).

There are several more points to be made in connection with the natural join operation. First of all, observe that intersection is a special case (i.e., $r 1$ INTERSECT $r 2$ is a special case of $r 1$ JOIN $r 2$, in Tutorial D terms). To be specific, it's the special case in which relations $r 1$ and $r 2$ aren't merely joinable but are actually of the same type (i.e., have the same heading). For example, the following expressions are logically equivalent:

```
P { CITY } INTERSECT S { CITY }
P { CITY } JOIN S { CITY }
```

However, I'll have more to say about INTERSECT as such later in this chapter.
Next, product is a special case, too (i.e., $r 1$ TIMES $r 2$ is a special case of $r 1$ JOIN $r 2$, in Tutorial D terms). To be specific, it's the special case in which relations $r 1$ and $r 2$ have no attribute names in common. Why? Because, in this case, (a) the set of common attributes is empty; (b) as noted earlier, every possible tuple has the same value for the empty set of attributes (namely, the 0 -tuple); thus, (c) every tuple in $r 1$ joins to every tuple in $r 2$, and so we get the product as stated. For example, the following expressions are logically equivalent:

```
P { ALL BUT CITY } TIMES S { ALL BUT CITY }
P { ALL BUT CITY } JOIN S { ALL BUT CITY }
```

For completeness, however, I'll give the definition anyway:
Definition: The cartesian product (or just product for short) of relations $r 1$ and $r 2, r 1$ TIMES $r 2$, where $r 1$ and $r 2$ have no common attribute names, is a relation with (a) heading the set theory union of the headings of $r 1$ and $r 2$ and (b) body the set of all tuples $t$ such that $t$ is the set theory union of a tuple from $r 1$ and a tuple from $r 2$.

Here's an example:

```
( P RENAME { CITY AS PCITY } )
    TIMES /* or JOIN */
( S RENAME { CITY AS SCITY } )
```

```
SELECT PNO , PNAME , COLOR ,
```

SELECT PNO , PNAME , COLOR ,
WEIGHT , P.CITY AS PCITY ,
WEIGHT , P.CITY AS PCITY ,
SNO , SNAME , STATUS ,
SNO , SNAME , STATUS ,
S.CITY AS SCITY
S.CITY AS SCITY
FROM P , S

```
FROM P , S
```

Note the need to rename at least one of the two CITY attributes in this example. The result heading has attributes or columns PNO, PNAME, COLOR, WEIGHT, PCITY, SNO, SNAME, STATUS, and SCITY (in that order, in SQL).

## 114 Chapter 6 / SQL and Relational Algebra I

Last, join is usually thought of as a dyadic operator specifically; however, it's possible, and useful, to define an $n$-adic version of the operator (and Tutorial $\mathbf{D}$ does), according to which we can write expressions of the form

```
JOIN {r1 , r2 , ... , rn }
```

to join any number of relations $r 1, r 2, \ldots, r n .^{9}$ For example, the join of parts and suppliers could alternatively be expressed as follows:

```
JOIN { P , S }
```

What's more, we can use this syntax to ask for "joins" of just a single relation, or even of no relations at all! The join of a single relation $r$, JOIN $\{r\}$, is just $r$ itself; this case is perhaps not of much practical importance (?). Perhaps surprisingly, however, the join of no relations at all, JOIN $\}$, is very important indeed!-and the result is TABLE_DEE. (Recall once again that TABLE_DEE is the unique relation with no attributes and just one tuple.) Why is the result TABLE_DEE? Well, consider the following:

- In ordinary arithmetic, 0 is what's called the identity (or identity value) with respect to " + "; that is, for all numbers $x$, the expressions $x+0$ and $0+x$ are both identically equal to $x$. As a consequence, the sum of no numbers is $0 .{ }^{10}$ (To see this claim is reasonable, consider a piece of code that computes the sum of $n$ numbers by initializing the sum to 0 and then iterating over those $n$ numbers. What happens if $n=0$ ?)

In like manner, 1 is the identity with respect to "*"; that is, for all numbers $x$, the expressions $x * 1$ and $1 * x$ are both identically equal to $x$. As a consequence, the product of no numbers is 1 .

In the relational algebra, TABLE_DEE is the identity with respect to JOIN; that is, for all relations $r$, the expressions $r$ JOIN TABLE_DEE and TABLE_DEE JOIN $r$ are both identically equal to $r$ (see the paragraph immediately following). As a consequence, the join of no relations is TABLE_DEE.

If you're having difficulty with this idea, don't worry about it too much for now. But if you come back to reread this section later, I do suggest you try to convince yourself that $r$ JOIN TABLE_DEE and TABLE_DEE JOIN $r$ are indeed both identically equal to $r$. It might help to point out that the joins in question are actually cartesian products (right?).

## Explicit JOINs in SQL

In SQL, the keyword JOIN can be used to express various kinds of join operations (although those operations can always be expressed without it, too). Simplifying slightly, the possibilities-I've numbered them for purposes of subsequent reference-are as follows ( $t 1$ and $t 2$ are tables, denoted by table expressions $t x l$ and $t x 2$, say; $b x$ is a boolean expression; and $C 1, C 2, \ldots, C n$ are columns appearing in both $t 1$ and $t 2$ ):

1. $t 1$ NATURAL JOIN $t 2$
2. $t 1$ JOIN $t 2$ ON bx

[^62]3. t1 JOIN t2 USING ( $C 1, C 2, \ldots, C n)$
4. $t 1$ CROSS JOIN t2

I'll elaborate on the four cases briefly, since the differences between them are a little subtle and can be hard to remember:

1. Case 1 has effectively already been explained. Note: Actually, Case 1 is logically identical to a Case 3 expression-see below-in which the specified columns $C 1, C 2, \ldots, C n$ are all of the common columns (i.e., all of the columns that appear in both $t 1$ and $t 2$ ), in the order in which they appear in $t 1$.
2. Case 2 is logically equivalent to the following:
```
( SELECT * FROM t1 , t2 WHERE bx )
```

3. Case 3 is logically equivalent to a Case 2 expression in which $b x$ takes the form
```
t1.C1 = t2.C1 AND t1.C2 = t2.C2 AND ... AND t1.Cn = t2.Cn
```

-except that columns $C 1, C 2, \ldots, C n$ appear once, not twice, in the result, and the column ordering in the heading of the result is (in general) different: Columns $C 1, C 2, \ldots, C n$ appear first, in that order; then the other columns of $t 1$ appear, in the order in which they appear in $t 1$; then the other columns of $t 2$ appear, in the order in which they appear in $t 2$. (Do you begin to see what a pain this left to right ordering business is?)
4. Finally, Case 4 is logically equivalent to the following:

```
( SELECT * FROM t1 , t2 )
```


## Recommendations:

1. Use Case 1 (NATURAL JOIN) in preference to other methods of formulating a join (but make sure columns with the same name are of the same type). Note that the NATURAL JOIN formulation will often be the most succinct if other recommendations in this book are followed. ${ }^{11}$
2. Avoid Case 2 (JOIN ON), because it's guaranteed to produce a result with duplicate column names (unless tables $t 1$ and $t 2$ have no common column names in the first place). But if you really do want to use Case $2-$ which you just might, if you want to formulate a greater-than join, say ${ }^{12}$-then make sure columns with the same name are of the same type, and make sure you do some appropriate renaming as well. For example:
[^63]
## 116 Chapter 6 / SQL and Relational Algebra I

```
SELECT TEMP.*
FROM ( SELECT * FROM S JOIN P ON S.CITY > P.CITY ) AS TEMP
    ( SNO , SNAME , STATUS , SCITY ,
        PNO , PNAME , COLOR , WEIGHT , PCITY )
```

It's not really clear why you'd ever want to use such a formulation, however, given that it's logically equivalent to the following slightly less cumbersome one:

```
SELECT SNO , SNAME , STATUS , S.CITY AS SCITY ,
    PNO , PNAME , COLOR , WEIGHT , P.CITY AS PCITY
FROM S , P
WHERE S.CITY > P.CITY
```

3. In Case 3, make sure columns with the same name are of the same type.
4. In Case 4, make sure there aren't any common column names.

Recall finally that as noted in Chapter 1 an explicit JOIN invocation isn't allowed in SQL as a "stand alone" table expression (i.e., one at the outermost level of nesting). Nor is it allowed as the table expression in parentheses that constitutes a subquery (see Chapter 12).

## UNION, INTERSECTION, AND DIFFERENCE

Union, intersection, and difference (UNION, INTERSECT, and MINUS in Tutorial D; UNION, INTERSECT, and EXCEPT in SQL) all follow the same general pattern. I'll start with union.

## Union

Definition: Let relations $r 1$ and $r 2$ be of the same type; then their union, $r 1$ UNION $r 2$, is a relation of the same type, with body consisting of all tuples $t$ such that $t$ appears in $r 1$ or $r 2$ or both.

For example (I'll assume for the sake of all of the examples in this section that parts have an extra attribute called STATUS, of type INTEGER):

```
P { STATUS , CITY } UNION
S { CITY , STATUS }
```

```
SELECT STATUS , CITY
FROM P
UNION CORRESPONDING
SELECT CITY , STATUS
FROM S
```

As with projection, it's worth noting explicitly in connection with union that "duplicates are eliminated." Note that we don't need to specify DISTINCT in the SQL version in order to achieve this effect; although UNION provides the same options as SELECT does (DISTINCT vs. ALL), the default for UNION is DISTINCT, not ALL (for SELECT it's the other way around, as you'll recall from Chapter 4). The result heading has attributes or columns STATUS and CITY-in that order, in SQL. As for the CORRESPONDING specification in the SQL formulation, that specification allows us to ignore the possibility that those columns might appear at different ordinal positions within the operand tables. Recommendations:

Make sure every column of the first operand table has the same name and type as some column of the second operand table and vice versa. ${ }^{13}$

Always specify CORRESPONDING if possible. ${ }^{14}$ If it isn't-in particular, if the SQL product you're using doesn't support it-then make sure columns line up properly, as in this revised version of the example:

```
SELECT STATUS , CITY FROM P
UNION
SELECT STATUS , CITY FROM S /* note the reordering */
```

- Don't include the "BY (column name commalist)" option in the CORRESPONDING specification, unless it makes no difference anyway (e.g., specifying BY (STATUS,CITY) would make no difference in the example). ${ }^{15}$ Note: This recommendation is perhaps a little debatable. At least the BY option might sometimes save keystrokes (though not always-see the example below). But it's misleading, because it means the union operands aren't the specified tables as such but certain projections of those tables; it's also unnecessary, because those projections could always be specified explicitly anyway. For example, the SQL expression

```
SELECT * FROM P
UNION CORRESPONDING BY ( CITY )
SELECT * FROM S
```

is logically equivalent to this (shorter!) one:

```
SELECT CITY FROM P
UNION
SELECT CITY FROM S
```

- Never specify ALL. Note: The usual reason for specifying ALL on UNION isn't that users want to see duplicate rows in the output; rather, it's that they know there aren't any duplicate rows in the input-i.e., the union is disjoint (see below)-and so they're trying to prevent the system from having to do the extra work of trying to eliminate duplicates that they know aren't there in the first place. In other words, it's a performance reason. See the discussion of such matters in Chapter 4, in the section "Avoiding Duplicates in SQL."

Tutorial D also supports "disjoint union" (D_UNION), which is a version of union that requires its operands to have no tuples in common. For example:

```
S { CITY } D_UNION P { CITY }
```

Given our usual sample data, this expression will produce a run time error, because supplier cities and part cities aren't disjoint. SQL has no direct counterpart to D_UNION.

Tutorial D also supports $n$-adic forms of both UNION and D_UNION. The syntax consists-with one small exception, explained below-of the operator name (i.e., UNION or D_UNION), followed by a commalist in braces

[^64]of relational expressions $r 1, r 2, \ldots, r n$. The relations denoted by $r 1, r 2, \ldots, r n$ must all be of the same type. For example, the foregoing D_UNION example could alternatively be expressed as follows:

```
D_UNION { S { CITY } , P { CITY } }
```

Note: The union or disjoint union of a single relation $r$ is just $r$. The union or disjoint union of no relations at all is the empty relation of the pertinent type-but that type needs to be specified explicitly, since there aren't any relational expressions from which the type can be inferred. Thus, for example, the expression

```
UNION { SNO CHAR , STATUS INTEGER } { }
```

denotes the empty relation of type RELATION \{SNO CHAR, STATUS INTEGER\}. Compare the answer to Exercise 3.15 in Chapter 3 (see Appendix F).

## Intersection

Definition: Let relations $r 1$ and $r 2$ be of the same type; then their intersection, $r 1$ INTERSECT $r 2$, is a relation of the same type, with body consisting of all tuples $t$ such that $t$ appears in both $r 1$ and $r 2$.

For example:

```
P { STATUS , CITY } INTERSECT
S { CITY , STATUS }
```

| SELECT | STATUS , CITY |
| :--- | :--- |
| FROM | P |
| INTERSECT | CORRESPONDING |
| SELECT | CITY, STATUS |
| FROM | S |

All comments and recommendations noted under "Union" apply here also, mutatis mutandis. Note: As we've already seen, intersect is really just a special case of join. Tutorial D and SQL both support it, however, if only for psychological reasons. As mentioned in a footnote earlier, Tutorial D also supports an $n$-adic form, but I'll skip the details here.

## Difference

Definition: Let relations $r 1$ and $r 2$ be of the same type; then their difference, $r 1$ MINUS $r 2$ (in that order), is a relation of the same type, with body consisting of all tuples $t$ such that $t$ appears in $r l$ and not $r 2$.

For example:

```
P { STATUS , CITY } MINUS
S { CITY , STATUS }
```

```
SELECT STATUS , CITY
FROM P
EXCEPT CORRESPONDING
SELECT CITY , STATUS
FROM S
```

All comments and recommendations noted under "Union" apply here also, mutatis mutandis. Note, however, that minus is strictly dyadic-Tutorial D doesn't support any kind of " $n$-adic minus" operation (see Exercise 6.17 at the end of the chapter). But it does support "included minus" (I_MINUS), which is a version of minus that requires the second operand to be included in the first (i.e., the second operand mustn't have any tuples that aren't also in the first operand). For example:

```
S { CITY } I_MINUS P { CITY }
```

Given our usual sample data, this expression will produce a run time error, because there's at least one part city that isn't also a supplier city. SQL has no direct counterpart to I_MINUS.

## WHICH OPERATORS ARE PRIMITIVE?

I've now covered all of the operators I want to cover in this chapter. As I've more or less said already, however, not all of those operators are primitive-some of them can be defined in terms of others. One possible primitive set is the set \{restrict, project, join, union, difference\}; another can be obtained by replacing join in this set by product. Note: You might be surprised to see no mention here of rename. In fact, however, rename isn't primitive, though I haven't covered enough groundwork yet to show why not (see Exercise 7.3 in Chapter 7). What this discussion does show, however, is that there's a difference between being primitive and being useful! I certainly wouldn't want to be without our useful rename operator, even if it isn't primitive.

## FORMULATING EXPRESSIONS ONE STEP AT A TIME

Consider the following Tutorial D expression (the query is "Get pairs of supplier numbers such that the suppliers concerned are colocated-i.e., are in the same city"):

```
( ( ( S RENAME { SNO AS SA } ) { SA , CITY } JOIN
    ( S RENAME { SNO AS SB } ) { SB , CITY } )
    WHERE SA < SB ) { SA , SB }
```

The result has two attributes, called SA and SB (it would have been sufficient to do just one attribute renaming; I did two for symmetry). The purpose of the condition $\mathrm{SA}<\mathrm{SB}$ is twofold: ${ }^{16}$

- It eliminates pairs of supplier numbers of the form $(a, a)$.

■ It guarantees that the pairs $(a, b)$ and $(b, a)$ won't both appear.
Be that as it may, I now show another formulation of the query in order to show how Tutorial D's WITH construct can be used to simplify the business of formulating what might otherwise be rather complicated expressions:

```
WITH ( R1 := ( S RENAME { SNO AS SA } ) { SA , CITY } ,
    R2 := ( S RENAME { SNO AS SB } ) { SB , CITY } ,
    R3 := R1 JOIN R2 ,
    R4 := R3 WHERE SA < SB ) :
R4 { SA, SB }
```

As the example suggests, a WITH clause in Tutorial D consists of the keyword WITH followed by a parenthesized commalist of specifications of the form name := expression, the whole commalist then being followed

[^65]by a colon. For each of those "name := expression" specifications, the expression is evaluated and the result effectively assigned to a temporary variable with the specified name. Also, those "name := expression" specifications are evaluated in sequence as written; as a consequence, any given specification in the commalist is allowed to refer to names introduced in specifications earlier in that same commalist.

Tutorial D allows WITH clauses on statements as well as expressions. For example:

```
WITH ( X := RELATION { TUPLE { SNO 'S5' , PNO 'P6', QTY 250 } } ) :
SP := SP UNION X ;
```

SQL too supports a WITH construct, with these differences:

- SQL uses the keyword AS in place of Tutorial D's assignment symbol (":=").
- SQL doesn't use the enclosing parentheses or colon separator.
- WITH in Tutorial D can be used at any level of nesting. By contrast, WITH in SQL can be used only at the outermost level.

■ WITH in Tutorial D can be used in connection with expressions of any kind. ${ }^{17}$ By contrast, WITH in SQL can be used only in connection with table expressions specifically.

- As already noted, Tutorial D allows WITH clauses on statements as well as expressions. SQL doesn't.

Also, in SQL, the name portion of a "name AS expression" specification can optionally be followed by a parenthesized column name commalist (much as in a range variable definition-see Chapter 12). However, it shouldn't be necessary to exercise this option very often if other recommendations in this book are followed. Here's an SQL version of the example:

```
WITH T1 AS ( SELECT SNO AS SA , CITY
        FROM S ) ,
    T2 AS ( SELECT SNO AS SB , CITY
        FROM S ) ,
        T3 AS ( SELECT *
        FROM T1 NATURAL JOIN T2 ) ,
    T4 AS ( SELECT *
                FROM T3
                        WHERE SA < SB )
SELECT SA , SB
FROM T4
```

In closing this section, I should make it clear that WITH isn't really an operator of the relational algebra as such-it's just a syntactic device to help with the formulation of complicated expressions (especially ones involving common subexpressions). I'll be making extensive use of it in the pages ahead.

[^66]
## WHAT DO RELATIONAL EXPRESSIONS MEAN?

Recall now from Chapter 5 that every relvar has a certain relvar predicate, which is, loosely, what the relvar means. For example, the predicate for the suppliers relvar S is:

Supplier SNO is under contract, is named SNAME, has status STATUS, and is located in city CITY.
What I didn't mention in Chapter 5, however, is that the foregoing notion extends in a natural way to apply to arbitrary relational expressions. For example, consider the projection of suppliers on all attributes but CITY:

```
S { SNO , SNAME , STATUS }
```

This expression denotes a relation containing all tuples of the form

```
TUPLE { SNO s , SNAME n , STATUS t }
```

such that a tuple of the form

```
TUPLE { SNO s , SNAME n , STATUS t , CITY c }
```

currently appears in relvar S for some CITY value $c$. In other words, the result represents the current extension of a predicate that looks like this (see Chapter 5 if you need to refresh your memory regarding the notion of a predicate's extension):

There exists some city CITY such that supplier SNO is under contract, is named SNAME, has status STATUS, and is located in city CITY.

This predicate thus represents the meaning of the relational expression $\mathrm{S}\{\mathrm{SNO}, \mathrm{SNAME}, \mathrm{STATUS}\}$. Observe that it has just three parameters and the corresponding relation has just three attributes-CITY isn't a parameter to that predicate but what logicians call a "bound variable" instead, owing to the fact that it's "quantified" by the phrase There exists some city (see Chapter 10 for further explanation of bound variables and quantifiers). ${ }^{18}$ Note: A possibly clearer way of making the same point-viz., that the predicate has just three parameters, not four-is to observe that the predicate in question is logically equivalent to this one:

Supplier SNO is under contract, is named SNAME, has status STATUS, and is located somewhere [in other words, in some city, but we don't know which].

Remarks analogous to the foregoing apply to every possible relational expression. To be specific: Every relational expression $r x$ always has an associated meaning, or predicate; moreover, the predicate for $r x$ can always be determined from the predicates for the relvars involved in that expression, together with the semantics of the relational operations involved. As an exercise, you might like to revisit some of the relational (or SQL) expressions

[^67]shown earlier in this chapter, with a view to determining what the corresponding predicate might look like in each case.

## EVALUATING SQL TABLE EXPRESSIONS

In addition to natural join, Codd originally defined an operator he called $\theta$-join, where $\theta$ denoted any of the usual scalar comparison operators (" $=$ ", " $\neq "$ ", " $<"$ ", and so on). Now, $\theta$-join isn't primitive; in fact, it’s defined to be a restriction of a product. Here by way of example is the "not equals" join of suppliers and parts on cities (so $\theta$ here is " $\neq$ "):

```
( ( S RENAME { CITY AS SCITY } )
    TIMES
    ( P RENAME { CITY AS PCITY } ) )
WHERE SCITY f PCITY
```

```
SELECT SNO , SNAME , STATUS ,
    S.CITY AS SCITY , PNO ,
    PNAME , COLOR , WEIGHT ,
    P.CITY AS PCITY
FROM S , P
WHERE S.CITY <> P.CITY
```

Now let's focus on the SQL formulation specifically. You can think of the expression constituting that formulation as being evaluated in three steps, as follows:

1. The FROM clause is evaluated and yields the product of tables $S$ and $P$. Note: If we were doing this relationally, we would have to rename at least one of the CITY attributes before that product could be computed. SQL gets away with renaming them afterward because its tables have a left to right ordering to their columns, meaning it can distinguish the two CITY columns by their ordinal position. For simplicity, let's ignore this detail.
2. Next, the WHERE clause is evaluated and yields a restriction of that product by eliminating rows in which the two city values are equal. Note: If $\theta$ had been " $=$ " instead of " $\neq$ " (or "<>", rather, in SQL), this step would have been: Restrict the product by retaining just the rows in which the two city values are equal-in which case we would now have formed what's called the equijoin of suppliers and parts on cities. In other words, an equijoin is a $\theta$-join for which $\theta$ is " $=$ ". Exercise: What's the difference between an equijoin and a natural join?
3. Finally, the SELECT clause is evaluated and yields a "projection" of that restriction on the columns specified in the SELECT clause-"projection" in quotes, because it won't actually eliminate duplicates, as true projection does, unless DISTINCT is specified. (Actually it's doing some renaming as well, in this particular example, and I mentioned earlier in this chapter that SELECT provides other functionality too, in general-but for now I want to ignore these details as well, for simplicity.)

At least to a first approximation, then, the FROM clause corresponds to a product, the WHERE clause to a restriction, and the SELECT clause to a projection; thus, the overall SELECT - FROM - WHERE expression denotes a projection of a restriction of a product. It follows that I've just given a loose, but reasonably formal, definition of the semantics of SQL's SELECT - FROM - WHERE expressions; equivalently, I've given a conceptual algorithm for evaluating such expressions. Now, there's no implication that the implementation has to use exactly that algorithm in order to evaluate such expressions; au contraire, it can use any algorithm it likes, just so long as whatever algorithm it does use is guaranteed to give the same result as the conceptual one. And there are often good reasons-usually performance reasons-for using a different algorithm, thereby (for example) evaluating the clauses in a different order or otherwise rewriting the original query. However, the implementation is free to do
such things only if it can be proved that the algorithm it does use is logically equivalent to the conceptual one. Indeed, one way to characterize the job of the optimizer is to find an algorithm that's guaranteed to be equivalent to the conceptual one but performs better ... which brings us to the next section.

## EXPRESSION TRANSFORMATION

In this section, I want to take a slightly closer look at what the optimizer does. More specifically, I want to consider what's involved in transforming some relational expression into another, logically equivalent, expression. Note: I mentioned this notion under the discussion of duplicates in Chapter 4, where I explained that such transformations are one of the things the optimizer does; in fact, such transformations constitute one of the two great ideas at the heart of relational optimization (the other, beyond the scope of this book, is the use of "database statistics" to do what's called cost based optimizing). ${ }^{19}$

I'll start with a trivial example. Consider the following Tutorial D expression (the query is "Get suppliers who supply part P2, together with the corresponding quantities," and I'll ignore the SQL analog for simplicity):

```
( ( S JOIN SP ) WHERE PNO = 'P2' ) { ALL BUT PNO }
```

Suppose there are 1,000 suppliers and $1,000,000$ shipments, of which 500 are for part P2. If the expression were simply evaluated by brute force (as it were), without any optimization at all, the sequence of events would be:

1. Join S and SP: This step involves reading the 1,000 supplier tuples; reading the $1,000,000$ shipment tuples 1,000 times each, once for each of the 1,000 suppliers; constructing an intermediate result consisting of $1,000,000$ tuples; and writing those $1,000,000$ tuples back out to the disk. (I'm assuming for simplicity that tuples are physically stored as such, and I'm also assuming I can take "number of tuple reads and writes" as a reasonable measure of performance. Neither of these assumptions is very realistic, but this fact doesn't materially affect my argument.)
2. Restrict the result of Step 1: This step involves reading $1,000,000$ tuples but produces a result containing only 500 tuples, which I'll assume can be kept in main memory. (By contrast, I was assuming for the sake of the example in Step 1, realistically or otherwise, that the $1,000,000$ intermediate result tuples couldn't be kept in main memory.)
3. Project the result of Step 2: This step involves no tuple reads or writes at all, so we can ignore it.

The following procedure is equivalent to the one just described, in the sense that it produces the same final result, but is obviously much more efficient:

1. Restrict SP to just the tuples for part P2: This step involves reading $1,000,000$ shipment tuples but produces a result containing only 500 tuples, which can be kept in main memory.
[^68]
## 124 Chapter 6 / SQL and Relational Algebra I

2. Join S and the result of Step 1: This step involves reading 1,000 supplier tuples (once only, not once per P2 shipment, because all the P2 shipments are in memory). The result contains 500 tuples (still in main memory).
3. Project the result of Step 2: Again we can ignore this step.

The first of these two procedures involves a total of $1,002,001,000$ tuple reads and writes, whereas the second involves only $1,001,000$; thus, it's clear the second procedure is likely to be over 1,000 times faster than the first. It's also clear we'd like the implementation to use the second rather than the first! If it does, then what it's doing (in effect) is transforming the original expression
( S JOIN SP ) WHERE $\mathrm{PNO}=$ 'P2'
-I'm ignoring the final projection now, since it isn't really relevant to the argument-into the expression

```
S JOIN ( SP WHERE PNO = 'P2' )
```

These two expressions are logically equivalent, but they have very different performance characteristics, as we've seen. If the system is presented with the first expression, therefore, we'd like it to transform it into the second before evaluating it-and of course it can. The point is, the relational algebra, being a high level formalism, is subject to various formal transformation laws; for example, there's a law that says, loosely, that a join followed by a restriction can always be transformed into a restriction followed by a join (this was the law I was using in the example). And a good optimizer will know those laws, and will apply them-because the performance of a query ideally shouldn't depend on the specific syntax used to express that query in the first place. Note: Actually it's an immediate consequence of the fact that not all of the algebraic operators are primitive that certain expressions can be transformed into others (for example, an expression involving intersect can be transformed into one involving join instead), but there's much more to the issue than that, as I hope is obvious from the example.

Now, there are many possible transformation laws, and this isn't the place for an exhaustive discussion. All I want to do is highlight a few important cases and key points. First, the law mentioned in the previous paragraph is actually a special case of a more general law, called the distributive law. In general, the monadic operator $f$ distributes over the dyadic operator $g$ if and only if $f(g(a, b))=g(f(a), f(b))$ for all $a$ and $b$. In ordinary arithmetic, for example, SQRT (nonnegative square root) distributes over multiplication, because

```
SQRT ( a * b ) = SQRT ( a ) * SQRT ( b )
```

for all $a$ and $b$ (take $f$ as SQRT and $g$ as "**"); thus, a numeric expression optimizer can always replace either of these expressions by the other when doing numeric expression transformation. As a counterexample, SQRT does not distribute over addition, because the square root of $a+b$ is not equal to the sum of the square roots of $a$ and $b$, in general.

In relational algebra, restriction distributes over intersect, union, and difference. It also distributes over join, provided the restriction condition consists, at its most complex, of the AND of two separate restriction conditions, one for each of the two join operands. In the case of the example discussed above, this requirement was satisfied-in fact, the restriction condition was very simple and applied to just one of the operands-and so we were able to use the distributive law to replace the expression by a more efficient equivalent. The net effect was that we were able to "do the restriction early." Doing restrictions early is almost always a good idea, because it serves, typically, (a) to reduce the number of tuples to be scanned in the next operation in sequence and (b) to reduce the number of tuples in the output from that operation as well.

Here are some other specific cases of the distributive law, this time involving projection. First, projection distributes over union, though not over intersection and difference. Second, it also distributes over join, so long as all of the joining attributes are included in the projection. These laws can be used to "do projections early," which again is usually a good idea, for reasons similar to those given above for restrictions.

Two more important general laws are the laws of commutativity and associativity:

- The dyadic operator $g$ is commutative if and only if $g(a, b)=g(b, a)$ for all $a$ and $b$. In ordinary arithmetic, for example, addition and multiplication are commutative, but subtraction and division aren't. In relational algebra, intersect, union, and join are all commutative, ${ }^{20}$ but difference isn't. So, for example, if a query involves a join of two relations $r 1$ and $r 2$, the commutative law tells us it doesn't matter which of $r 1$ and $r 2$ is taken as the "outer" relation and which the "inner." The system is therefore free to choose (say) the smaller relation as the outer one in computing the join.
- The dyadic operator $g$ is associative if and only if $g(a, g(b, c))=g(g(a, b), c)$ for all $a, b, c$. In arithmetic, addition and multiplication are associative, but subtraction and division aren't. In relational algebra, intersect, union, and join are all associative, but difference isn't. So, for example, if a query involves a join of three relations $r 1, r 2$, and $r 3$, the associative and commutative laws taken together tell us we can join the relations pairwise in any order we like. The system is thus free to decide which of the various possible sequences is most efficient.

Note, incidentally, that all of these transformations can be performed without any regard for either actual data values or physical access paths (indexes and the like) in the database as physically stored. In other words, such transformations represent optimizations that are virtually guaranteed to be good, regardless of what the database looks like physically. Perhaps I should add, however, that while many such transformations are available for sets, not so many are available for bags (as indeed we saw in Chapter 4); and fewer still are available if column ordinal position has to be taken into account; and far fewer still are available if nulls and 3VL have to be taken into account as well. What do you conclude?

## THE RELIANCE ON ATTRIBUTE NAMES

There's one question that might have been bothering you but hasn't been addressed in this chapter so far. The operators of the relational algebra, at least as described in this book, all rely heavily on attribute naming. For example, the Tutorial D expression $R 1$ JOIN $R 2$-where I'll suppose, just to be definite, that $R 1$ and $R 2$ are base relvars-is defined to do the join on the basis of those attributes of $R 1$ and $R 2$ that have the same names. But the question often arises: Isn't this approach rather fragile? For example, what happens if we later add a new attribute to relvar $R 2$, say, that has the same name as one already existing in relvar $R 1$ ?

Well, first let me clarify one point. It's true that the operators do rely, considerably, on proper attribute naming. However, they also require attributes of the same name to be of the same type (and hence in fact to be the very same attribute, formally speaking); equivalently, they require attributes of different types to have different names. Thus, for example, an error would occur-at compile time, too, I would hope-if, in the expression R1 JOIN

[^69]$R 2, R 1$ and $R 2$ both had an attribute called $A$ but the two $A$ 's were of different types. ${ }^{21}$ Note that this requirement (that attributes of different types have different names) imposes no serious limitation on functionality, thanks to the availability of the attribute RENAME operator.

Now to the substance of the question. In fact, there's a popular misconception here, and I'm very glad to have this opportunity to dispel it. In today's SQL systems, application program access to the database is provided either through a call level interface or through an embedded, but conceptually distinct, data sublanguage ("embedded SQL"). But embedded SQL is really just a call level interface with a superficial dusting of syntactic sugar, so the two approaches come to the same thing from the DBMS's point of view, and indeed from the host language's point of view as well. In other words, SQL and the host language are typically only loosely coupled in most systems today. As a result, much of the advantage of using a well designed, well structured programming language is lost in today's database environment. Here's a quote: ${ }^{22}$ "Most programming errors in database applications would show up as type errors [if the database definition were] part of the type structure of the program."

Now, the fact that the database definition is not "part of the type structure of the program" in today's systems can be traced back to a fundamental misunderstanding that was prevalent in the database community in the early 1960s or so. The perception at that time was that, in order to achieve data independence (more specifically, logical data independence-see Chapter 9), it was necessary to move the database definition out of the program so that, in principle, that definition could be changed later without changing the program. But that perception was at least partly incorrect. What was, and is, really needed is two separate definitions, one inside the program and one outside; the one inside would represent the programmer's perception of the database (and would provide the necessary compile time checking on queries, etc.), the one outside would represent the database "as it really is." Then, if it subsequently becomes necessary to change the definition of the database "as it really is," logical data independence is preserved by changing the mapping between the two definitions.

Here's how the mechanism I've just described might look in SQL. First let me introduce the notion of a public table, which represents the application's perception of some portion of the database. For example:

```
CREATE PUBLIC TABLE X /* hypothetical syntax! */
    ( SNO VARCHAR(5) NOT NULL ,
        SNAME VARCHAR(25) NOT NULL ,
        CITY VARCHAR(20) NOT NULL ,
        UNIQUE ( SNO ) ) ;
CREATE PUBLIC TABLE Y /* hypothetical syntax! */
    ( SNO VARCHAR(5) NOT NULL ,
        PNO VARCHAR(6) NOT NULL ,
        UNIQUE ( SNO , PNO ) ) ,
        FOREIGN KEY ( SNO ) REFERENCES X ( SNO ) ) ;
```

These definitions effectively assert that "the application believes" there are tables in the suppliers-and-parts database called $X$ and $Y$, with columns and keys as specified. Such is not the case, of course-but there are database tables called S and SP (with columns and keys as specified for X and Y , respectively, but with one additional column in each case), and we can define mappings as follows:

[^70]```
X def SELECT SNO , SNAME , CITY FROM S ; /* hypothetical syntax! */
Y def SELECT SNO , PNO FROM SP ; /* hypothetical syntax! */
```

These mappings are defined outside the application (the symbol "def " means "is defined as").
Now consider the SQL expression X NATURAL JOIN Y. Clearly, the join here is being done on the basis of the common column SNO. And if, say, a column SNAME is added to the database table SP, all we have to do is change the mapping-actually no change is required at all, in this particular example!-and everything will continue to work as before; in other words, logical data independence will be preserved.

Unfortunately, today's SQL products don't work this way. Thus, for example, the SQL expression S NATURAL JOIN SP is, sadly, subject to exactly the "fragility" problem mentioned in the original question (but then so too is the simpler expression SELECT * FROM S, come to that). However, you can reduce that problem to more manageable proportions by adopting the strategy suggested under the discussion of column naming in Chapter 3. For convenience, I repeat that strategy here:

- For every base table, define a view identical to that base table except possibly for some column renaming.
- Make sure the set of views so defined abides by the naming discipline described in that same discussion (i.e., of column naming) in Chapter 3.
- Operate in terms of those views instead of the underlying base tables.

Now, if the base tables change subsequently, all you'll have to do is change the view definitions accordingly.

## EXERCISES

6.1 What if anything is wrong with the following SQL expressions (from a relational perspective or otherwise)?
a. SELECT * FROM S , SP
b. SELECT SNO , CITY FROM S
c. SELECT SNO , PNO , 2 * QTY FROM SP
d. SELECT S.SNO FROM S , SP
e. SELECT S.SNO, S.CITY FROM S NATURAL JOIN P
f. SELECT CITY FROM S UNION SELECT CITY FROM P
g. SELECT S.* FROM S NATURAL JOIN SP
h. SELECT * FROM S JOIN SP ON S.SNO = SP.SNO
i. SELECT * FROM ( S NATURAL JOIN P ) AS TEMP

## 128 Chapter 6 / SQL and Relational Algebra I

j. SELECT * FROM S CROSS JOIN SP CROSS JOIN P
6.2 Closure is important in the relational model for the same kind of reason that numeric closure is important in ordinary arithmetic. In arithmetic, however, there's one situation where the closure property breaks down, in a sense-namely, division by zero. Is there any analogous situation in the relational algebra?
6.3 Given the usual suppliers-and-parts database, what's the value of the Tutorial $\mathbf{D}$ expression JOIN \{S,SP,P\}? What's the corresponding predicate? And how would you express this join in SQL?
6.4 Why do you think the project operator is so called?
6.5 For each of the following Tutorial $\mathbf{D}$ expressions on the suppliers-and-parts database, give both (a) an SQL analog and (b) an informal interpretation of the expression (i.e., a corresponding predicate) in natural language. Also show the result of evaluating the expressions, given our usual sample values for relvars $\mathrm{S}, \mathrm{P}$, and SP .
a. ( S JOIN ( SP WHERE PNO = 'P2' ) ) \{ CITY \}
b. ( P \{ PNO \} MINUS ( SP WHERE SNO = 'S2' ) \{ PNO \} ) JOIN P
c. $S$ \{ CITY \} MINUS P \{ CITY \}
d. ( S \{ SNO , CITY \} JOIN P \{ PNO , CITY \} ) \{ SNO , PNO \}
e. JOIN \{ ( S RENAME \{ CITY AS SC \} ) \{ SC \} ,
( P RENAME \{ CITY AS PC \} ) \{ PC \} \}
6.6 Union, intersection, product, and join are all both commutative and associative. Verify these claims. Are they valid in SQL?
6.7 Which of the operators described in this chapter (if any) have a definition that doesn't rely on tuple equality?
6.8 The SQL FROM clause FROM $t 1, t 2, \ldots, t n$ (where each $t i$ denotes a table) returns the product of its arguments. But what if $n=1$ ?-what's the product of just one table? And by the way, what's the product of $t 1$ and $t 2$ if $t 1$ and $t 2$ both contain duplicate rows?
6.9 Write Tutorial $\mathbf{D}$ and/or SQL expressions for the following queries on the suppliers-and-parts database:
a. Get all shipments.
b. Get supplier numbers for suppliers who supply part P1.
c. Get suppliers with status in the range 15 to 25 inclusive.
d. Get part numbers for parts supplied by a supplier in London.
e. Get part numbers for parts not supplied by any supplier in London.
f. Get all pairs of part numbers such that some supplier supplies both of the indicated parts.
g. Get supplier numbers for suppliers with a status lower than that of supplier S1.
h. Get part numbers for parts supplied by all suppliers in London.
i. Get (SNO,PNO) pairs such that the indicated supplier does not supply the indicated part.
j. Get suppliers who supply at least all parts supplied by supplier S2.
6.10 Prove the following statements (making them more precise where necessary):
a. A sequence of restrictions of a given relation can be transformed into a single restriction.
b. A sequence of projections of a given relation can be transformed into a single projection.
c. A restriction of a projection can be transformed into a projection of a restriction.
6.11 Union is said to be idempotent, because $r$ UNION $r$ is identically equal to $r$ for all $r$. (Is this true in SQL?) As you might expect, idempotence can be useful in expression transformation. Which other relational operators, if any, are idempotent?
6.12 Let $r$ be a relation. What does the Tutorial $\mathbf{D}$ expression $r\}$ mean (i.e., what's the corresponding predicate)? What does it return? Also, what does the Tutorial D expression $r$ \{ALL BUT $\}$ mean, and what does it return?
6.13 The boolean expression $x>y$ AND $y>3$ (which might be part of a query) is equivalent to-and can therefore be transformed into-the boolean expression $x>y$ AND $y>3$ AND $x>3$. (The equivalence is based on the fact that the comparison operator " $>$ " is transitive; i.e., $a>b$ and $b>c$ together imply $a>c$.) Note that the transformation is certainly worth making if $x$ and $y$ are from different relations, because it enables the system to perform an additional restriction (using $x>3$ ) before doing the greater-than join implied by $x>y$. As we saw in the body of the chapter, doing restrictions early is generally a good idea; having the system infer additional "early" restrictions, as here, is also a good idea. Do you know of any SQL products that actually perform this kind of optimization?

### 6.14 Consider the following Tutorial D expression:

```
WITH ( PP := P WHERE COLOR = 'Purple' ,
    T := SP RENAME { SNO AS X } ) :
S WHERE ( T WHERE X = SNO ) { PNO } ` PP { PNO }
```

What does this expression mean? Given our usual sample data values, show the result returned. Does that result accord with your intuitive understanding of what the expression means? Justify your answer.
6.15 SQL has no direct counterpart to either D_UNION or I_MINUS. How best might the D_UNION and I_MINUS examples from the body of the chapter-i.e., S \{CITY\} D_UNION P $\{$ CITY $\}$ and S $\{$ CITY $\}$ I_MINUS $\mathrm{P}\{$ CITY $\}$-be simulated in SQL?

130 Chapter 6 / SQL and Relational Algebra I
6.16 What do you understand by the term joinable? How could the definition of the term be extended to cover the case of $n$ relations for arbitrary $n$ (instead of just $n=2$, which was the case discussed in the body of the chapter)?
6.17 What exactly is it that makes it possible to define $n$-adic versions of JOIN and UNION (and D_UNION)? Does SQL have anything analogous? Why doesn't an $n$-adic version of MINUS (or I_MINUS) make sense?
6.18 I claimed earlier in the book that TABLE_DEE meant TRUE and TABLE_DUM meant FALSE. Substantiate and/or elaborate on these claims.
6.19 What exactly does the following SQL expression return?

```
SELECT DISTINCT S.*
FROM S , P
```

Warning: There's a trap here.

## Chapter 7

## SQL and Relational Algebra II:

## Additional Operators

Algebra is the part of advanced mathematics<br>that is not calculus.<br>-John Derbyshire:<br>Unknown Quantity: A Real and Imaginary History of Algebra (2006)

As I've said several times already, an operator of the relational algebra is an operator that takes one or more relations as input and produces another relation as output. As I observed in Chapter 1, however, any number of operators can be defined that conform to this simple characterization. Chapter 6 described the original operators (join, project, etc.); the present chapter describes some of the many additional operators that have been defined since the relational model was first invented. It also considers how those operators might best be realized in SQL.

Note: By its nature, this chapter is necessarily something of a miscellany. Thus, you might want just to skim it lightly on a first pass, and come back to it later if you need to gain a deeper understanding of any of the topics discussed. Perhaps it would help to say that from a practical point of view, the most important topics are probably these:

- Semijoin and semidifference (MATCHING and NOT MATCHING)

EXTEND
Image relations

- Aggregate operators

But I'll begin with a brief discussion of exclusive union.

## EXCLUSIVE UNION

In set theory, union is inclusive; that is, given sets $s 1$ and $s 2$, an element appears in their union if and only if it appears in either or both of $s 1$ or $s 2$. Thus, UNION can be seen as the set theory counterpart to logical OR, which is inclusive in a similar sense. But logic additionally defines an exclusive version of OR (XOR), and so we can define an exclusive union operator analogously: The exclusive union (XUNION) of two sets $s l$ and $s 2$ is the set of elements appearing in $s 1$ or $s 2$ but not both. And, of course, we can define a relational version of this operator as well:

Definition: Let relations $r 1$ and $r 2$ be of the same type; then their exclusive union, $r 1$ XUNION $r 2$, is a relation of the same type, with body consisting of all tuples $t$ such that $t$ appears in $r 1$ or $r 2$ but not both.

For example (assuming as we did in Chapter 6, in the section "Union, Intersection, and Difference," that parts have an extra attribute called STATUS, of type INTEGER):

```
P { STATUS , CITY } XUNION
S { CITY , STATUS }
```

```
SELECT STATUS , CITY
FROM P
WHERE ( STATUS , CITY ) NOT IN
    ( SELECT STATUS , CITY
        FROM S )
UNION CORRESPONDING
SELECT CITY , STATUS
FROM S
WHERE ( CITY , STATUS ) NOT IN
    ( SELECT CITY , STATUS
        FROM S )
```

Tutorial D also supports an $n$-adic form of XUNION. However, the details are a little tricky; for that reason, I'll just give a definition here, without further discussion. You can find more details, if you're interested, in the paper " $N$-adic vs. Dyadic Operators: An Investigation" (see Appendix G).

Definition: Let relations $r 1, r 2, \ldots, r n(n \geq 0)$ all be of the same type $T$. Then the expression XUNION $\{r 1, r 2, \ldots, r n\}$ denotes a relation of type $T$ with body the set of all tuples $t$ such that $t$ appears in exactly $m$ of $r 1, r 2, \ldots, r n$, where $m$ is odd (and is possibly different for different tuples $t$ ).

The exclusive union of a single relation $r$ is just $r$. The exclusive union of no relations at all is the empty relation of the pertinent type-but that type needs to be specified explicitly, since there aren't any relational expressions from which the type can be inferred. Thus, for example, the expression

XUNION \{ SNO CHAR , STATUS INTEGER \} \{ \}
denotes the empty relation of type RELATION \{SNO CHAR, STATUS INTEGER\}.
Note: In set theory, exclusive union is more usually known as symmetric difference. Thus, the keyword XMINUS might be acceptable as an alternative to XUNION.

## SEMIJOIN AND SEMIDIFFERENCE

Join is one of the most familiar of all of the relational operators. In practice, however, it turns out that queries that require the join operator at all often really require an extended form of that operator called semijoin (you might not have heard of semijoin before, but in fact it's quite important). Here's the definition:

Definition: The semijoin of relations $r 1$ and $r 2$ (in that order), $r 1$ MATCHING $r 2$, is equivalent to ( $r 1$ JOIN $r 2)\{H 1\}$, where $\{H 1\}$ is the heading of $r 1$.

In other words, $r 1$ MATCHING $r 2$ is the join of $r 1$ and $r 2$, projected back on the attributes of $r 1$ (and so the heading of the result is the same as that of $r 1$ ). Here's an example ("Get suppliers who currently supply at least one part"):

```
SELECT S.* FROM S
WHERE SNO IN
    ( SELECT SNO FROM SP )
```

Note that the expressions $r 1$ MATCHING $r 2$ and $r 2$ MATCHING $r 1$ aren't equivalent, in general-the first returns some subset of $r 1$, the second returns some subset of $r 2$. Note too that we could replace IN by MATCH in the SQL version; interestingly, however, we can't replace NOT IN by NOT MATCH in the semidifference analog (see below), because there's no "NOT MATCH" operator in SQL.

Turning now to semidifference: If semijoin is in some ways more important than join, a similar remark applies here also, but with even more force-in practice, most queries that require difference at all really require semidifference. ${ }^{1}$ Here's the definition:

Definition: The semidifference between relations $r 1$ and $r 2$ (in that order), $r 1$ NOT MATCHING $r 2$, is equivalent to $r 1$ MINUS ( $r 1$ MATCHING $r 2$ ).

Here's an example ("Get suppliers who currently supply no parts at all"):

```
S NOT MATCHING SP
```

```
SELECT S.* FROM S
WHERE SNO NOT IN
    ( SELECT SNO FROM SP )
```

As with MATCHING, the heading of the result is the same as that of $r 1$. Note: If $r 1$ and $r 2$ are of the same type, $r 1$ NOT MATCHING $r 2$ degenerates to $r 1$ MINUS $r 2$; in other words, difference (MINUS) is a special case of semidifference, relationally speaking. By contrast, join isn't a special case of semijoin-they're really different operators, though it's true that (loosely speaking) some joins are semijoins and some semijoins are joins. See Exercise 7.19 at the end of the chapter.

## EXTEND

You might have noticed that the algebra as I've described it so far in this book doesn't have any conventional computational capabilities. Now, SQL does; for example, we can write queries in SQL along the lines of SELECT A + B AS C ... (for example). However, as soon as we write that "+" sign, we've gone beyond the bounds of the algebra as originally defined. So we need to add something to the algebra in order to provide this kind of functionality, and that's what EXTEND is for. By way of example, suppose part weights (in relvar P) are given in pounds, and we want to see those weights in grams. There are 454 grams to a pound, and so we can write:

```
EXTEND P :
{ GMWT := WEIGHT * 454 }
SELECT P.* , WEIGHT * 454 AS GMWT
FROM P
```

Given our usual sample values, the result looks like this:

[^71]| PNO | PNAME | COLOR | WEIGHT | CITY | GMWT |
| :--- | :--- | :--- | ---: | :--- | :--- |
| P1 | Nut | Red | 12.0 | London | 5448.0 |
| P2 | Bolt | Green | 17.0 | Paris | 7718.0 |
| P3 | Screw | Blue | 17.0 | Oslo | 7718.0 |
| P4 | Screw | Red | 14.0 | London | 6356.0 |
| P5 | Cam | Blue | 12.0 | Paris | 5448.0 |
| P6 | Cog | Red | 19.0 | London | 8626.0 |

Important: Relvar $P$ is not changed in the database! EXTEND is not an SQL-style ALTER TABLE; the EXTEND expression is just an expression, and like any expression it simply denotes a value. In particular, therefore, it can be nested inside other expressions. Here's an example (the query is "Get part number and gram weight for parts with gram weight greater than 7000 grams"):

```
( ( EXTEND P :
    { GMWT := WEIGHT * 454 } )
        WHERE GMWT > 7000.0 )
    { PNO , GMWT }
\begin{tabular}{ll} 
SELECT & PNO, \\
& WEIGHT * 454 AS GMWT \\
FROM & P \\
WHERE & WEIGHT * \(454>7000.0\)
\end{tabular}
```

As you can see, there's an interesting difference between the Tutorial D and SQL versions of this example. To be specific, the (sub)expression WEIGHT * 454 appears once in the Tutorial D version but twice in the SQL version. In the SQL version, therefore, we have to hope the implementation will be smart enough to recognize that it need evaluate that subexpression just once per tuple (or row, rather) instead of twice.

The problem this example illustrates is that SQL's SELECT - FROM - WHERE template is too rigid. What we need to do, as the Tutorial D formulation makes clear, is form a restriction of an extension; in SQL terms, we need to apply the WHERE clause to the result of the SELECT clause, as it were. But the SELECT - FROM WHERE template forces the WHERE clause to apply to the result of the FROM clause, not the SELECT clause (see the section "Evaluating SQL Table Expressions" in Chapter 6). To put it another way: In many respects, it's the whole point of the algebra that (thanks to closure) relational operations can be combined and nested in arbitrary ways; but SQL's SELECT - FROM - WHERE template effectively means that queries must be expressed as a product, followed by a restrict, followed by some combination of project and/or extend and/or rename ${ }^{2}$-and many queries just don't fit this pattern.

Incidentally, you might be wondering why I didn't formulate the SQL version like this:

```
SELECT PNO , WEIGHT * 454 AS GMWT
FROM P
WHERE GMWT > 7000.0
```

(The change is in the last line.) The reason is that GMWT is the name of a column of the final result; table P has no such column, the WHERE clause thus makes no sense, and the expression fails at compile time.

Actually, the SQL standard does allow the query under discussion to be formulated in a style that's a little closer to that of Tutorial D (and now I'll make all of the otherwise implicit dot qualifications explicit, for clarity):

```
SELECT TEMP.PNO , TEMP.GMWT
FROM ( SELECT PNO , (WEIGHT * 454 ) AS GMWT
        FROM P ) AS TEMP
WHERE TEMP.GMWT > 7000.0
```

[^72]But I'm not sure all SQL products allow nested subqueries to appear in the FROM clause in this manner. Note too that this kind of formulation inevitably leads to a need to reference certain variables (TEMP, in the example) before they're defined-quite possibly a long way before they're defined, in fact, in real SQL queries.

Note: I need to say a little more about the FROM clause in the foregoing example. As you can see, it takes the form

```
FROM ( ... ) AS TEMP
```

Formally speaking, it's the parenthesized portion of this FROM clause that constitutes the nested subquery (see Chapter 12). And-here comes the point-SQL has a syntax rule to the effect that a nested subquery in the FROM clause must be accompanied by an explicit AS clause that defines a name for the table denoted by that subquery, ${ }^{3}$ even if that name is never explicitly referenced elsewhere in the overall expression. In fact, in the example at hand, we could omit all of the explicit references to the name TEMP (i.e., all of the explicit "TEMP." dot qualifications) if we wanted to, thus:

```
SELECT PNO , GMWT
FROM ( SELECT PNO , ( WEIGHT * 454 ) AS GMWT
        FROM P ) AS TEMP
WHERE GMWT > 7000.0
```

But the TEMP definition (i.e., that AS TEMP specification) is still needed nonetheless.
I'll close this section with a formal definition of the EXTEND operator:
Definition: Let $r$ be a relation, and let $r$ not have an attribute named $X$. Then the extension EXTEND $r$ : $\{X:=\exp \}$ is a relation with (a) heading the heading of $r$ extended with attribute $X$ and (b) body the set of all tuples $t$ such that $t$ is a tuple of $r$ extended with a value for attribute $X$ that's computed by evaluating $\exp$ on that tuple of $r$. Observe that the result has cardinality equal to that of $r$ and degree equal to that of $r$ plus one. The type of $X$ in that result is the type of exp.

## IMAGE RELATIONS

An image relation is, loosely, the "image" within some relation of some tuple (usually a tuple within some other relation). For example, given the suppliers-and-parts database and our usual sample values, the following is the image within the shipments relation of the supplier tuple for supplier S4:

| PNO | QTY |
| :---: | :---: |
| P2 | 200 |
| P4 | 300 |
| P5 | 400 |

Clearly, this particular image relation can be obtained by means of the following Tutorial D expression:

[^73]
## 136 Chapter 7 / SQL and Relational Algebra II

```
( SP WHERE SNO = 'S4' ) { ALL BUT SNO }
```

Here's a formal definition of image relations in general:
Definition: Let relations $r 1$ and $r 2$ be joinable (i.e., such that attributes with the same name are of the same type); let $t l$ be a tuple of $r 1$; let $t 2$ be a tuple of $r 2$ that has the same values for those common attributes as tuple $t 1$ does; let relation $r 3$ be that restriction of $r 2$ that contains all and only such tuples $t 2$; and let relation $r 4$ be the projection of $r 3$ on all but those common attributes. Then $r 4$ is the image relation (with respect to $r 2)$ corresponding to $t 1$.

Here's an example that illustrates the usefulness of image relations:

```
S WHERE ( !!SP ) { PNO } = P { PNO }
```


## Explanation:

- First of all, the roles of $r 1$ and $r 2$ from the definition are being played by the suppliers relation and the shipments relation, respectively (where by "the suppliers relation" I mean the current value of relvar S, and similarly for "the shipments relation").

■ Next, observe that the boolean expression in the WHERE clause involves an equality comparison between two relations (actually two projections). We can imagine that boolean expression being evaluated for each tuple $t l$ in $r l$ (i.e., each tuple in the suppliers relation) in turn.

- Consider one such tuple, say that for supplier $\mathrm{S} x$. For that tuple, then, the expression !!SP-pronounced "bang bang SP" or "double bang SP"-denotes the corresponding image relation $r 4$ within $r 2$; in other words, it denotes the set of (PNO,QTY) pairs within SP for parts supplied by that supplier Sx. ${ }^{4}$ The expression !!SP is an image relation reference.

■ The expression (!!SP) \{PNO\}-i.e., the projection of the image relation on $\{\mathrm{PNO}\}$-thus denotes the set of part numbers for parts supplied by supplier $S x$.

- The expression overall (i.e., S WHERE ...) thus denotes suppliers from S for whom that set of part numbers is equal to the set of all part numbers in the projection of P on $\{\mathrm{PNO}\}$. In other words, it represents the query "Get suppliers who supply all parts" (speaking a little loosely).

Note: Since the concept of an image relation is defined in terms of some given tuple ( $t 1$, in the formal definition), it's clear that an image relation reference can appear, not in all possible contexts in which relational expressions in general can appear, but only in certain specific contexts: namely, those in which the given tuple $t l$ is understood. WHERE clauses are one such context, as the foregoing example indicates, and we'll see another in the section "Image Relations bis," later in this chapter.

Aside: SQL has no direct support for image relations as such. Here for interest is an SQL formulation of the query "Get suppliers who supply all parts" (I show it for your consideration, but I'm not going to discuss it in detail, except to note that it can obviously (?) be improved in a variety of ways):

[^74]```
SELECT *
FROM S
WHERE NOT EXISTS
    ( SELECT PNO
        FROM SP
        WHERE SP.SNO = S.SNO
        EXCEPT CORRESPONDING
        SELECT PNO
        FROM P )
AND NOT EXISTS
    ( SELECT PNO
        FROM P
        EXCEPT CORRESPONDING
        SELECT PNO
        FROM SP
        WHERE SP.SNO = S.SNO )
```

End of aside.

To get back to image relations as such, it's worth noting that the "!!" operator can be defined in terms of MATCHING. For example, the example discussed above-

```
S WHERE ( !!SP ) { PNO } = P { PNO }
```

-is logically equivalent to the following:

```
S WHERE
( SP MATCHING RELATION { TUPLE { SNO SNO } } ) { PNO } = P { PNO }
```

Explanation: Again consider some tuple of S, say that for supplier $\mathrm{S} x$. For that tuple, then, the expression TUPLE \{SNO SNO\}-which is a tuple selector invocation-denotes a tuple containing just the SNO value $\mathrm{S} x$ (the first SNO in that expression is an attribute name, the second denotes the value of the attribute of that name in the tuple for $\mathrm{S} x$ within relvar S ). So the expression

```
RELATION { TUPLE { SNO SNO } }
```

-which is a relation selector invocation-denotes the relation that contains just that tuple. Hence, the expression

```
SP MATCHING RELATION { TUPLE { SNO SNO } }
```

denotes a certain restriction of SP: namely, that restriction that contains just those shipment tuples that have the same SNO value as the supplier tuple for supplier $S x$ does. It follows that, in the context under consideration, the expression shown ("SP MATCHING ...") is logically equivalent to the image relation reference "!!SP", and the overall result follows.

By way of another example, suppose we're given a revised version of the suppliers-and-parts database-one that's simultaneously both extended and simplified, compared to our usual version-that looks like this (in outline):

| S | SNO |  | /* suppliers |
| :---: | :---: | :---: | :---: |
| SP | SNO | PNO | /* supplier supplies part |
| P J | PNO | JNO | /* part is used in project |
| J | JNO |  | /* projects |

Relvar J here represents projects (JNO stands for project number), and relvar PJ indicates which parts are used in which projects. Now consider the query "Get all (sno,jno) pairs such that sno is an SNO value currently appearing in relvar S, jno is a JNO value currently appearing in relvar J, and supplier sno supplies all parts used in project jno." This is a complicated query!-but a formulation using image relations is almost trivial:

```
( S JOIN J ) WHERE !!PJ \subseteq !!SP
```

Exercise: Give an SQL analog of this expression.
Reverting now to the usual suppliers-and-parts database, here's another example ("Delete shipments from suppliers in London"-and this time I'll show an SQL analog as well):

```
DELETE SP WHERE IS_NOT_EMPTY
    ( !!( S WHERE
    CITY = 'London' ) ) ;
```

```
DELETE FROM SP
WHERE SNO IN
    ( SELECT SNO FROM S
    WHERE CITY = 'London' ) ;
```

For a given shipment, the relation denoted by the specified image relation reference ("!!(S WHERE ...") is either empty, if the corresponding supplier isn't in London, or contains exactly one tuple otherwise.

## DIVIDE

I include the following discussion of divide in this chapter only to show why (contrary to conventional wisdom, perhaps) I don't think it's very important; in fact, I think it should be dropped. You can skip this section if you like.

I have several reasons (three at least) for wanting to drop divide. One is that any query that can be formulated in terms of divide can alternatively, and much more simply, be formulated in terms of image relations instead, as I'll demonstrate in just a moment. Another is that there are at least seven different divide operators anyway!-that is, there are, unfortunately, at least seven different operators all having some claim to be called "divide," and I certainly don't want to explain all of them. Instead, I'll limit my attention here to the original and simplest one.

Definition: Let relations $r 1$ and $r 2$ be such that the heading $\{Y\}$ of $r 2$ is some subset of the heading of $r 1$ and the set $\{X\}$ is the other attributes of $r 1$. Then the division of $r 1$ by $r 2, r 1$ DIVIDEBY $r 2,{ }^{5}$ is shorthand for the following:

```
r1 { X } NOT MATCHING ( ( r1 { X } JOIN r2 ) NOT MATCHING r1 )
```

For example, the expression
SP \{ SNO , PNO \} DIVIDEBY P \{ PNO \}
(given our usual sample data values) yields:

[^75]| SNO |
| :---: |
| $\overline{S 1}$ |

The expression can thus be loosely characterized as a representation of the query "Get supplier numbers for suppliers who supply all parts" (I'll explain the reason for that qualifier "loosely" in a few moments). In practice, however, we're more likely to want full supplier details (not just supplier numbers) for the suppliers in question, in which case the division will need to be followed by a join:

```
( SP { SNO , PNO } DIVIDEBY P { PNO } ) JOIN S
```

But we already know how to formulate this query more simply using image relations:

```
S WHERE ( !!SP ) { PNO } = P { PNO }
```

This latter formulation is (a) more succinct, (b) easier to understand (at least, it seems so to me), and (c) correct. This last point is the crucial one, of course, and I'll explain it below. First, however, I want to explain why divide is called divide, anyway. The reason is that if $r 1$ and $r 2$ are relations with no attribute names in common and we form the product $r 1$ TIMES $r 2$, and then divide the result by $r 2$, we get back to $r 1$. (At least, we do so just as long as $r 2$ isn't empty. What happens if it is?) In other words, product and divide are inverses of each other, in a sense.

As I've said, the expression

```
SP { SNO , PNO } DIVIDEBY P { PNO }
```

can loosely be characterized as a formulation of the query "Get supplier numbers for suppliers who supply all parts"; indeed, this very example is often used as a basis for explaining, and justifying, the divide operator in the first place. Unfortunately, however, that characterization isn't quite correct. Rather, the expression is a formulation of the query "Get supplier numbers for suppliers who supply at least one part and in fact supply all parts." ${ }^{6}$ In other words, the divide operator not only suffers from problems of complexity and lack of succinctness-it doesn't even solve the problem it was originally, and explicitly, intended to address.

## AGGREGATE OPERATORS

In a sense this section is a bit of a digression, because the operators to be discussed aren't relational but scalar-they return a scalar result. ${ }^{7}$ But I do need to say something about them before I can get back to the main theme of the chapter.

[^76]
## 140 Chapter 7 / SQL and Relational Algebra II

An aggregate operator in the relational model is an operator that derives a single value from the "aggregate" (i.e., the bag or set) of values appearing within some attribute within some relation-or, in the case of COUNT, which is slightly special, from the "aggregate" that's the entire relation. Here are two examples:

```
X := COUNT ( S ) ;
Y := COUNT ( S { STATUS } ) ;
```

```
SELECT COUNT ( * ) AS X
```

SELECT COUNT ( * ) AS X
SELECT COUNT ( DISTINCT STATUS )
SELECT COUNT ( DISTINCT STATUS )
FROM

```
FROM
```

I'll focus on the Tutorial D statements on the left first. Given our usual sample values, the first assigns the value 5 (the number of tuples in the current value of relvar $S$ ) to the variable X ; the second assigns the value 3 (the number of tuples in the projection of the current value of relvar $S$ on $\{$ STATUS\}, which is to say the number of distinct STATUS values in that current value) to the variable Y.

In general, a Tutorial $\mathbf{D}$ aggregate operator invocation looks like this:

```
<agg op name> ( <relation exp> [, <exp> ] )
```

Legal <agg op name>s include COUNT, SUM, AVG, MAX, MIN, AND, OR, and XOR. ${ }^{8}$ Within the $<\exp >$, an <attribute ref> can appear wherever a literal would be allowed. That <exp> must be omitted if the <agg op name $>$ is COUNT; otherwise, it can be omitted only if the <relation exp $>$ denotes a relation of degree one, in which case an $<\exp >$ consisting of a reference to the sole attribute of that relation is assumed. Here are some examples:

```
1. SUM ( SP , QTY )
```

This expression denotes the sum of all quantities in relvar SP (given our usual sample values, the result is 3100).
2. SUM ( SP \{ QTY \} )

This expression is shorthand for $\operatorname{SUM}(\operatorname{SP}\{\mathrm{QTY}\}, \mathrm{QTY})$, and it denotes the sum of all distinct quantities in SP (i.e., 1000).
3. AVG ( SP , 3 * QTY )

This expression effectively asks what the average shipment quantity would be if quantities were all triple their current value (the answer is 775). More generally, the expression

```
agg ( rx , x )
```

[^77](where $x$ is some expression more complicated than a simple <attribute ref>) is essentially shorthand for the following:

```
agg ( EXTEND rx : { y := x } , y )
```

Now I turn to SQL. For convenience, let me first repeat the examples:

```
X := COUNT ( S ) ;
Y := COUNT ( S { STATUS } ) ;
```

```
SELECT COUNT ( * ) AS X
```

SELECT COUNT ( * ) AS X
SELECT COUNT ( DISTINCT STATUS )
SELECT COUNT ( DISTINCT STATUS )
FROM

```
FROM
```

Now, you might be surprised to hear me claim that SQL doesn't really support aggregate operators at all! I say this knowing full well that most people would consider expressions like those on the right above to be, precisely, SQL aggregate operator invocations. ${ }^{9}$ But they aren't. Let me explain. As we know, the counts are 5 and 3, respectively. But those SQL expressions don't evaluate to those counts as such, as true aggregate operator invocations would; rather, they evaluate to tables that contain those counts. More precisely, each yields a table with one row and one column, and the sole value in that row is the actual count: ${ }^{10}$


As you can see, therefore, the SELECT expressions really don't represent aggregate operator invocations as such; at best, they represent only approximations to such invocations. In fact, aggregation is treated in SQL as if it were a special case of summarization. Of course, I haven't discussed summarization yet; for present purposes, however, you can regard it as what's represented in SQL by a SELECT expression with a GROUP BY clause. Now, the foregoing SQL expressions don't have a GROUP BY clause-but they're defined to be shorthand for the following, which do (and do therefore represent summarizations as claimed):

```
SELECT COUNT ( * ) AS X
FROM
GROUP BY ( )
SELECT COUNT ( DISTINCT STATUS ) AS Y
FROM S
GROUP BY ( )
```

Note: In case these expressions look strange to you, I should explain that SQL does in fact allow both (a) GROUP BY clauses with an empty operand commalist and (b) GROUP BY clauses with the operand commalist

[^78]enclosed in parentheses. What's more, specifying a GROUP BY clause with an empty operand commalist (with or without parentheses) is equivalent to omitting the GROUP BY clause entirely. Here's why:
a. Such a GROUP BY clause effectively means "group by no columns."
b. Every row has the same value for no columns-namely, the 0-row (despite the fact that SQL doesn't actually support the 0 -row!).
c. Every row in the table is thus part of the same group; in other words, the entire table is treated as a single group, and that's effectively what happens when the GROUP BY clause is omitted entirely.

So SQL does support summarization-but it doesn't support aggregation as such. Sadly, the two concepts are often confused, and perhaps you can begin to see why. What's more, the picture is confused still further by the fact that, in SQL, it's common in practice for the table that results from an "aggregation" to be coerced to the single row it contains, or even doubly coerced to the single value that row contains: two separate errors (of judgment, if nothing else) thus compounding to make the SQL-style "aggregation" look more like a true aggregation after all! Such double coercion occurs in particular when the SELECT expression is enclosed in parentheses to form a scalar subquery, as in the following SQL assignments:

```
SET X = ( SELECT COUNT ( * ) FROM S ) ;
SET Y = ( SELECT COUNT ( DISTINCT STATUS ) FROM S ) ;
```

But assignment as such is far from being the only context in which such coercions occur (see Chapters 2 and 12).
Aside: Actually there's another oddity arising in connection with SQL-style aggregation (I include this observation here because this is where it logically belongs, but it does rely on a detailed understanding of SQL-style summarization, and you can skip it if you like):

■ In general, an expression of the form SELECT - FROM $T$ - WHERE - GROUP BY - HAVING delivers a result containing exactly one row for each group in $G$, where $G$ is the "grouped table" resulting from applying the WHERE, GROUP BY, and HAVING clauses to table $T$.

- Omitting the WHERE and HAVING clauses, as in a "straightforward" SQL-style aggregation, is equivalent to specifying WHERE TRUE and HAVING TRUE, respectively. For present purposes, therefore, we need consider the effect of the GROUP BY clause, only, in determining the grouped table $G$.
- Suppose table $T$ has $n T$ rows. Then arranging those rows into groups can produce at most $n T$ groups; in other words, the grouped table $G$ has $n G$ groups for some $n G(n G \leq n T)$, and the overall result, obtained by applying the SELECT clause to $G$, thus has $n G$ rows.

■ Now suppose $n T$ is zero (i.e., table $T$ is empty); then $n G$ must clearly be zero as well (i.e., table $G$, and hence the result of the SELECT expression overall, must both be empty as well).

- In particular, therefore, the expression

```
SELECT COUNT ( * ) AS X
FROM
GROUP BY ( )
```

-which is, recall, the expanded form of SELECT COUNT(*) AS X FROM S—ought logically to produce the result shown on the left, not the one shown on the right, if table S happens to be empty:
$\square$


In fact, however, it produces the result on the right. How? Answer: By special casing. Here's a direct quote from the standard: "If there are no grouping columns, then the result of the <group by clause> is the grouped table consisting of $T$ as its only group." In other words, while grouping an empty table in SQL does indeed (as argued above) produce an empty set of groups in general, the case where the set of grouping columns is empty is special; in that case, it produces a set containing exactly one group, that group being identical to the empty table $T$. In the example, therefore, the COUNT operator is applied to an empty group, and thus "correctly" returns the value zero.

Now, you might be thinking the discrepancy here is hardly earth shattering; you might even be thinking the result on the right above is somehow "better" than the one on the left. But (to state the obvious) there's a logical difference between the two, and-to quote Wittgenstein again-all logical differences are big differences. Logical mistakes like the one under discussion are simply unacceptable in a system that's meant to be solidly based on logic, as relational systems are. End of aside.

## Empty Arguments

The foregoing aside does raise another issue, however. Let $a g g$ be an aggregate operator. What should happen if agg is invoked on an empty argument? For example, given our usual sample data values, what value should the following statement assign to X ?

```
X := SUM ( SP WHERE SNO = 'S5' , QTY ) ;
```

The answer, of course, is zero; as explained in Chapter 6 under the discussion of $n$-adic join, zero is the identity value with respect to addition, and the sum of no numbers is therefore zero. More generally, in fact, if:
a. An aggregate operator is invoked on an empty argument, and
b. That invocation is essentially just shorthand for repeated invocation of some dyadic operator (e.g., the dyadic operator is " + " in the case of SUM), and
c. An identity value exists for that dyadic operator, and
d. The semantics of the aggregate operator in question do not require the result of an invocation to be a value that actually appears in the aggregate in question,
then the result of that invocation is that identity value. For the aggregate operators discussed in this section, identity values (and hence the result returned if the argument is empty) are as follows: ${ }^{11}$

- AND: TRUE.
- OR and XOR: FALSE.
- COUNT and SUM: Zero. Note: The type of the result in these cases is INTEGER (for COUNT) and the type of the specified argument expression (for SUM). By way of example, if relvar P is currently empty, COUNT(P) returns 0 and SUM(P,WEIGHT) returns 0.0.
- AVG: Since asking for the average of an empty set is effectively asking for zero to be divided by zero, the only reasonable response is to raise an exception (and careful coding might sometimes be called for, therefore).
- MAX and MIN: By definition, asking for the maximum or minimum of some set of values is asking for some specific value from within that set. If the set in question happens to be empty, therefore, the only reasonable response is, again, to raise an exception (and careful coding might again sometimes be called for, therefore).

Note: For AVG, MAX, and MIN, we're currently investigating the possibility of providing additional operators called (say) AVGX, MAXX, and MINX, respectively, each of which takes a further argument $X$ in addition to the aggregate argument as such (the idea being that $X$ denotes the value to be returned if the aggregate argument is empty). For the purposes of the present book, I'll assume this scheme has indeed been implemented. Be aware, however, that the idea is still only a tentative one at this time (in particular, it might be better to provide a means of handling exceptions in general, instead of just a means of handling these special cases in particular).

## IMAGE RELATIONS bis

In this section, I just want to present a series of examples that show the usefulness of image relations in connection with aggregate operators as discussed in the previous section.

Example 1: Get suppliers for whom the total shipment quantity, taken over all shipments for the supplier in question, is less than 1000 .

S WHERE SUM ( !!SP , QTY ) < 1000
For any given supplier, the expression SUM(!!SP,QTY) denotes, precisely, the total shipment quantity for the supplier in question. An equivalent formulation without the image relation is:

S WHERE SUM ( SP MATCHING RELATION \{ TUPLE \{ SNO SNO \} \} , QTY ) < 1000

[^79]Here for interest is an SQL "analog"-"analog" in quotes because actually there's a trap in this example; the SQL expression shown is not quite equivalent to the Tutorial D expressions shown previously (why not?):

```
SELECT S.*
FROM S , SP
WHERE S.SNO = SP.SNO
GROUP BY S.SNO , S.SNAME , S.STATUS , S.CITY
HAVING SUM ( SP.QTY ) < 1000
```

Incidentally, I can't resist pointing out in passing that (as this example suggests) SQL lets us say "S.*" in the SELECT clause but not in the GROUP BY clause, where it would make just as much sense.

Example 2: Get suppliers with fewer than three shipments.

S WHERE COUNT ( !!SP ) < 3

Example 3: Get suppliers for whom the minimum shipment quantity is less than half the maximum shipment quantity (taken over all shipments for the supplier in question in both cases).

S WHERE MINX ( !!SP , QTY , 0 ) < 0.5 * MAXX ( !!SP , QTY , 0 )

Example 4: Get shipments such that at least two other shipments involve the same quantity.

```
SP WHERE COUNT ( !!( SP RENAME { SNO AS SN , PNO AS PN } ) ) > 2
```

This example is very contrived, but it illustrates the point that we might occasionally need to do some attribute renaming in connection with image relation references. In the example, the renaming is needed in order to ensure that the image relation we want, in connection with a given shipment tuple, is defined in terms of attribute QTY only. The introduced names SN and PN are arbitrary.

I remark in passing that the RENAME invocation in this example-

```
SP RENAME { SNO AS SN , PNO AS PN }
```

-illustrates the "multiple" form of the RENAME operator. The individual renamings in such a RENAME invocation are effectively executed in parallel. Note: As a consequence of this fact, a RENAME of the following form can be used to switch the names of the specified attributes:

```
R RENAME { A AS B, B AS A }
```

Similar "multiple" forms are defined for various other operators, too, including EXTEND in particular (I'll give an example later).

Example 5: Update suppliers for whom the total shipment quantity, taken over all shipments for the supplier in question, is less than 1000 , reducing their status to half its previous value.

```
UPDATE S WHERE SUM ( !!SP , QTY ) < 1000 : { STATUS := 0.5 * STATUS } ;
```


## SUMMARIZATION

Definition: Let relations $r 1$ and $r 2$ be such that $r 2$ has the same heading as some projection of $r 1$, and let the attributes of $r 2$ be $A, B, \ldots, C$. Then the summarization SUMMARIZE $r 1$ PER ( $r 2$ ) : $\{X:=$ summary $\}$ is a relation with (a) heading the heading of $r 2$ extended with attribute $X$ and (b) body the set of all tuples $t$ such that $t$ is a tuple of $r 2$ extended with a value $x$ for attribute $X$. That value $x$ is computed by evaluating summary over all tuples of $r l$ that have the same value for attributes $A, B, \ldots, C$ as tuple $t$ does. Observe that the result has cardinality equal to that of $r 2$ and degree equal to that of $r 2$ plus one. The type of $X$ in that result is the type of summary. Note: I assume for simplicity that relations $r 1$ and $r 2$ don't already have an attribute named $X$.

Here's an example (which I'll label SX1-"SUMMARIZE Example 1"-for purposes of subsequent reference):

SUMMARIZE SP PER ( S \{ SNO \} ) : \{ PCT := COUNT ( PNO ) \}
Given our usual sample values, the result looks like this:

| SNO | PCT |
| ---: | ---: |
| S1 | 6 |
| S2 | 2 |
| S3 | 1 |
| S4 | 3 |
| S5 | 0 |

In other words, the result contains one tuple for each tuple in the PER relation-i.e., one tuple for each of the five supplier numbers, in the example-extended with the corresponding count.

Aside: Note carefully that the "summary" COUNT(PNO)-I deliberately call it a "summary" and not an expression, because it isn't an expression (at least, not in the usual Tutorial D sense)-in the foregoing SUMMARIZE is not an invocation of the COUNT aggregate operator. That aggregate operator takes a relation as its argument. By contrast, the argument to COUNT in the foregoing SUMMARIZE is an attribute: an attribute of some relation, of course, but just which relation is specified only indirectly. In fact, the syntactic construct COUNT(PNO) is really very special-it has no meaning outside the context of an appropriate SUMMARIZE, and it can't be used outside that context. (Note, therefore, that my earlier criticisms of COUNT and the rest in SQL, to the effect that they can't appear "stand alone," apply with just as much force to Tutorial D's "summaries.") All of which begins to make it look as if SUMMARIZE might be not quite respectable, in a way, and it might be nice if we could replace it by something better ... See the section "Summarization bis," later. End of aside.

As a shorthand, if relation $r 2$ doesn't merely have the same heading as some projection of relation $r 1$ but actually is such a projection, the PER specification can be replaced by a BY specification, as in this example ("Example SX2"):

SUMMARIZE SP BY \{ SNO \} : \{ PCT := COUNT (PNO ) \}

Here's the result:

| SNO | PCT |
| ---: | ---: |
| S1 | 6 |
| S2 | 2 |
| S3 | 1 |
| S4 | 3 |

As you can see, this result differs from the previous one in that it contains no tuple for supplier S5. That's because $\mathrm{BY}\{\mathrm{SNO}\}$ in the example is defined to be shorthand for PER (SP $\{\mathrm{SNO}\}$ )-SP, because SP is what we want to summarize-and relvar SP doesn't contain a tuple for supplier S5.

Now, Example SX2 can be expressed in SQL as follows:

```
SELECT SNO , COUNT ( ALL PNO ) AS PCT
FROM
GROUP BY SNO
```

As this example suggests, summarizations-as opposed to "aggregations"-are typically formulated in SQL by means of a SELECT expression with an explicit GROUP BY clause (but see later!). Points arising:

You can think of such expressions as being evaluated as follows. First, the table specified by the FROM clause is partitioned into set of disjoint "groups"-actually tables-as specified by the grouping column(s) in the GROUP BY clause; result rows are then obtained, one for each group, by computing the specified summary (or summaries, plural) for that group and appending other items as specified by the SELECT item commalist. Note: The SQL analog of the term summary is "set function"; the term is doubly inappropriate, however, because (a) the argument to such a function isn't a set but a bag, in general, and (b) the result isn't a set either.

- It's safe to specify just SELECT, not SELECT DISTINCT, in the example because (a) the result table is guaranteed to contain just one row for each group, by definition, and (b) each group contains just one value for the grouping column(s), again by definition.
- The ALL specification could be omitted from the COUNT invocation in this example, because for set functions ALL is the default. (In the example, in fact, it makes no difference whether ALL or DISTINCT is specified, because the combination of supplier number and part number is a key for table SP.)
- The set function $\operatorname{COUNT}\left({ }^{*}\right)$ is a special case-it applies, not to values in some column (as, e.g., SUM(...) does), but to rows in some table. (In the example, the specification COUNT(PNO) could be replaced by COUNT(*) without changing the result.)

Now let's get back to Example SX1. Here's a possible SQL formulation of that example:

```
SELECT S.SNO , ( SELECT COUNT ( PNO )
    FROM SP
    WHERE SP.SNO = S.SNO ) AS PCT
FROM S
```

The important point here is that the result now does contain a row for supplier S5, because by definition (thanks to the FROM clause, which takes the form FROM S) that result contains one row for each supplier number in table S, not table SP. As you can see, this formulation differs from the one given for Example SX2-the one that missed supplier S5-in that it doesn't include a GROUP BY clause, and it doesn't do any grouping (at least, not overtly).

Aside: By the way, there's another trap for the unwary here. As you can see, the second item in the SELECT item commalist in the foregoing SQL expression-i.e., the subexpression (SELECT ... S.SNO) AS PCT-is of the form subquery AS name (and the subquery in question is in fact a scalar one). Now, if that very same text were to appear in a FROM clause, the "AS name" specification would be understood as defining a name for the table denoted by that subquery. ${ }^{12}$ In the SELECT clause, however, that same "AS name" specification is understood as defining a name for the pertinent column of the overall result. It follows that the following SQL expression is not logically equivalent to the one shown above:

```
SELECT S.SNO , ( SELECT COUNT ( PNO ) AS PCT
    FROM SP
    WHERE SP.SNO = S.SNO )
FROM S
```

With this formulation, the table $t$ that's returned by evaluation of the subquery has a column called PCT. That table $t$ is then doubly coerced to the sole scalar value it contains, producing a column value in the overall result-but (believe it or not) that column in the overall result is not called PCT; instead, it has no name. End of aside.

To revert to the main thread of the discussion: As a matter of fact, Example SX2 could also be expressed in SQL without using GROUP BY, as follows:

```
SELECT DISTINCT SPX.SNO , ( SELECT COUNT ( SPY.PNO )
    FROM SP AS SPY
    WHERE SPY.SNO = SPX.SNO ) AS PCT
FROM SP AS SPX
```

As these examples suggest, SQL's GROUP BY clause is in fact logically redundant-any relational expression that can be represented with it can also be represented without it. Be that as it may, there's another point that needs to be made here. Suppose Example SX1 had requested, not the count of part numbers, but the sum of quantities, for each supplier:

```
SUMMARIZE SP PER ( S { SNO } ) : { TOTQ := SUM ( QTY ) }
```

Given our usual sample values, the result looks like this:

[^80]| SNO | TOTQ |
| ---: | ---: |
| S1 | 1300 |
| S2 | 700 |
| S3 | 200 |
| S4 | 900 |
| S5 | 0 |

By contrast, this SQL expression-

```
SELECT S.SNO , ( SELECT SUM ( QTY )
    FROM SP
    WHERE SP.SNO = S.SNO ) AS TOTQ
FROM S
```

-gives a result in which the TOTQ value for supplier S5 is shown as null, not zero. That's because (as mentioned earlier) if any SQL set function other than COUNT(*) or COUNT is invoked on an empty argument, the result is incorrectly defined to be null. To get the correct result, therefore, we need to use COALESCE, as follows:

```
SELECT S.SNO , ( SELECT COALESCE ( SUM ( QTY ) , 0 )
    FROM SP
    WHERE SP.SNO = S.SNO ) AS TOTQ
FROM S
```

Suppose now that Example SX1 had asked for the sum of quantities for each supplier, but only where that sum is greater than 250 :

```
( SUMMARIZE SP PER ( S { SNO } ) : { TOTQ := SUM ( QTY ) } )
```

                                    WHERE TOTQ > 250
    Result:

| SNO | TOTQ |
| ---: | ---: |
| S1 | 1300 |
| S2 | 700 |
| S4 | 900 |

The "natural" SQL formulation of this query would be:

```
SELECT SNO , SUM ( QTY ) AS TOTQ
FROM SP
GROUP BY SNO
HAVING SUM ( QTY ) > 250 /* not TOTQ > 250 !!! */
```

But it could also be formulated like this:

```
SELECT DISTINCT SPX.SNO , ( SELECT SUM ( SPY.QTY )
    FROM SP AS SPY
    WHERE SPY.SNO = SPX.SNO ) AS TOTQ
FROM SP AS SPX
WHERE ( SELECT SUM ( SPY.QTY )
        FROM SP AS SPY
        WHERE SPY.SNO = SPX.SNO ) > 250
```

As this example suggests, HAVING, like GROUP BY, is also logically redundant-any relational expression that can be represented with it can also be represented without it. So GROUP BY and HAVING could both be dropped from SQL without any loss of relational functionality! And while it might be true that the GROUP BY and HAVING versions of some query are often more succinct, ${ }^{13}$ it's also true that they sometimes deliver the wrong answer. For example, consider what would happen in the foregoing example if we had wanted the sum to be less than, instead of greater than, 250. Simply replacing " $>$ " by "<" in the GROUP BY / HAVING formulation does not work. (Does it work in the other formulation?) Recommendations: If you do use GROUP BY or HAVING, make sure the table you're summarizing is the one you really want to summarize (typically suppliers rather than shipments, in terms of the examples in this section). Also, be on the lookout for the possibility that some summarization is being done on an empty set, and use COALESCE wherever necessary.

There's one more thing I need to say about GROUP BY and HAVING. Consider the following SQL expression:

```
SELECT SNO , CITY , SUM ( QTY ) AS TOTQ
FROM S NATURAL JOIN SP
GROUP BY SNO
```

Observe that CITY appears in the SELECT item commalist here but isn't one of the grouping columns. That appearance is legitimate, however, because table $S$ is subject to a certain functional dependency-see Chapter 8-according to which each SNO value in that table has just one corresponding CITY value (again, in that table); what's more, the SQL standard includes rules according to which the system will in fact be aware of that functional dependency. As a consequence, even though it isn't a grouping column, CITY is still known to be single valued per group, and it can therefore indeed appear in the SELECT clause as shown (also in the HAVING clause, if there is one).

Of course, it's not logically wrong-though there might be negative performance implications-to specify the column as a grouping column anyway, as here:

```
SELECT SNO, CITY, SUM ( QTY ) AS TOTQ
FROM S NATURAL JOIN SP
GROUP BY SNO , CITY
```


## SUMMARIZATION bis

The SUMMARIZE operator has been part of Tutorial D since its inception. With the introduction of image relations, however, that operator became logically redundant-and while there might be reasons (perhaps pedagogic ones) to retain it, the fact is that most summarizations can be more succinctly expressed by means of EXTEND. ${ }^{14}$

[^81]Recall Example SX1 from the previous section ("For each supplier, get the supplier number and a count of the number of parts supplied"). The SUMMARIZE formulation looked like this:

```
SUMMARIZE SP PER ( S { SNO } ) : { PCT := COUNT ( PNO ) }
```

Here by contrast is an equivalent EXTEND formulation:

```
EXTEND S { SNO } : { PCT := COUNT ( !!SP ) }
```

(Since the combination $\{\mathrm{SNO}, \mathrm{PNO}\}$ is a key for relvar SP , there's no need to project the image relation on $\{\mathrm{PNO}\}$ before computing the count.) As the example suggests, EXTEND is certainly another context in which image relations make sense; in fact, they're arguably even more useful in this context than they are in WHERE clauses.

The rest of this section consists of more examples. I've continued the numbering from the examples in the section "Image Relations bis."

Example 6: For each supplier, get supplier details and total shipment quantity, taken over all shipments for the supplier in question.

```
EXTEND S : { TOTQ := SUM ( !!SP , QTY ) }
```

Example 7: For each supplier, get supplier details and total, maximum, and minimum shipment quantity, taken over all shipments for the supplier in question.

```
EXTEND S : { TOTQ := SUM ( !!SP , QTY ) ,
    MAXQ := MAXX ( !!SP , QTY , O ) ,
    MINQ := MINX ( !!SP , QTY , 0 ) }
```

Note the use of the multiple form of EXTEND in this example.

Example 8: For each supplier, get supplier details, total shipment quantity taken over all shipments for the supplier in question, and total shipment quantity taken over all shipments for all suppliers.

```
EXTEND S : { TOTQ := SUM ( !!SP , QTY ) ,
    GTOTQ := SUM ( SP , QTY ) }
```

Result:

| SNO | TOTQ | GTOTQ |
| ---: | ---: | ---: |
| S1 | 1300 | 3100 |
| S2 | 700 | 3100 |
| S3 | 200 | 3100 |
| S4 | 900 | 3100 |
| S5 | 0 | 3100 |

Example 9: For each city $c$, get $c$ and the maximum and minimum shipment quantities for all shipments for which the supplier city and part city are both $c$.

```
WITH ( TEMP := S JOIN SP JOIN P ) :
EXTEND TEMP { CITY } : { MAXQ := MAXX ( !!TEMP , QTY , 0 ) ,
    MINQ := MINX ( !!TEMP , QTY , O ) }
```

The point of this rather contrived example is to illustrate the usefulness of WITH, in connection with "SUMMARIZE-type" EXTENDs in particular, in avoiding the need to write out some possibly lengthy subexpression several times. Note: This book generally has little to say about performance matters, but I think it's worth pointing out that we would surely expect the system, in examples like this one, to evaluate the pertinent subexpression once instead of several times. In other words, the use of WITH can be one of those nice win-win situations that are good for both the user and the DBMS.

## GROUP, UNGROUP, AND RELATION VALUED ATTRIBUTES

Recall from Chapter 2 that relations with relation valued attributes (RVAs for short) are legal. Fig. 7.1 below shows relations R1 and R4 from Figs. 2.1 and 2.2 in that chapter; R4 has an RVA and R1 doesn't, but the two relations clearly represent the same information.

R1 | SNO | PNO |
| :---: | :---: |
| S2 | P1 |
| S2 | P2 |
| S3 | P2 |
| S4 | P2 |
| S4 | P4 |
| S4 | P5 |

| SNO | PNO_REL |
| :---: | :---: |
| S2 | PNO |
|  | P1 P2 |
| S3 | PNO |
|  | P2 |
| S 4 | PNO |
|  | P2 P4 P5 |

Fig. 7.1: Relations R1 and R4 from Figs. 2.1 and 2.2 in Chapter 2
Now, we obviously need a way to map between relations without RVAs and relations with them, and that's the purpose of the GROUP and UNGROUP operators. I don't want to go into a lot of detail on those operators here; let me just say that, given the relations shown in Fig. 7.1, the expression

```
R1 GROUP ( { PNO } AS PNO_REL )
```

will produce R4, and the expression

```
R4 UNGROUP ( PNO_REL )
```

will produce R1.
By the way, it's worth noting that the following expression-

```
EXTEND R1 { SNO } : { PNO_REL := !!R1 }
```

-will produce exactly the same result as the GROUP example shown above. In other words, GROUP can be defined in terms of EXTEND and image relations. Now, I'm not suggesting that we get rid of our useful GROUP operator; quite apart from anything else, a language that had an explicit UNGROUP operator (as Tutorial D does) but no explicit GROUP operator could certainly be criticized on ergonomic grounds, if nothing else. But it's at least interesting, and perhaps pedagogically helpful, to note that the semantics of GROUP can so easily be explained in terms of EXTEND and image relations.

And by the way again: If R4 includes exactly one tuple for supplier number $S x$, say, and if the PNO_REL value in that tuple is empty, then the result of the foregoing UNGROUP will contain no tuple at all for supplier number Sx. For further details, I refer you to my book An Introduction to Database Systems (see Appendix G) or the book Databases, Types, and the Relational Model: The Third Manifesto (again, see Appendix G), by Hugh Darwen and myself.

The SQL counterparts to GROUP and UNGROUP are quite complex, and I don't propose to go into details here. However, I will at least show SQL analogs of the Tutorial D examples above. Here first is the GROUP example: ${ }^{15}$

```
SELECT DISTINCT X.SNO ,
    TABLE ( ( SELECT Y.PNO
                                FROM R1 AS Y
                                WHERE Y.SNO = X.SNO ) ) AS PNO_REL
FROM R1 AS X
```

And here's the UNGROUP example:

```
SELECT Y.SNO , X.PNO
FROM R4 AS Y , UNNEST ( ( SELECT Z.PNO REL
    FROM R4 AS Z
    WHERE Z.SNO = Y.SNO ) ) AS X
```

Note: I can't help pointing out a certain irony in SQL's version of the GROUP example. As you can see, the SQL expression in that example involves a subquery in the SELECT clause. Of course, a subquery denotes a table; in SQL, however, that table is often coerced-in the context of a SELECT clause in particular-to a single row, or even to a single column value from within that single row. In the case at hand, however, we don't want any such coercion; so we have to tell SQL explicitly, by means of the TABLE keyword, not to do what it normally would do (by default, as it were) in such a context.

## RVAs Make Outer Join Unnecessary

There are several further points worth making in connection with relation valued attributes. First of all, RVAs make outer join unnecessary! Second, it turns out they're sometimes necessary even in base relvars. Third, they're

[^82]conceptually necessary anyway in order to support relational comparison operations. And fourth, they make it desirable to support certain additional aggregate operators. I'll elaborate on each of these points in turn.

I'll begin by showing a slightly more complicated example of an RVA. Consider the following Tutorial D expression:

```
EXTEND S : { PQ := !!SP }
```

Suppose we evaluate this expression and assign the result to a relvar SPQ. A sample value for SPQ , corresponding to our usual sample values for relvars S and SP , is shown (in outline) in Fig. 7.2 below. Attribute PQ is relation valued.


Fig. 7.2: Relvar SPQ (sample value)

Now consider the following SQL expression:

```
SELECT SNO , SNAME , STATUS , CITY , PNO , QTY
FROM S NATURAL LEFT OUTER JOIN SP
```

The result of evaluating this expression is shown (again in outline) in Fig. 7.3 opposite.
Observe now that with our usual sample values, the set of shipments for supplier S5 is empty, and that:

- In Fig. 7.2, that empty set of shipments is represented by an empty set.
- In Fig. 7.3, by contrast, that empty set is represented by nulls (indicated by shading in the figure).

To represent an empty set by an empty set seems like such an obviously good idea! In fact, as I said earlier, there would be no need for outer join at all if RVAs were properly supported. Thus, one advantage of RVAs is that they
deal more elegantly with the problem that outer join is intended to solve than outer join itself does-and I'm tempted to say that this fact all by itself, even if there were no other advantages, is a big argument in favor of RVAs.

| SNO | SNAME | STATUS | CITY | PNO | QTY |
| :--- | :--- | ---: | :--- | :--- | :--- |
| S1 | Smith | 20 | London | P1 | 300 |
| S1 | Smith | 20 | London | P2 | 200 |


| Si | Smith | 20 | London | P6 |
| :--- | :--- | :--- | :--- | :--- |
| S2 | Jones | 10 | Paris | P1 |
| S2 | Jones | 10 | Paris | P2 |
| So | 400 |  |  |  |

S.
S5 Adams

Fig. 7.3: Left outer join of S and SP (sample value)
At the risk of laboring the obvious, I'd like to say too that if there aren't any shipments for supplier S5, it means, to repeat, that the set of shipments for supplier S5 is empty (and that's exactly what the relation in Fig. 7.2 says). It certainly doesn't mean that supplier S5 supplies some unknown part in some unknown quantity; and yet unknown is-and in fact was originally and explicitly intended to be-the way null is usually interpreted. So Fig. 7.3 not only involves nulls (which as we saw in Chapter 4 is bad news for all kinds of reasons), it actually misrepresents the semantics of the situation.

## RVAs in Base Relvars

Let's look at some typical operations involving relvar SPQ (Fig. 7.2). Consider first the following queries:

- Get supplier numbers for suppliers who supply part P2.

```
( ( SPQ UNGROUP ( PQ ) ) WHERE PNO = 'P2' ) { SNO }
```

- Get part numbers for parts supplied by supplier S2.

```
( ( SPQ WHERE SNO = 'S2' ) UNGROUP ( PQ ) ) { PNO }
```

As you can see, the natural language versions of these two queries are symmetric, but the Tutorial D formulations on the RVA design (Fig. 7.2) aren't. By contrast, Tutorial D formulations of the same queries against our usual (non RVA) design are symmetric, as well as being simpler than their RVA counterparts:

```
( SP WHERE PNO = 'P2' ) { SNO }
( SP WHERE SNO = 'S2' ) { PNO }
```

In fact, the queries on the RVA design effectively involve mapping that design to the non RVA design anyway (that's what the UNGROUPs do).

Similar remarks apply to updates and constraints. For example, suppose we need to update the database to show that supplier S2 supplies part P5 in a quantity of 500. Here are Tutorial D formulations on (a) the non RVA design, (b) the RVA design:

```
INSERT SP RELATION { TUPLE { SNO 'S2' , PNO 'P5' , QTY 500 } } ;
UPDATE SPQ WHERE SNO = 'S2' :
    { INSERT PQ RELATION { TUPLE { PNO 'P5' , QTY 500 } } } ;
```

Once again, the natural language requirement is stated in a symmetric fashion; its formulation in terms of the non RVA design is symmetric too; but its formulation in terms of the RVA design isn't (in fact, it's quite cumbersome). And, of course, the reason for this state of affairs is that the non RVA design itself is asymmetric-in effect, it regards parts as subordinate to suppliers, instead of giving parts and suppliers equal treatment, as it were.

Examples like the ones discussed above tend to suggest that RVAs in base relvars are probably a bad idea (certainly relvar SPQ in particular isn't very well designed). But this position might better be seen as a guideline rather than an absolute limitation, because in fact there are cases-comparatively rare ones perhaps-where a base relvar with an RVA is exactly the right design. A sample value for such a relvar (SIBLING) is shown in Fig. 7.4 below. The intended interpretation is that the persons identified within any given PERSONS value are all siblings of one another, and have no other siblings. Thus, Amy and Bob are siblings; Cal, Don, and Eve are siblings; and Fay is an only child. Note that the relvar has just one attribute (an RVA) and three tuples. Note too that the sole key involves an RVA.


Fig. 7.4: Relvar SIBLING (sample value)
Note: It's important to understand that no non RVA relation exists that represents exactly the same information, no more and no less, as the relation shown in Fig. 7.4 does. In particular, if we ungroup that relation as follows-

```
SIBLING UNGROUP ( SIBS )
```

-we lose the information as to who's the sibling of whom.

## RVAs Are Necessary for Relational Comparisons

Consider once again this example from the section on image relations earlier in this chapter:

```
S WHERE ( !!SP ) { PNO } = P { PNO }
```

("suppliers who supply all parts"). Clearly, the boolean expression in the WHERE clause here involves a relational comparison (actually an equality comparison). Recall now from Chapter 6 that an expression of the form $r$ WHERE $b x$ denotes a restriction as such only if $b x$ is a restriction condition, and $b x$ is a restriction condition if and only if
every attribute reference in $b x$ identifies some attribute of $r$ and there aren't any relvar references. In the example, therefore, the boolean expression isn't a genuine restriction condition, because (a) it involves some references to attributes that aren't attributes of S and (b) it also involves some relvar references (to relvars SP and P). In fact, the example overall is really shorthand for something that might look like this:

```
WITH ( R1 := EXTEND S : { X := ( !!SP ) { PNO } } ,
    R2 := EXTEND R1 : { Y := P { PNO } } ) :
R2 WHERE X = Y
```

Now the boolean expression in the WHERE clause (in the last line) is indeed a genuine restriction condition. Observe, however, that attributes X and Y are both RVAs. As the example suggests, therefore, RVAs are always involved, at least implicitly, whenever relational comparisons are performed.

## Aggregate Operators

Consider again relvar SPQ, with sample value as shown in Fig. 7.2. Attribute PQ is relation valued. And just as it makes sense (and is useful) to define, e.g., numeric aggregate operators such as SUM on numeric attributes, so it makes sense, and is useful, to define relational aggregate operators on relation valued attributes. For example, the following expression returns the union of all of the relations currently appearing as values of attribute PQ in relvar SPQ:

UNION ( SPQ , PQ )
Or equivalently (why exactly is this equivalent?):

```
UNION ( SPQ { PQ } )
```

Tutorial D supports the following relation valued aggregate operators: UNION, D_UNION, and INTERSECT. And SQL has analogs of UNION and INTERSECT (though not D_UNION); however, they're called, not UNION and INTERSECT as one might reasonably have expected, but FUSION and INTERSECTION [sic], respectively. (It would be very naughty of me to suggest that if union is called FUSION, then intersection ought surely to be called FISSION, so I won't.)

## "WHAT IF" QUERIES

"What if" queries are a frequent requirement; they're used to explore the effect of making certain changes without actually having to make (and subsequently unmake, possibly) the changes in question. Here's an example ("What if parts in Paris were in Nice instead and their weight was doubled?"):

```
```

EXTEND P WHERE CITY = 'Paris' : $\mid$ WITH T1 AS

```
```

EXTEND P WHERE CITY = 'Paris' : $\mid$ WITH T1 AS
\{ CITY := 'Nice' ,
\{ CITY := 'Nice' ,
WEIGHT : = 2 * WEIGHT \}

```
        WEIGHT : = 2 * WEIGHT \}
```

WITH T1 AS
( SELECT P.*
FROM P

```
            WHERE CITY = 'Paris' ) ,
```

            WHERE CITY = 'Paris' ) ,
    T2 AS
    T2 AS
    ( SELECT P.* , 'Nice' AS NC ,
    ( SELECT P.* , 'Nice' AS NC ,
                                    2 * WEIGHT AS NW
                                    2 * WEIGHT AS NW
            FROM T1 )
            FROM T1 )
    SELECT PNO, PNAME , COLOR ,
SELECT PNO, PNAME , COLOR ,
NW AS WEIGHT ,
NW AS WEIGHT ,
NC AS CITY
NC AS CITY
FROM
FROM
T2

```
    T2
```

As you can see, the Tutorial D expression on the left here makes use of EXTEND once again. Note, however, that the target attributes in the assignments in braces aren't "new" attributes, as they normally are for EXTEND; instead, they're attributes already existing in the specified relation. What the expression does is this: It yields a relation containing exactly one tuple $t 2$ for each tuple $t l$ in the current value of relvar P for which the city is Paris-except that, in that tuple $t 2$, the weight is double that in tuple $t 1$ and the city is Nice, not Paris. ${ }^{16}$ In other words, the expression overall is shorthand for the following (and this expansion should help you understand the SQL version of the query):

```
WITH ( R1 := P WHERE CITY = 'Paris' ,
    R2 := EXTEND R1 : { NC := 'Nice' , NW := 2 * WEIGHT } ,
    R3 := R2 { ALL BUT CITY , WEIGHT } ) :
R3 RENAME { NC AS CITY , NW AS WEIGHT }
```

And now I can take care of some unfinished business from Chapter 5. In that chapter, I said the relational UPDATE operator was shorthand for a certain relational assignment, but the details were a little more complicated than they were for INSERT and DELETE. Now I can explain those details. By way of example, consider the following UPDATE statement:

```
UPDATE P WHERE CITY = 'Paris' :
    { CITY := 'Nice' , WEIGHT := 2 * WEIGHT } ;
```

This statement is logically equivalent to the following relational assignment:

```
P := ( P WHERE CITY f 'Paris' )
    UNION
    ( EXTEND ( P WHERE CITY = 'Paris' ) :
    { CITY := 'Nice' , WEIGHT := 2 * WEIGHT } ) ;
```

Alternatively, recall from Chapter 5 that "updating relvar $R$ " really means we're replacing the relation $r 1$ that's the original value of $R$ by another relation $r 2$, where $r 2$ is computed as ( $r 1$ MINUS $s 1$ ) UNION $s 2$ for certain relations $s l$ and $s 2$. In the case at hand, using " "der" to denote "is defined as," we have:

```
s1 \def P WHERE CITY = 'Paris'
s2 \stackrel{def EXTEND ( P WHERE CITY = 'Paris' ) :}{=}=0
    { CITY := 'Nice' , WEIGHT := 2 * WEIGHT } )
```

[^83]Thus, the expanded form of the UPDATE becomes:

```
P := ( P MINUS s1 ) UNION s2 ;
```

Note: Actually, we could safely replace MINUS and UNION here by I_MINUS and D_UNION, respectively, and we could safely drop the parentheses. (In both cases, why?)

## A NOTE ON RECURSION

Consider the following edited extract from Exercise 5.16 in Chapter 5:

The well known bill of materials application involves a relvar-PP, say-showing which parts contain which parts as immediate components. Of course, immediate components are themselves parts, and they can have further immediate components of their own.

Fig. 7.5 below shows (a) a sample value for that relvar PP and (b) the corresponding transitive closure TC. ${ }^{17}$ The predicates are as follows:

- PP: Part PX contains part PY as an immediate component.
- TC: Part PX contains part PY as a component at some level (not necessarily immediate).

| $P \mathrm{PX}$ | PY |
| :---: | :---: |
| $=\mathrm{P1}$ | P 2 |
| P 1 | P 3 |
| P 2 | P 4 |
| P 3 | P 4 |
| P 4 | P 5 |
| P 5 | P 6 |


| PX | PY |
| :---: | :---: |
| P1 | P2 |
| P1 | P3 |
| P2 | P4 |
| P3 | P4 |
| P4 | P5 |
| P5 | P6 |
| P1 | P 4 |
| P2 | P5 |
| P3 | P5 |
| P4 | P6 |
| P1 | P5 |
| P2 | P6 |
| P3 | P6 |

Fig. 7.5: Relvars PP and TC (sample values)

Given a (relation) value $p p$ for relvar PP , the relation $t c$ that's the transitive closure of $p p$ can be defined as follows:

Definition: The pair $(p x, p y)$ appears in $t c$ if and only if:

[^84]a. It appears in $p p$, or
b. There exists some $p z$ such that the pair $(p x, p z)$ appears in $p p$ and the pair $(p z, p y)$ appears in $t c$.

In other words, if we think of $p p$ as representing a directed graph, with a node for each part and an arc from each node to each corresponding immediate component node, then $(p x, p y)$ appears in the transitive closure if and only if there's a path in that graph from node $p x$ to node $p y$. Observe that the definition involves a recursive reference to relation $t c$.

Aside: In practice relvar PP would probably have a QTY attribute as well (showing how many instances of the immediate component part PY are needed to make one instance of part PX), and we would probably want to compute, not just the transitive closure as such, but also the total number of instances of part PY needed to make one instance of part PX: the gross requirements problem. I ignore this refinement for simplicity. End of aside.

It's also possible to define the transitive closure procedurally (and iteratively):

```
TC := PP ;
do until TC reaches a "fixpoint" ;
    WITH ( R1 := PP RENAME { PY AS PZ } ,
            R2 := TC RENAME { PX AS PZ } ,
            R3 := ( R1 JOIN R2 ) { PX , PY } ) :
    TC := TC UNION R3 ;
end ;
```

Loosely speaking, this code works by repeatedly forming an intermediate result consisting of the union of (a) the previous intermediate result and (b) a relation computed on the current iteration. The process is repeated until that intermediate result reaches a fixpoint (i.e., until it ceases to grow). Note: It's easy to see the code is very inefficient!-in effect, each iteration repeats the entire computation of the previous one. In fact, it's little more than a direct implementation of the original (recursive) definition. However, it could clearly be made more efficient if desired. Similar remarks apply to all of the code samples in the present section.

Turning now to Tutorial $\mathbf{D}$, we could define a recursive operator (TCLOSE) to compute the transitive closure as follows: ${ }^{18}$

```
OPERATOR TCLOSE ( XY RELATION { X ... , Y ... } )
            RETURNS RELATION { X ... , Y ... } ;
    RETURN ( WITH ( R1 := XY RENAME { Y AS Z } ,
                R2 := XY RENAME { X AS Z } ,
                R3 := ( R1 JOIN R2 ) { X , Y } ,
                R4 := XY UNION R3 ) :
            IF R4 = XY THEN R4 /* unwind recursion */
            ELSE TCLOSE ( R4 ) /* recursive invocation */
            END IF ) ;
```

END OPERATOR ;

[^85]Now, e.g., the expression $\operatorname{TCLOSE}(p p)$ will return the transitive closure of $p p$. Hence, for example, the expression

```
( TCLOSE ( PP ) WHERE PX = 'P1') { PY }
```

will give the "bill of materials" for part P1, and the expression

```
( TCLOSE ( PP ) WHERE PY = 'P6') { PX }
```

will give the "where used" list for part P6. Note: Computing the bill of materials for a given part is sometimes referred to as part explosion; likewise, computing the "where used" list for a given part is referred to as part implosion.

Now, SQL too supports what it calls "recursive queries." Here's an SQL expression to compute the transitive closure of PP:

```
WITH RECURSIVE TC ( PX , PY ) AS
    ( SELECT PP.PX , PP.PY
        FROM PP
        UNION CORRESPONDING
        SELECT PP.PX , TC.PY
        FROM PP , TC
        WHERE PP.PY = TC.PX )
SELECT PX , PY
FROM TC
```

As you can see, this expression too is a more or less direct transliteration of the original recursive definition.
Note: This book deliberately has very little to say about commercial SQL products. However, I'd like to offer a brief remark here regarding Oracle specifically. As you might know, Oracle has had some recursive query support for many years. By way of example, the query "Explode part P1" can be expressed in Oracle as follows:

```
SELECT LEVEL , PY
FROM PP
CONNECT BY PX = PY
START WITH PX = 'P1'
```

I don't want to explain in detail how this expression is evaluated-but I do want to show the result it produces, given the sample data of Fig. 7.5. Here it is:

| LEVEL | PY |
| ---: | :---: |
| 1 | P 2 |
| 2 | P 4 |
| 3 | P 5 |
| 4 | P 6 |
| 1 | P 3 |
| 2 | P 4 |
| 3 | P 5 |
| 4 | P 6 |

Note carefully that this result isn't a relation (and the relational closure property has thereby been violated). First of all, it contains some duplicate rows; for example, the row ( $2, \mathrm{P} 4$ ) appears twice. More important, those duplicate rows are not duplicate rows as we usually understand them in SQL (as they might appear in, say, the result of evaluating some SQL expression in the standard); that is, they aren't just "saying the same thing twice," as I put it in Chapter 4. To spell the point out, one of those two ( $2, \mathrm{P} 4$ ) rows reflects the path in the graph from part P1 to part P 4 via part P2; the other reflects the path in the graph from part P1 to part P4 via part P3. Thus, if we deleted one of those rows, we would lose information.

> Aside: Actually the same kind of problem can arise in the SQL standard if the recursive query in question uses UNION ALL instead of UNION DISTINCT-as in practice such queries very typically do. Further details are beyond the scope of this book; however, if you try to code the gross requirements problem in SQL you might see for yourself why it's tempting, at least superficially, to use UNION ALL. End of aside.

Note too that in addition to the foregoing violations, the ordering of the rows in the result is significant as well. For example, the reason we know the first ( $2, \mathrm{P} 4$ ) row corresponds to the path from P1 to P4 via P2 specifically is because it immediately follows the row corresponding to the path from P1 to its immediate component P2. Thus, if we reordered the rows, again we would lose information.

## Cycles

Consider Fig. 7.5 once again. Suppose the relation $p p$ shown as a value for relvar PP in that figure additionally contained a tuple representing, say, the pair (P5,P1). Then there would be a cycle in the data (actually two cycles, one involving parts P1-P2-P4-P5-P1 and one involving parts P1-P3-P4-P5-P1). In the case of bill of materials, such cycles should presumably not be allowed to occur, since they make no sense. Sometimes, however, they do make sense; the classic example is a transportation network, in which there are routes from, say, New York (JFK) to London (LHR), London to Paris (CDG), and Paris back to New York again (as well as routes in all of the reverse directions, of course).

Now, the existence of a cycle in the data has no effect on the transitive closure as such. But it does have the potential to cause an infinite loop in certain kinds of processing. For example, a query to find travel routes from New York to Paris might-if we're not careful-produce results as follows:

```
JFK - LHR - CDG
JFK - LHR - JFK - LHR - CDG
JFK - LHR - JFK - LHR - JFK - LHR - CDG
etc., etc. etc.
```

Of course, it might at least be possible to formulate the query in such a way as to exclude segments in which the destination city is JFK (since we certainly don't want a route that takes us back to where we started). But even this trick will still allow routes such as:

```
JFK - ORD - LHR - ORD - LHR - ORD - LHR - ... - CDG
```

(ORD = Chicago). Moreover, it still won't prevent an infinite loop. Now, we might prevent the infinite loop as such by rejecting routes involving, say, more than four segments; but under such a scheme we could still get, e.g., the route JFK-ORD-LHR-ORD-CDG. Clearly, what we need is a more general mechanism that will allow the query to recognize when a given node in the graph has been previously visited. And SQL does in fact include a feature, the CYCLE clause, that can be used in recursive queries to achieve such an effect. The specifics are a little complicated, and I don't want to get into details here; suffice it to say that the CYCLE clause provides a means of
tagging nodes (i.e., rows) as they're visited, and then stopping the recursion if a tagged node is subsequently encountered again. For more details, I refer you to the standard document itself.

## WHAT ABOUT ORDER BY?

The last topic I want to address in this chapter is ORDER BY (just ORDER, in Tutorial D). Now, despite the title of this chapter, ORDER BY isn't actually part of the relational algebra; in fact, as I pointed out in Chapter 1, it isn't a relational operator at all, because it produces a result that isn't a relation (it does take a relation as input, but it produces something else-namely, a sequence of tuples-as output). Please don't misunderstand me here; I'm not saying ORDER BY isn't useful; however, I am saying it can't sensibly appear in a relational expression ${ }^{19}$ (unless it's treated simply as a "no op," I suppose). By definition, therefore, the following expressions, though legal, aren't relational expressions as such:

```
S MATCHING SP ORDER ( ASC SNO )
```

```
SELECT DISTINCT S.*
FROM S , SP
WHERE S.SNO = SP.SNO
ORDER BY SNO ASC
```

That said, I'd like to point out that for a couple of reasons ORDER BY is actually a rather strange operator. First, it effectively works by sorting tuples into some specified sequence-and yet "<" and ">" aren't defined for tuples, as we know from Chapter $3 .^{20}$ Second, it's not a function. All of the operators of the relational algebra described in this book-in fact, all read-only operators, as that term is usually understood-are functions, meaning there's always just one possible output for any given input. By contrast, ORDER BY can produce several different outputs from the same input. As an illustration of this point, consider the effect of the operation ORDER BY CITY on our usual suppliers relation. Clearly, this operation can return any of four distinct results, corresponding to the following sequences (I'll show just the supplier numbers, for simplicity):

Note: It would be remiss of me not to mention in passing that although the operators of the relational algebra described in this book are indeed functions, most of them have counterparts in SQL that aren't. This state of affairs is due to the fact that, as explained in Chapter 2, SQL sometimes defines the result of the comparison $v 1=v 2$ to be TRUE even when $v 1$ and $v 2$ are distinct. For example, consider the character strings 'Paris' and 'Paris ', respectively (note the trailing space in the latter); these values are clearly distinct, and yet SQL sometimes regards them as equal. As explained in Chapter 2, therefore, certain SQL expressions are "possibly nondeterministic." Here's a simple example:

[^86]```
SELECT DISTINCT CITY FROM S
```

If one supplier has CITY 'Paris' and another 'Paris', then the result will include either 'Paris' or 'Paris ' (or possibly both), but which result we get might not be defined. We could even legitimately get one result on one day and another on another, even if the database hasn't changed at all in the interim. You might like to meditate on the implications of this state of affairs.

## EXERCISES

7.1 For each of the following Tutorial D expressions on the suppliers-and-parts database, give both (a) an SQL analog and (b) an informal interpretation of the expression (i.e., a corresponding predicate) in natural language. Also show the result of evaluating the expressions, given our usual sample values for relvars $\mathrm{S}, \mathrm{P}$, and SP .

```
a. S MATCHING ( SP WHERE PNO = 'P2' )
b. S NOT MATCHING ( SP WHERE PNO = 'P2' )
c. P WHERE ( !!SP ) { SNO } = S { SNO }
d. P WHERE SUM ( !!SP , QTY ) < 500
e. P WHERE TUPLE { CITY CITY } \in S { CITY }
f. EXTEND S : { TAG := 'Supplier' }
g. EXTEND ( S MATCHING ( SP WHERE PNO = 'P2' ) ) :
                                    { TRIPLE_STATUS := 3 * STATUS }
h. EXTEND ( P JOIN SP ) : { SHIPWT := WEIGHT * QTY }
i. EXTEND P : { GMWT := WEIGHT * 454 , OZWT := WEIGHT * 16 }
j. EXTEND P : { SCT := COUNT ( !!SP ) }
k. EXTEND S :
    { NP := COUNT ( ( SP RENAME { SNO AS X } ) WHERE X = SNO ) }
1. SUMMARIZE S BY { CITY } : { SUM_STATUS := SUM ( STATUS ) }
m. SUMMARIZE ( S WHERE CITY = 'London' ) PER ( TABLE_DEE ) :
    { N := \overline{COUNT ( SNO ) }}
```

Note: Tutorial D allows the PER clause to be omitted, in which case PER (TABLE_DEE) is assumed by default. The foregoing SUMMARIZE could therefore be simplified slightly, thus:

```
SUMMARIZE ( S WHERE CITY = 'London' ) : { N := COUNT ( SNO ) }
```

```
n. EXTEND SP WHERE SNO = 'S1' : { SNO := 'S7' , QTY = 0.5 * QTY }
```

7.2 In what circumstances (if any) are $r 1$ MATCHING $r 2$ and $r 2$ MATCHING $r 1$ equivalent?
7.3 Show that RENAME isn't primitive.
7.4 Give an expression involving EXTEND instead of SUMMARIZE that's logically equivalent to the following:

```
SUMMARIZE SP PER ( S { SNO } ) : { NP := COUNT ( PNO ) }
```

7.5 Consider the following Tutorial D expressions. Which if any are equivalent to which of the others? Show an SQL analog in each case.
a. SUMMARIZE $r \operatorname{PER}(r$ \{ \} ) : \{ CT := SUM ( 1 ) \}
b. SUMMARIZE $r$ PER ( TABLE_DEE ) : \{ CT := SUM ( 1 ) \}
c. SUMMARIZE $r$ BY $\}:\{C T:=\operatorname{SUM}(1)\}$
d. EXTEND TABLE_DEE : \{ CT $:=\operatorname{COUNT}(r)\}$
7.6 Consider the relational aggregate operators UNION and INTERSECT. What do you think these operators should return if their argument (a set of relations) happens to be empty?
7.7 Let relation R4 in Fig. 7.1 denote the current value of some relvar. If R4 is as described in Chapter 2, what's the predicate for that relvar?
7.8 Let $r$ be the relation denoted by the following Tutorial $\mathbf{D}$ expression:

```
SP GROUP ( { } AS X )
```

What does $r$ look like, given our usual sample value for SP? Also, what does the following expression yield?

```
r UNGROUP ( X )
```

7.9 Write Tutorial D and/or SQL expressions for the following queries on the suppliers-and-parts database:
a. Get the total number of parts supplied by supplier S 1 .
b. Get supplier numbers for suppliers whose city is first in the alphabetic list of such cities.
c. Get city names for cities in which at least two suppliers are located.
d. Get city names for cities in which at least one supplier or at least one part is located, but not both.
e. Get part numbers for parts supplied by all suppliers in London.
f. Get suppliers who supply at least all parts supplied by supplier S2.
7.10 Let relation $p p$ be as defined in the section "A Note on Recursion" and let TCLOSE be the transitive closure operator. What does the expression TCLOSE(TCLOSE $(p p)$ ) denote?
7.11 Given our usual sample values for the suppliers-and-parts database, what does the following Tutorial D expression denote?

```
EXTEND S : { PNO_REL := ( !!SP ) { PNO } }
```

7.12 Let the relation returned by the expression in the previous exercise be kept as a relvar called SSP. What do the following updates do?

```
INSERT SSP RELATION
    { TUPLE { SNO 'S6' , SNAME 'Lopez' , STATUS 30 , CITY 'Madrid' ,
        PNO_REL RELATION { TUPLE { PNO 'P5' } } } } ;
UPDATE SSP WHERE SNO = 'S2' :
    { INSERT PNO_REL RELATION { TUPLE { PNO 'P5' } } } ;
```

7.13 Using relvar SSP from the previous exercise, write expressions for the following queries:
a. Get pairs of supplier numbers for suppliers who supply exactly the same set of parts.
b. Get pairs of part numbers for parts supplied by exactly the same set of suppliers.
7.14 A quota query is a query that specifies a desired limit, or quota, on the cardinality of the result: for example, the query "Get the two heaviest parts," for which the quota is two. Give Tutorial D and SQL formulations of this query. Given our usual data values, what exactly do these formulations return?
7.15 Using the explicit SUMMARIZE operator, how would you deal with the query "For each supplier, get the supplier number and the sum of distinct shipment quantities for shipments by that supplier"?
7.16 Given a revised version of the suppliers-and-parts database that looks like this-

```
S { SNO } /* suppliers */
SP { SNO , PNO } /* supplier supplies part */
SJ { SNO , JNO } /* supplier supplies project */
```

-give Tutorial D and SQL formulations of the query "For each supplier, get supplier details, the number of parts supplied by that supplier, and the number of projects supplied by that supplier." For Tutorial D, give both EXTEND and SUMMARIZE formulations.
7.17 What does the following Tutorial D expression mean?

```
S WHERE ( !!(!!SP) ) { PNO } = P { PNO }
```

7.18 Is there a logical difference between the following two Tutorial $\mathbf{D}$ expressions? If so, what is it?

```
EXTEND TABLE_DEE : { NSP := COUNT ( SP ) }
EXTEND TABLE_DEE : { NSP := COUNT ( !!SP ) }
```

7.19 Give an example of a join that's not a semijoin and a semijoin that's not a join. When exactly are the expressions $r 1$ JOIN $r 2$ and $r 1$ MATCHING $r 2$ equivalent?
7.20 Let relations $r 1$ and $r 2$ be of the same type, and let $t 1$ be a tuple in $r 1$. For that tuple $t 1$, then, what exactly does the expression !! $r 2$ denote? And what happens if $r 1$ and $r 2$ aren't just of the same type but are in fact the very same relation?
7.21 What's the logical difference, if any, between the following SQL expressions?

```
SELECT COUNT ( * ) FROM S
SELECT SUM ( 1 ) FROM S
```

7.22 By definition, ORDER BY can't appear in a relational expression (or table expression, rather, in SQL). So where can it appear?

## Chapter 8

## SQL and Constraints

A foolish consistency is the hobgoblin of little minds.<br>—Ralph Waldo Emerson: "Self Reliance" (1841)

I've touched on the topic of integrity constraints here and there in previous chapters, but it's time to get more specific. Here's a rough definition, repeated from Chapter 1: An integrity constraint (constraint for short) is basically just a boolean expression that must evaluate to TRUE. Constraints in general are so called because they constrain the values that can legally appear as values of some variable; but the ones we're interested in here are the ones that apply to variables in the database (i.e., relvars) specifically. ${ }^{1}$ Such constraints fall into two broad categories, type constraints and database constraints; in essence, a type constraint defines the values that constitute a given type, and a database constraint further constrains the values that can appear in a given database (where "further" means over and above the constraints imposed by the pertinent type constraints). As usual, in what follows I'll discuss these ideas in both relational and SQL terms.

By the way, it's worth noting that constraints in general can be regarded as a formal version of what some people call business rules. Now, this latter term doesn't really have a precise definition (at least, not one that's universally accepted); in general, however, a business rule is a declarative statement-emphasis on declarative-of some aspect of the enterprise the database is meant to serve, and statements that constrain the values of variables in the database certainly fit that loose definition. In fact, I'll go further. In my opinion, constraints are really what database management is all about. The database is supposed to represent some aspect of the enterprise in question; that representation is supposed to be as faithful as possible, in order to guarantee that decisions made on the basis of what the database says are right ones; and constraints are the best mechanism we have for ensuring that the representation is indeed as faithful as possible. Constraints are crucial, and proper DBMS support for them is crucial as well.

A note on terminology: Let constraint $C$ apply to relvar $R$ (e.g., $C$ might be the constraint that a certain subset of the heading of $R$ constitutes a key for $R$ and thus has the uniqueness property). Then we say relvar $R$ is subject to constraint $C$; equivalently, we say constraint $C$ holds in relvar $R$. Further, let $r$ be a relation of the same type as $R$. If evaluating constraint $C$ on relation $r$ yields TRUE, we say $r$ satisfies $C$; otherwise we say it violates $C$. Of course, if $r$ violates $C$, it can't be assigned to $R$; at all times, therefore, the current value of $R$ satisfies all constraints to which $R$ is subject, necessarily and by definition.

## TYPE CONSTRAINTS

As we saw in Chapter 2, one of the things we have to do when we define a type is specify the values that make up that type-and that's effectively what a type constraint does. Now, in the case of system defined types, it's the system that carries out this task, and there's not much more to be said. In the case of user defined types, by contrast, there certainly is more to say, much more. So let's suppose for the sake of the example that shipment quantities,

[^87]instead of being of the system defined type INTEGER, are of some user defined type (QTY, say). Here then is a possible Tutorial D definition for that type:

```
TYPE QTY
    POSSREP QPR
            { Q INTEGER
                CONSTRAINT Q \geq 0 AND Q \leq5000 } ;
```


## Explanation:

Line 1 just says we're defining a type called QTY.

- Line 2 says quantities have a "possible representation" called QPR. Now, physical representations are always hidden from the user, as we know from Chapter 2. However, Tutorial D requires every TYPE statement to include at least one POSSREP specification, ${ }^{2}$ indicating that values of the type in question can possibly be represented in some specific way; and unlike physical representations, possible representations-which we usually abbreviate to just possreps-definitely are visible to the user (in the example, users do definitely know that quantities have a possrep called QPR). Note carefully, however, that there's no suggestion that the specified possible representation is the same as any physical representation, whatever that happens to be; it might be or it might not, but either way it makes no difference to the user.

Line 3 says the possrep QPR has a single component, called Q, which is of type INTEGER; in other words, values of type QTY can possibly be represented by integers (and users are aware of this fact).

Finally, line 4 says those integers must lie in the range 0 to 5000 inclusive. Thus, lines 2-4 together define valid quantities to be, precisely, values that can possibly be represented by integers in the specified range, and it's that definition that constitutes the type constraint for type QTY. Observe, therefore, that such constraints are specified not in terms of the type as such but, rather, in terms of a possrep for the type. Indeed, one of the reasons the possrep concept is required in the first place is precisely to serve as a vehicle for formulating type constraints, as I think the example shows.

Here's a slightly more complicated example:

```
TYPE POINT
    POSSREP CARTESIAN { X RATIONAL , Y RATIONAL
    CONSTRAINT SQRT ( X ** 2 + Y ** 2 ) \leq 100.0 } ;
```

Type POINT denotes geometric points in two-dimensional space; it has a possrep called CARTESIAN with two components called X and Y (corresponding, presumably, to cartesian coordinates); those components are both of type RATIONAL; and there's a CONSTRAINT specification that (in effect) says the only points we're interested in are those that lie on or inside a circle with center the origin and radius 100 (SQRT = nonnegative square root). Note: I used a type called POINT in an example in Chapter 2, as you might recall, but I deliberately didn't show the POSSREP and CONSTRAINT specifications for that type at that time; tacitly, however, I was assuming the type had a possrep called POINT, not CARTESIAN. See the subsection immediately following.

[^88]
## Selectors and THE_Operators

Before I continue with my discussion of type constraints as such, I need to digress for a few moments in order to clarify a few issues raised by the QTY and POINT examples.

Recall from Chapter 2 that types generally have certain associated selector and THE_ operators. Well, those operators are intimately related to the possrep notion; in fact, selector operators correspond one to one to possreps, and THE_ operators correspond one to one to possrep components. Here are some examples:

1. QPR ( 250 )

This expression is a selector invocation for type QTY. The selector has the same name, QPR, as the sole possrep for that type; it takes an argument that corresponds to, and is of the same type as, the sole component of that possrep; and it returns a quantity (that is, a value of type QTY). Note: In practice, possreps often have the same name as the associated type (I used different names in the QTY example to make it clear there's a logical difference between the possrep and the type, but it would be much more usual not to). In fact, Tutorial D has a syntax rule that says we can omit the possrep name from the TYPE statement entirely if we want to, in which case it defaults to the associated type name. So let's simplify the QTY type definition accordingly:

TYPE QTY POSSREP \{ Q INTEGER CONSTRAINT Q $\geq 0$ AND Q $\leq 5000$ \} ;

Now the possrep and the corresponding selector are both called QTY, and the selector invocation shown above becomes just QTY(250)-which is the style I used for selectors in Chapter 2, if you care to go back and look. I'll assume this revised definition for type QTY from this point forward, barring explicit statements to the contrary.
2. QTY ( A + B )

The argument to a QTY selector invocation can be specified as an expression of arbitrary complexity (just so long as it's of type INTEGER, of course). If that expression is a literal, as it was in the previous example, then the selector invocation is a literal in turn; thus, a literal is a special case of a selector invocation (as in fact we already know from Chapter 2).
3. THE_Q ( QZ )

This expression is a THE_ operator invocation for type QTY. The operator is named THE_Q because Q is the name of the sole component of the sole possrep for type QTY; it takes an argument (specified as an arbitrarily complex expression) of type QTY; and it returns the integer that's the Q component of the possrep for that specific argument.

As for type POINT, let's first redefine that type so that the possrep has the same name as the type, as in the QTY example above:

TYPE POINT POSSREP \{ X RATIONAL , Y RATIONAL CONSTRAINT ... \} ;

Now continuing with the examples:

## 172 Chapter $8 / S Q L$ and Constraints

4. POINT ( $5.7,-3.9$ )

This expression is a POINT selector invocation (actually a POINT literal).
5. THE_X ( P )

This expression returns the RATIONAL value that's the X coordinate of the cartesian possible representation of the point that's the current value of variable P (which must be of type POINT).

Just as an aside, let me draw your attention to the fact that (as I said earlier) Tutorial D requires a TYPE statement to include at least one POSSREP specification. The fact is, Tutorial D does allow a type to have several distinct possreps. POINT is a good example-we might well want to define two distinct possreps for points, to reflect the fact that points in two-dimensional space can possibly be represented by either cartesian or polar coordinates. Temperatures provide another example-again, we might want to define two possreps, to reflect the fact that temperatures can be possibly represented in either degrees Celsius or degrees Fahrenheit. Further details don't belong in a book of this nature; I'll just note for the record that SQL has no analogous feature.

## More on Type Constraints

Now let's get back to type constraints as such. Suppose I had defined type QTY as follows, with no explicit CONSTRAINT specification:

```
TYPE QTY POSSREP { Q INTEGER } ;
```

This definition is defined to be shorthand for the following:

```
TYPE QTY POSSREP { Q INTEGER CONSTRAINT TRUE } ;
```

Given this definition, anything that could possibly be represented by an integer would be a legitimate QTY value, and so type QTY would necessarily still have an associated type constraint, albeit a rather weak one. In other words, the specified possrep defines an a priori constraint for the type, and the CONSTRAINT specification effectively imposes an additional constraint, over and above that a priori one. (Informally, however, we often take the term "type constraint" to refer to what's stated in the CONSTRAINT specification as such.)

Now, one important issue I've ducked so far is the question of when type constraints are checked. In fact, they're checked whenever some selector is invoked. Assume again that values of type QTY are such that they must be possibly representable as integers in the range 0 to 5000 inclusive. Then the expression QTY(250) is an invocation of the QTY selector, and that invocation succeeds. By contrast, the expression QTY(6000) is also such an invocation, but it fails. In fact, it should be obvious that we can never tolerate an expression that's supposed to denote a value of some type $T$ but in fact doesn't; after all, "a value of type $T$ that's not a value of type $T$ ' is a contradiction in terms. Since, ultimately, the only way any expression can yield a value of type $T$ is by means of some invocation of some selector for type $T$, it follows that no variable-in particular, no relvar-can ever be assigned a value that's not of the right type.

One last point to close this section: Declaring anything to be of some particular type imposes a constraint on that thing, by definition. ${ }^{3}$ In particular, declaring attribute QTY of relvar SP (for example) to be of type QTY

[^89]imposes the constraint that no tuple in relvar SP will ever contain a value in the QTY position that fails to satisfy the QTY type constraint. (As an aside, I note that this constraint on attribute QTY is an example of what's sometimes called an attribute constraint.)

## TYPE CONSTRAINTS IN SQL

As I'm sure you noticed, I didn't give SQL versions of the examples in the previous section. That's because, believe it or not, SQL doesn't support type constraints at all!-apart from the rather trivial a priori ones, of course. For example, although SQL would certainly let you create a user defined type called QTY and specify that quantities must be representable as integers, it wouldn't let you say those integers must lie in a certain range. In other words, an SQL definition for that type might look like this:

```
CREATE TYPE QTY AS INTEGER FINAL ;
```

(The keyword FINAL here just means type QTY doesn't have any proper subtypes. Subtypes in general are beyond the scope of this book.)

With the foregoing SQL definition, all available integers (including negative ones!) will be regarded as denoting valid quantities. If you want to constrain quantities to some particular range, therefore, you'll have to specify an appropriate database constraint-in practice, probably a base table constraint (see the section "Database Constraints in SQL")-on each and every use of the type. For example, if column QTY in base table SP is defined to be of type QTY instead of type INTEGER, then you might need to extend the definition of that table as follows (note the CONSTRAINT specification at the end):

```
CREATE TABLE SP
    ( SNO VARCHAR(5) NOT NULL ,
        PNO VARCHAR(6) NOT NULL,
        QTY QTY NOT NULL ,
        UNIQUE ( SNO , PNO ) ,
        FOREIGN KEY ( SNO ) REFERENCES S ( SNO ) ,
        FOREIGN KEY ( PNO ) REFERENCES P ( PNO ) ,
        CONSTRAINT SPQC CHECK ( QTY >= QTY(0) AND
                                QTY <= QTY(5000) ) ) ;
```

The expressions $\operatorname{QTY}(0)$ and $\operatorname{QTY}(5000)$ in the CONSTRAINT specification here can be regarded as QTY selector invocations. I remind you, however, that selector isn't an SQL term (and nor is THE_operator); as indicated in Chapter 2, in fact, the situation regarding selectors and THE_ operators in SQL is too complicated to describe in detail in this book. Suffice it to say that analogs of those operators are usually available, though they aren't always provided "automatically" as they are in Tutorial D.

For interest, I also show an SQL definition for type POINT (and here I've specified NOT FINAL instead of FINAL, just to illustrate the possibility):

CREATE TYPE POINT AS
( X NUMERIC $(5,1)$, $\mathrm{Y} \operatorname{NUMERIC}(5,1)$ ) NOT FINAL ;
To say it again, then, SQL doesn't really support type constraints. The reasons for the omission are complex-they have to do with type inheritance and are therefore beyond the scope of this book-but the implications are serious. Recommendation: Wherever possible, use database constraints to make up for the omission, as in the QTY example above. Of course, this recommendation might lead to a lot of duplicated effort,
but such duplication is better than the alternative: namely, bad data in the database. See Exercise 8.8 at the end of the chapter.

Aside: Although I've said type inheritance in general is beyond the scope of this book, I can't resist pointing out one implication of SQL's lack of support for type constraints in particular: namely, that SQL has to permit absurdities such as nonsquare squares (by which I mean, more precisely, values of a user defined type SQUARE whose sides are of different lengths and are thus not in fact squares at all). End of aside.

## DATABASE CONSTRAINTS

A database constraint constrains the values that can appear in a given database. In Tutorial D, such constraints are specified by means of a CONSTRAINT statement (or some logically equivalent shorthand); in SQL, they're specified by means of a CREATE ASSERTION statement (or, again, some equivalent shorthand). I don't want to get into details of those shorthands-at least, not yet-because they're essentially just a matter of syntax; for now, let me stay with the "longhand" forms. Here then are some examples (Tutorial D on the left and SQL on the right as usual):

## Example 1:

```
CONSTRAINT CX1 IS_EMPTY
    ( S WHERE STATU\overline{S}<1
        OR STATUS > 100 ) ;
```

CREATE ASSERTION CX1 CHECK
( NOT EXISTS
( SELECT * FROM S
WHERE STATUS < 1
OR STATUS > 100 ) ) ;

Constraint CX1 says: Supplier status values must be in the range 1 to 100 inclusive. This constraint involves just a single attribute of a single relvar. Note in particular that it can be checked for a given supplier tuple by examining just that tuple in isolation-there's no need to look at any other tuples in the relvar or any other relvars in the database. For that reason, such constraints are sometimes referred to, informally, as tuple constraints, or row constraints in SQL-though this latter term is also used in SQL to mean, more specifically, a row constraint that can't be formulated as a column constraint (see the section "Database Constraints in SQL"). Now, all such usages ought really to be deprecated, because constraints constrain updates, and as we saw in Chapter 5 there's no such thing as a tuple or row level update in the relational world. (By the same token, there's no such thing as a tuple variable, or row variable, in a relational database.) However, the terms can sometimes be convenient, and so-somewhat against my own better judgment-I'll be using them occasionally myself in what follows.

Recall now that (as mentioned in a footnote in Chapter 7) tuple constraints can alternatively be formulated in terms of the aggregate operator AND. Here by way of example is such a formulation for constraint CX1:

```
CONSTRAINT CX1 AND ( S , STATUS \geq 1 AND STATUS \leq 100 ) ;
```

This formulation says, in effect, that the expression STATUS $\geq 1$ AND STATUS $\leq 100$ must evaluate to TRUE for every tuple in S . As you can see, the desired constraint ("Status values must be greater than or equal to 1 and less than or equal to $100^{\prime \prime}$ ) is stated a little more directly with this formulation than it was with the IS_EMPTY version, where it had to be stated in the negative ("Status values mustn't be less than 1 or greater than 100 ").

What about SQL? Well, SQL's analog of AND is called EVERY. Here's an SQL formulation of constraint CX1 that makes use of that operator:

```
CREATE ASSERTION CX1 CHECK
    ( ( SELECT COALESCE ( EVERY ( STATUS >= 1 AND STATUS <= 100 ) ,
        FROM S ) = TRUE ) ;
```

As you can see, however, this formulation isn't as user friendly as the Tutorial $\mathbf{D}$ version, for at least two reasons:

- First, EVERY, unlike Tutorial D's AND, returns null, not TRUE, if its argument is empty; hence the need for that COALESCE.

Second, I pointed out in Chapter 7 that SQL doesn't really support aggregate operators anyway, and the present example brings that point home. To be specific, the parenthesized subexpression SELECT ... FROM S is, of course, a table expression; hence it denotes, not a truth value as such, but rather a one-row, onecolumn table that contains such a truth value. In fact, that subexpression, parentheses included, is a scalar subquery. As explained in Chapter 2, then, asking for that subquery and the literal value TRUE to be tested for equality causes a double coercion to occur; in other words, the truth value is effectively extracted from the table and then tested to see whether it's equal to TRUE.

The net of this discussion is that EVERY isn't nearly as useful for the formulation of row constraints in SQL as AND is for the formulation of tuple constraints in Tutorial D.

Aside: The foregoing might be a little unfair to SQL. To be specific, I think-according to my own reading of the standard-that it would be possible to simplify the example by omitting both the COALESCE and the explicit comparison with TRUE, thereby reducing the CHECK clause portion of the assertion to just:

```
CHECK ( ( SELECT EVERY ( STATUS >= 1 AND STATUS <= 100 ) FROM S ) ) ;
```

But these simplifications rely on several aspects of SQL that are, to put matters politely, hardly very respectable. First of all, note that the double enclosing parentheses are necessary-the outer parentheses enclose a subquery, which requires parentheses of its own. Second, the subquery in question is in fact a scalar subquery, and the table it returns gets doubly coerced to the single value-actually a truth value-in the single column of the single row of the table in question (see Chapter 12). Third, if the EVERY invocation in fact returns a null, that null is considered to stand for the truth value UNKNOWN (see Chapter 4). Fourth, if the boolean expression in a CHECK clause evaluates to UNKNOWN, that UNKNOWN gets coerced to TRUE! (See the answer to Exercise 8.20 g in Appendix F for further discussion of this last point.) Speaking for myself, therefore, I would far rather include both the COALESCE and the comparison with TRUE, in the interest of explicitness if nothing else. End of aside.

## Example 2:

```
CONSTRAINT CX2 IS EMPTY
    ( S WHERE CITY = 'London',
```

CREATE ASSERTION CX2 CHECK
( NOT EXISTS
( SELECT * FROM S
WHERE CITY = 'London'
AND STATUS <> 20) ;

Constraint CX2 says: Suppliers in London must have status 20. This constraint involves two distinct attributes; however, it's still the case, as it was with constraint CX1, that the constraint can be checked for a given
supplier tuple by examining just that tuple in isolation (hence it too is a tuple or row constraint). Here for interest are AND and EVERY formulations:

```
CONSTRAINT CX2
    AND ( S , CITY f 'London'
    OR STATUS = 20 ) ;
```

```
CREATE ASSERTION CX2 CHECK
( ( SELECT COALESCE
    ( EVERY ( CITY <> 'London' OR
                                    STATUS = 20 ) ,
                                TRUE )
        FROM S ) = TRUE ) ;
```


## Example 3:

CONSTRAINT CX3

```
```

    COUNT ( S ) =
    ```
```

    COUNT ( S ) =
            COUNT ( S { SNO } ) ;
    ```
```

            COUNT ( S { SNO } ) ;
    ```
```

```
CREATE ASSERTION CX3 CHECK 
CREATE ASSERTION CX3 CHECK 
CREATE ASSERTION CX3 CHECK 
```

                1
    Constraint CX3 says: Every supplier has a unique supplier number; in other words, $\{\mathrm{SNO}\}$ is a superkey-actually, of course, it's a key-for relvar S (recall from Chapter 5 that a superkey is a superset of a key, loosely speaking). Like constraints CX1 and CX2, this constraint still involves just one relvar; however, it can't be checked for a given supplier tuple by examining just that tuple in isolation, and so it isn't a tuple or row constraint. Points arising:

- In practice, of course, it's very unlikely that constraint CX3 would be specified in longhand as shown-some kind of explicit KEY shorthand is almost certainly preferable. I give the longhand form merely to make the point that such shorthands are indeed, in the final analysis, just shorthands. ${ }^{4}$
- As you can see, the SQL formulation of constraint CX3 involves an invocation of the SQL UNIQUE operator. That operator returns TRUE if and only if every row within its argument table is distinct; in the example, therefore, the UNIQUE invocation returns TRUE if and only if no two rows in table S have the same supplier number. Note, incidentally, that the SELECT expression in that invocation must-for once-definitely not specify DISTINCT! (Why not?) I'll have more to say about SQL's UNIQUE operator in Chapter 10.

Here for interest is an SQL formulation of constraint CX3 that more closely resembles the Tutorial D formulation: ${ }^{5}$

CREATE ASSERTION CX3 CHECK
( ( SELECT COUNT ( ALL SNO ) FROM S ) =
( SELECT COUNT ( DISTINCT SNO ) FROM S ) ;

[^90]
## Example 4:

CONSTRAINT CX4
COUNT ( S \{ SNO \} ) =
COUNT ( S \{ SNO , CITY \} ) ;

```
CREATE ASSERTION CX4 CHECK
( ( SELECT COUNT ( SNO )
        FROM S ) =
    ( SELECT COUNT ( * )
        FROM (\underset{ SELECT SNO , CITY}{\mathrm{ FROM S ) ) ; ;}}=\mp@code{M}
```

Constraint CX4 says: Whenever two suppliers have the same supplier number, they also have the same city. In other words, a certain functional dependency (FD) holds in relvar S—namely, an FD from \{SNO\} to \{CITY\}. In practice, as I'm sure you know, that FD would more usually be expressed as follows:
$\{$ SNO \} $\rightarrow$ \{ CITY \}

Here's a precise definition:
Definition: Let $A$ and $B$ be subsets of the heading of relvar $R$. Then the functional dependency (FD) $A \rightarrow B$ holds in $R$ if and only if, in every relation that's a legal value for $R$, whenever two tuples have the same value for $A$, they also have the same value for $B$.

The FD $A \rightarrow B$ is read as " $B$ is functionally dependent on $A$," or " $A$ functionally determines $B$," or, more simply, just " $A$ arrow $B$." As the example shows, however, a functional dependency is basically just another integrity constraint (though, like constraint CX3, it isn't a tuple or row constraint).

Now, as noted in Chapter 5, the fact that relvar S is subject to this particular FD is a logical consequence of the fact that $\{\mathrm{SNO}\}$ is a key for that relvar. For that reason, there's no need to state it explicitly, just so long as the fact that $\{\mathrm{SNO}\}$ is a key is stated explicitly. But not all FDs are consequences of keys. For example, suppose it's the case that if two suppliers are in the same city, then they must have the same status. This hypothetical new constraint (which is not satisfied by our usual sample values, please note) is clearly an FD:
\{ CITY \} $\rightarrow$ \{ STATUS \}

It can thus be stated in the style of constraint CX4 (see Exercise 8.22 at the end of the chapter).
Now, you might be thinking some shorthand syntax would be desirable for stating FDs, similar to the shorthand we already have for stating keys. Myself, I don't think so, because although not all FDs are consequences of keys in general, all FDs will almost certainly be consequences of keys if the database is well designed. In other words, the very fact that FDs are hard to state if the database is badly designed might be seen as a small argument in favor of not designing the database badly in the first place! Note: By "well designed" here, I really mean fully normalized. Normalization as such is beyond the scope of this book (it's covered in depth in the book Normal Forms and All That Jazz, which is a companion to the present book-see Appendix G). Of course, relational (or SQL) statements and expressions will work regardless of whether or not the relvars (or tables) are fully normalized. But I should at least point out that those statements and expressions will often be easier to formulate (and, contrary to popular opinion, will often perform better too) if the tables are fully normalized. However, normalization as such is primarily a database design issue, not a relational model or SQL issue.

## Example 5:

```
CONSTRAINT CX5 IS_EMPTY
    ( ( S JOIN SP )
        WHERE STATUS < 20
        AND PNO = 'P6' ) ;
```

```
CREATE ASSERTION CX5 CHECK
( NOT EXISTS
( SELECT *
        FROM S NATURAL JOIN SP
        WHERE STATUS < 20
    AND PNO = 'P6' ) ) ;
```

Constraint CX5 says: No supplier with status less than 20 can supply part P6. Observe that this constraint involves (better: interrelates) two distinct relvars, S and SP. In general, a database constraint might involve, or interrelate, any number of distinct relvars. Terminology: A constraint that involves just a single relvar is known, informally, as a relvar constraint (sometimes a single relvar constraint, for emphasis); a constraint that involves two or more distinct relvars is known, informally, as a multirelvar constraint. (Thus, constraints CX1-CX4 were single relvar constraints, while constraint CX5 is a multirelvar constraint.) All of these terms are somewhat deprecated, however, for reasons to be discussed in the next chapter in connection with what's called The Principle of Interchangeability.

## Example 6:

```
CONSTRAINT CX6 CREATE ASSERTION CX6 CHECK
    SP { SNO } \subseteq S { SNO } ;
```

```
    ( NOT EXISTS
```

    ( NOT EXISTS
        ( SELECT SNO FROM SP
        ( SELECT SNO FROM SP
        EXCEPT CORRESPONDING
        EXCEPT CORRESPONDING
        SELECT SNO FROM S ) ) ;
    ```
        SELECT SNO FROM S ) ) ;
```

Constraint CX6 says: Every supplier number in SP must appear in S. As you can see, the Tutorial D formulation involves a relational comparison; SQL doesn't support relational comparisons, however, and so we have to indulge in some circumlocution in the SQL formulation. However, given that $\{\mathrm{SNO}\}$ is a key-in fact, the sole key-for relvar S, it's clear that constraint CX6 is basically just the foreign key constraint from SP to S . The usual FOREIGN KEY syntax can thus be regarded as shorthand for constraints like CX6.

## DATABASE CONSTRAINTS IN SQL

Any constraint that can be formulated by means of a CONSTRAINT statement in Tutorial D can be formulated by means of a CREATE ASSERTION statement in SQL, as examples CX1-CX6 in the previous section should have been sufficient to suggest. ${ }^{6}$ Unlike Tutorial D, however, SQL has a feature according to which any such constraint can alternatively be specified as part of the definition of some base table-i.e., as a base table constraint. For example, here again is the SQL version (using CREATE ASSERTION) of constraint CX5 from the previous section:

```
CREATE ASSERTION CX5 CHECK
    ( NOT EXISTS ( SELECT *
        FROM S NATURAL JOIN SP
        WHERE STATUS < 20
        AND PNO = 'P6' ) ) ;
```

[^91]This example could have been stated in slightly different form as a base table constraint as part of the definition of base table SP, like this:

```
CREATE TABLE SP
    ( ... ,
        CONSTRAINT CX5 CHECK /* "base table" constraint */
            ( PNO <> 'P6' OR ( SELECT STATUS FROM S
                        WHERE SNO = SP.SNO ) >= 20 ) ) ;
```

Note, however, that a logically equivalent formulation could have been specified as part of the definition of base table S instead—or base table P , or absolutely any base table in the database, come to that (see Exercise 8.17 at the end of the chapter).

Now, this alternative style can be useful for row constraints (i.e., constraints that can be checked for an individual row in isolation), because it's a little simpler than its CREATE ASSERTION counterpart. Here, for example, are constraints CX1 and CX2 from the previous section, reformulated as base table constraints on base table S:

```
CREATE TABLE S
    ( ... ,
        CONSTRAINT CX1 CHECK ( STATUS >= 1 AND STATUS <= 100 ) ) ;
CREATE TABLE S
    ( ... ,
        CONSTRAINT CX2 CHECK ( STATUS = 20 OR CITY <> 'London' ) ) ;
```

For a constraint involving more than one base table, however, CREATE ASSERTION is usually better, because it avoids having to make an arbitrary choice as to which table to attach the constraint to.

Note: Certain constraints-for example, NOT NULL constraints and key constraints for keys that involve just one column-can optionally be formulated as "column constraints" in SQL. A column constraint is one that's specified, not just as part of the definition of the base table in question, but as part of the definition of some specific column of that base table. For simplicity, I'll ignore this possibility in this book, except for NOT NULL constraints in particular.

Two last points to close this section:

- Be aware that any constraint stated as part of the CREATE TABLE for base table $T$ is automatically satisfied if $T$ is empty-even if the constraint is of the form " $T$ mustn't be empty"! (Or even if it's of the form " $T$ must contain -5 rows," or the form " $1=0$," come to that.) See Exercises 8.15 and 8.16 at the end of the chapter.
(Important!) While most current SQL products do support key and foreign key constraints, they don't support CREATE ASSERTION at all, and they don't support base table constraints any more complicated than simple row constraints. (Formally, they don't permit base table constraints to contain a subquery.) Recommendation: Specify constraints declaratively whenever you can. In practice, however, many constraints (perhaps most) will, regrettably, have to be enforced by means of procedural code (possibly triggered procedures)-and that code can be quite difficult to write, too. ${ }^{7}$ This state of affairs represents a serious defect in today's products, and it needs to be remedied, urgently.

[^92]
## TRANSACTIONS

Despite the defect identified at the end of the previous section, I do need to assume for the rest of the chapter (just as the relational model does, in fact) that database constraints of arbitrary complexity can be stated declaratively. The question now arises: When are such constraints checked? Conventional wisdom has it that single relvar constraint checking is immediate (meaning it's done whenever the relvar in question is updated), while multirelvar constraint checking is deferred to end of transaction ("commit time"). I want to argue, however, that all checking should be immediate, and deferred checking-which is supported in the SQL standard, and indeed in at least one SQL product to my knowledge-is a logical mistake. In order to explain this unorthodox view, I need to digress for a moment to discuss transactions.

Transaction theory is a large topic in its own right. But it doesn't have much to do with the relational model as such (at least, not directly), and for that reason I don't want to discuss it in detail here. In any case, you're a database professional, and I'm sure you're familiar with basic transaction concepts. ${ }^{8}$ All I want to do here is briefly review the so called ACID properties of transactions. ACID is an acronym, standing for atomicity - consistency isolation - durability, where:

- Atomicity means that transactions are "all or nothing."
- Consistency means that any given transaction transforms a consistent state of the database into another consistent state, without necessarily preserving consistency at all intermediate points. Note: A database state is consistent if and only if it satisfies all defined constraints (consistency in this context is just another word for integrity).
- Isolation means that any given transaction's updates are concealed from all other transactions until such time as the given transaction commits.

Durability means that once a given transaction commits, its updates survive in the database, even if there's a subsequent system crash.

Now, one argument in favor of transactions has always been that they're supposed to act as "a unit of integrity" (that's what the consistency property is all about). But I don't believe that argument. Rather, as I've more or less said already, I believe statements have to be that unit; in other words, I believe database constraints must be satisfied at statement boundaries. The section immediately following gives my justification for this position.

## WHY DATABASE CONSTRAINT CHECKING MUST BE IMMEDIATE

I have at least five reasons for taking the position I do (viz., that database constraints must be satisfied at statement boundaries). The first and biggest one is this: As we know from Chapter 5, a database can be regarded as a collection of propositions, propositions we believe to be true ones. And if that collection is ever allowed to include any inconsistencies, then all bets are off; as I'll show in the section "Constraints and Predicates" later, we can never trust the answers we get from an inconsistent database. And while it might be true, thanks to the isolation property, that no more than one transaction ever sees any particular inconsistency, the fact remains that that particular transaction does see the inconsistency and can therefore produce wrong answers.

[^93]Now, I think this first argument is strong enough to stand on its own, but for completeness I'll give the other arguments as well. Second, then, I don't agree that any given inconsistency can be seen by only one transaction, anyway; that is, I don't really believe in the isolation property. Part of the problem here is that the word isolation doesn't mean quite the same in the world of transactions as it does in ordinary English-in particular, it doesn't mean that transactions can't communicate with one another. For if transaction $T X 1$ produces some result, in the database or elsewhere, that's subsequently read by transaction $T X 2$, then $T X 1$ and $X T 2$ aren't truly isolated from each other (and this remark applies regardless of whether $T X 1$ and $T X 2$ run concurrently or otherwise). In particular, therefore, if (a) TXI sees an inconsistent state of the database and therefore produces an incorrect result, and (b) that result is then seen by $T X 2$, then (c) the inconsistency seen by $T X 1$ has effectively been propagated to $T X 2$. In other words, it can't be guaranteed that a given inconsistency, if permitted, will be seen by just one transaction, anyway. Note: Similar remarks apply if $T X 1$ (a) sees an inconsistency and therefore assigns an incorrect value to some local variable $V$ and then (b) transmits the value of that variable $V$ to some outside user (since local variables aren't and can't possibly be subject to the jurisdiction of the transaction management subsystem).

Third, we surely don't want every program (or other "code unit") to have to deal with the possibility that the database might be inconsistent when it's invoked. There's a severe loss of orthogonality if some piece of code that assumes consistency can't be used safely while constraint checking is deferred. In other words, I want to be able to specify code units independently of whether they're to be executed as a transaction as such or just as part of a transaction. (In fact, I'd like support for nested transactions, but that's a topic for another day.)

Fourth, The Principle of Interchangeability (of base relvars and views-see the next chapter) implies that the very same constraint might be a single relvar constraint with one design for the database and a multirelvar constraint with another. For example, suppose we have two virtual relvars, or views, with Tutorial D definitions as follows ( $\mathrm{LS}=$ London suppliers, NLS $=$ non London suppliers):

VAR LS VIRTUAL ( S WHERE CITY = 'London' ) ;
VAR NLS VIRTUAL ( $S$ WHERE CITY $\neq$ 'London' ) ;
These views are subject to the constraint that no supplier number appears in both. However, there's no need to state that constraint explicitly, because it's implied by the fact that every supplier has exactly one city-i.e., the FD $\{\mathrm{SNO}\} \rightarrow\{\mathrm{CITY}\}$ holds in base relvar $\mathrm{S}-$ together with the real world fact that any given city is either equal to London or not equal to London. But suppose we made LS and NLS base relvars and then defined their union as a view called S . Then the constraint would have to be stated explicitly:

```
CONSTRAINT CX7 IS EMPTY
    ( LS { SNO } JŌIN
        NLS { SNO } ) ;
```

```
CREATE ASSERTION CX7 CHECK
    ( NOT EXISTS
    ( SELECT *
        FROM LS , NLS
        WHERE LS.SNO = NLS.SNO ) ) ;
```

Now what was previously a single relvar constraint has become a multirelvar constraint instead. Thus, if we agree that single relvar constraints must be checked immediately, we must surely agree that multirelvar constraints must be checked immediately as well (since, logically, there's no real difference between the two, as the example demonstrates).

Fifth and last, there's an optimization technique called semantic optimization (it involves expression transformation, but I deliberately didn't discuss it in the section of that name in Chapter 6). By way of example, consider the expression (SP JOIN S) $\{\mathrm{PNO}\}$. Now, the join here is based on the correspondence between a foreign key in a referencing relvar, SP, and the relevant candidate key in the referenced relvar, S. As a consequence, every SP tuple does join to some S tuple, and every SP tuple thus does contribute a part number to the projection that's the overall result. So there's no need to do the join!-the expression can be simplified to just SP $\{\mathrm{PNO}\}$. Note carefully,
however, that this transformation is valid only because of the semantics of the situation; with join in general, each operand will include some tuples that have no counterpart in the other and so don't contribute to the overall result, and transformations such as the one just shown therefore won't be valid. But in the case at hand every SP tuple necessarily does have a counterpart in S, because of the integrity constraint-actually a foreign key constraint-that says that every shipment must have a supplier, and so the transformation is valid after all. A transformation that's valid only because a certain integrity constraint is in effect is called a semantic transformation, and the resulting optimization is called a semantic optimization.

Now, in principle, any constraint whatsoever can be used in semantic optimization; we're not limited to foreign key constraints as in the example. ${ }^{9}$ For example, suppose the suppliers-and-parts database is subject to the constraint "All red parts must be stored in London," and consider the query:

Get suppliers who supply only red parts and are located in the same city as at least one of the parts they supply.

This is a fairly complex query; but thanks to the integrity constraint, we see that it can be transformed-by the optimizer, I mean, not by the user-into this much simpler one:

Get London suppliers who supply only red parts.
We could easily be talking about several orders of magnitude improvement in performance here. And so, while commercial products do comparatively little in the way of semantic optimization at the time of writing (as far as I know), I certainly expect them to do more in the future, because the payoff is so dramatic.

To get back to the main thread of the discussion, I now observe that if a given constraint is to be usable in semantic optimization, then that constraint must be satisfied at all times (or rather, and more precisely, at statement boundaries), not just at transaction boundaries. As we've just seen, semantic optimization means using constraints to simplify queries in order to improve performance. Clearly, then, if some constraint is violated at some time, then any simplification based on that constraint won't be valid at that time, and query results based on that simplification will be wrong at that time (in general). Note: Alternatively, we could adopt the weaker position that "deferred constraints" (meaning constraints for which the checking is deferred) can't be used in semantic optimization-but I think such a position would effectively just mean we've shot ourselves in the foot, that's all.

To sum up: Database constraints must be satisfied-that is, they must evaluate to TRUE, given the values currently appearing in the database-at statement boundaries (or, very informally, "at semicolons"); in other words, they must be checked at the end of any statement that might cause them to be violated. If any such check fails, the effects on the database of the offending statement must be undone and an exception raised.

## BUT DOESN'T SOME CHECKING HAVE TO BE DEFERRED?

The arguments of the previous section notwithstanding, the conventional wisdom is that multirelvar constraint checking, at least, does have to be deferred to commit time. By way of example, suppose the suppliers-and-parts database is subject to the following constraint:

[^94]```
CONSTRAINT CX8
    COUNT ( ( S WHERE SNO = 'S1' ) { CITY }
                    UNION
                    ( P WHERE PNO = 'P1' ) { CITY } ) < 2 ;
```

This constraint says that supplier S1 and part P1 must never be in different cities. To elaborate: If relvars S and $P$ contain tuples for supplier $S 1$ and part $P 1$, respectively, then those tuples must contain the same CITY value (if they didn't, the COUNT invocation would return the value two); however, it's legal for relvar S to contain no tuple for S 1 , or relvar P to contain no tuple for P 1 , or both (in which case the COUNT invocation will return either one or zero). Given this constraint and our usual sample values, then, each of the following SQL UPDATEs will fail under immediate checking:

```
UPDATE S SET CITY = 'Paris' WHERE SNO = 'S1' ;
UPDATE P SET CITY = 'Paris' WHERE PNO = 'P1' ;
```

I show these UPDATEs in SQL rather than Tutorial D precisely because checking is immediate in Tutorial $\mathbf{D}$ and the conventional solution to the problem therefore doesn't work in Tutorial D. What is that conventional solution? Answer: We defer the checking of the constraint to commit time, ${ }^{10}$ and we make sure the two UPDATEs are part of the same transaction, as in this SQL code:

```
START TRANSACTION ;
    UPDATE S SET CITY = 'Paris' WHERE SNO = 'S1' ;
    UPDATE P SET CITY = 'Paris' WHERE PNO = 'P1' ;
COMMIT ;
```

In this conventional solution, the constraint is checked at end of transaction, and the database is inconsistent between the two UPDATEs. In particular, if the transaction were to ask the question "Are supplier S1 and part P1 in different cities?" between the two UPDATEs (and assuming rows for S1 and P1 do exist), it would get the answer yes.

## Multiple Assignment

A better solution to the foregoing problem is to support a multiple form of assignment, which allows any number of individual assignments to be performed "simultaneously," as it were. For example (switching back now to Tutorial D):

```
UPDATE S WHERE SNO = 'S1' : { CITY := 'Paris' } ,
UPDATE P WHERE PNO = 'P1' : { CITY := 'Paris' } ;
```

Explanation: First, note the comma separator, which means the two UPDATEs are part of the same overall statement. Second, UPDATE is really assignment, as we know, and the foregoing "double UPDATE" is thus just shorthand for a double assignment of the following form:

[^95]
## 184 Chapter $8 /$ SQL and Constraints

```
S := .. , P := .. ;
```

This double assignment assigns one value to relvar S and another to relvar P , all as part of the same overall operation. In general, the semantics of multiple assignment are as follows:

First, all of the source expressions on the right sides are evaluated.
Second, the individual assignments (to the variables on the left sides) are executed.
(Actually this definition requires a slight refinement in the case where two or more of the individual assignments specify the same target variable, but that refinement needn't concern us here.) Observe that, precisely because all of the source expressions are evaluated before any of the individual assignments are executed, none of those individual assignments can depend on the result of any other (and so the sequence in which they're executed is irrelevant; in fact, you can think of them as being executed in parallel, or "simultaneously"). Moreover, since multiple assignment is considered to be a semantically atomic operation, no integrity checking is performed "in the middle of" any such assignment-indeed, this fact is the major rationale for supporting the operation in the first place. In the example, therefore, the double assignment succeeds where the two separate single assignments failed. Note in particular that there's now no way for the transaction to see an inconsistent state of the database between the two UPDATEs, because the notion of "between the two UPDATEs" now has no meaning. Note further that there's now no need for deferred checking at all.

Aside: Perhaps I should state for the record here that all statements are semantically atomic in the relational model. In fact, most statements are syntactically atomic too; multiple assignment is an exception, because it's semantically atomic but not syntactically so. End of aside.

So what about multiple assignment in SQL? Well, SQL does have some support for this operation; in fact, it's had some such support for many years. First of all, referential actions such as CASCADE imply, in effect, that a single DELETE or UPDATE statement can cause several base tables to be updated "simultaneously," as part of a single operation. Second, the ability to update (for example) certain join views, if it's supported, implies the same thing. Third, FETCH INTO and SELECT INTO are both multiple assignment operations, of a kind. Fourth, SQL explicitly supports a multiple assignment form of the SET statement (indeed, that's exactly what row assignment is-see Chapters 2 and 3). And so on (this isn't an exhaustive list). However, the one kind of multiple assignment that SQL doesn't currently support is an explicit "simultaneous" assignment to several different tables-which is precisely the case illustrated by the foregoing example, and precisely what we need in order to avoid having to do deferred checking. ${ }^{11}$

One last point: Please understand that support for multiple assignment doesn't mean we can discard support for transactions. Transactions are still necessary for recovery and concurrency purposes, if nothing else. All I'm saying is that transactions aren't the "unit of integrity" they're usually supposed to be.

Recommendation: In SQL, use immediate checking whenever you can. Given the state of today's products, however, some checking (especially for constraints that involve more than one table) will almost certainly have to be deferred. In such a case, you should do whatever it takes-which in practice might mean terminating the transaction-to force the check to be done before any operation is executed that might rely on the constraint being satisfied.

[^96]
## CONSTRAINTS AND PREDICATES

Recall from Chapter 5 that the predicate for any given relvar is the intended interpretation-loosely, the meaning-for that relvar. For example, the predicate for relvar S looks like this:

Supplier SNO is under contract, is named SNAME, has status STATUS, and is located in city CITY.

In an ideal world, then, this predicate would serve as "the criterion for acceptability of updates" on relvar $\mathrm{S}-$ that is, it would dictate whether a given update operation on that relvar can be accepted. But of course this goal is unachievable:

- For one thing, the system can't know what it means for a "supplier" to be "under contract" or to be "located" somewhere; to repeat, these are matters of interpretation. For example, if the supplier number S1 and the city name London happen to appear together in the same tuple, then the user can interpret that fact to mean that supplier S 1 is located in London, ${ }^{12}$ but there's no way the system can do anything analogous.
- For another, even if the system could know what it means for a supplier to be under contract or to be located somewhere, it still couldn't know a priori whether what the user tells it is true! If the user asserts to the system (by means of some update operation) that there's a supplier S6 named Lopez with status 30 and city Madrid, then there's no way for the system to know whether that assertion is true. All the system can do is check that the user's assertion doesn't cause any integrity constraint to be violated. Assuming it doesn't, the system will accept the user's assertion and will treat it as true from that point forward (until such time as the user tells the system, by executing another update, that it isn't true any more).

Thus, the pragmatic "criterion for acceptability of updates," as opposed to the ideal one, is not the predicate but the corresponding set of constraints, which might thus be regarded as the system's approximation to the predicate. Equivalently:

## The system can't enforce truth, only consistency.

Sadly, truth and consistency aren't the same thing. To be specific, if the database contains only true propositions, then it's consistent, but the converse isn't necessarily so; if it's inconsistent, then it contains at least one false proposition, but the converse isn't necessarily so. Or to put it another way, correct implies consistent (but not the other way around), and inconsistent implies incorrect (but not the other way around)-where to say the database is correct is to say it faithfully reflects the true state of affairs in the real world, no more and no less.

Now let me try to pin down these notions a little more precisely. Let $R$ be a base relvar, and let $C 1, C 2, \ldots$, $C m(m \geq 0)$ be all of the database constraints, single relvar or multirelvar, that mention $R$. Assume for simplicity that each $C i$ is just a boolean expression (i.e., ignore the constraint names, for simplicity). Then the boolean expression

```
( C1 ) AND ( C2 ) AND ... AND ( Cm ) AND TRUE
```

[^97]is the total relvar constraint for relvar $R$ (but I'll refer to it for the purposes of this book as just the constraint for $R$ ). Note that final "AND TRUE," by the way; the implication is that in the unlikely event that no constraints at all are defined for a given relvar (i.e., $m=0$ ), then the default is just TRUE. ${ }^{13}$

Now let $R C$ be "the" relvar constraint for relvar $R$. Clearly, $R$ must never be allowed to have a value that causes $R C$ to evaluate to FALSE. This state of affairs is the motivation for (the first version of) what I like to call The Golden Rule:

No update operation must ever cause the relvar constraint for any relvar to evaluate to FALSE.
Now let $D B$ be a database, and let $R 1, R 2, \ldots, R n(n \geq 0)$ be all of the relvars in $D B$. Let the constraints for those relvars be $R C 1, R C 2, \ldots, R C n$, respectively. Then the boolean expression
( RC1 ) AND ( RC2 ) AND ... AND ( $R C n$ ) AND TRUE
is the total database constraint for $D B$ (but I'll refer to it for the purposes of this book as just the constraint for $D B$ ). And here's a correspondingly extended-in fact, the final-version of The Golden Rule:

No update operation must ever cause the database constraint for any database to evaluate to FALSE.
Observe in particular that, in accordance with my position that all integrity checking must be immediate, The Golden Rule talks in terms of update operations, not transactions.

Now I can take care of a piece of unfinished business. I've said we can never trust the answers we get from an inconsistent database; here's the proof. As we know, a database can be regarded as a collection of propositions. Suppose that collection is inconsistent; that is, suppose it implies that both $p$ and NOT $p$ are true, where $p$ is some proposition. Now let $q$ be any arbitrary proposition. Then:

- From the truth of $p$, we can infer the truth of $p$ OR $q$.
- From the truth of $p$ OR $q$ and the truth of NOT $p$, we can infer the truth of $q$.

But $q$ was arbitrary! It follows that any proposition whatsoever (even ones that are obviously false, like $1=$ 0 ) can be shown to be "true" in an inconsistent system. Note: In case you're still not convinced, let me refer you to the further discussion of this issue in Chapter 10.

## MISCELLANEOUS ISSUES

There are a few further points to do with integrity that I need to cover somewhere but don't fit very well into any of the preceding sections.

First of all, a constraint, since it's basically a boolean expression that must evaluate to TRUE, is in fact a proposition (I more or less suggested as much in the previous section, but I never came out and stated it explicitly). To see that this is so, consider constraint CX1 once again from the section "Database Constraints":

CONSTRAINT CX1 IS_EMPTY ( S WHERE STATUS < 1 OR STATUS > 100 ) ;

[^98]The relvar name " S " here constitutes what logicians call a designator; when the constraint is checked, it designates a specific value-namely, the value of that relvar at the time in question. By definition, that value is a relation ( $s$, say), and so the constraint effectively becomes:

```
CONSTRAINT CX1 IS_EMPTY ( s WHERE STATUS < 1 OR STATUS > 100 ) ;
```

Clearly, the boolean expression here-which is really the constraint as such, "CONSTRAINT CX1" being little more than window dressing-is certainly either true or false, unequivocally, and that's the definition of what it means to be a proposition (see Chapter 5).

Second, suppose relvar $S$ already contains a tuple that violates constraint CX1 when the CONSTRAINT statement just shown is executed; then that execution must fail. More generally, whenever we try to define a new database constraint, the system must first check to see whether that constraint is satisfied by the database at that time; if it isn't, the constraint must be rejected, otherwise it can be accepted and enforced from that point forward.

Third, relational databases are supposed to satisfy the referential integrity rule, which says there mustn't be any unmatched foreign key values. Now, in Chapter 1, I referred to this rule as a "generic integrity constraint." However, it should be clear by now that it's somewhat different in kind from the constraints we've been examining in this chapter. It's really a metaconstraint, in a sense; what it says is that every specific database must satisfy the specific referential constraints that apply to that particular database. In the case of the suppliers-and-parts database, for example, it says the referential constraints from SP to S and P must be satisfied-because if they aren't, then that database will violate the referential integrity metaconstraint. Likewise, in the case of the departments-andemployees database from Chapter 1, the referential constraint from EMP to DEPT must be satisfied, because if it isn't, then again that database will violate the referential integrity metaconstraint.

Fourth, I remind you from Chapter 5 that update operators are always set level, and hence that constraint checking mustn't be done until all of the updating has been done; i.e., a set level update mustn't be treated as a sequence of individual tuple level updates (or row level updates, in SQL). I also said in that chapter that the SQL standard does conform to this requirement, but that products might not. Indeed, the last time I looked, there was at least one major product that didn't conform but (on foreign key constraints, at least) did "inflight checking" instead. One problem with this state of affairs is that it can lead to undesirable and possibly complex prohibitions against certain operations. For example, suppose there's a cascade delete rule from suppliers to shipments. Then the product in question won't allow the following apparently innocuous, and reasonable, DELETE statement:

```
DELETE
FROM S
WHERE SNO NOT IN
    ( SELECT SNO
        FROM SP ) ;
```

(an attempt to delete suppliers with no shipments).
Another issue I didn't mention previously is the possibility of supporting what are called transition constraints. A transition constraint is a constraint on the legal transitions that variables of some kind-relvars in particular-can make from one value to another (by contrast, a constraint that isn't a transition constraint is sometimes said to be a state constraint). For example, a person's marital status can change from "never married" to "married" but not the other way around. Here's a database example ("No supplier's status must ever decrease"):

```
CONSTRAINT CX9 IS EMPTY
    ( ( ( S' { SNNO , STATUS } RENAME { STATUS AS OLD } )
                JOIN
                ( S { SNO , STATUS } RENAME { STATUS AS NEW } ) )
        WHERE OLD > NEW ) ;
```

Explanation: I'm adopting the convention that a primed relvar name such as $\mathrm{S}^{\prime}$ refers to the pertinent relvar as it was prior to the update under consideration. Constraint CX9 thus says: If we join the old value of S and the new one on $\{\mathrm{SNO}\}$ and restrict the result to just those tuples where the old status is greater than the new one, the final result must be empty. (Since the join is on \{SNO\}, any tuple in the join for which the old status is greater than the new one would represent a supplier whose status had decreased.)

Transition constraints aren't currently supported in either Tutorial D or SQL (other than procedurally).
Last, I hope you agree from everything we've covered in this chapter that constraints are vital-and yet they seem to be very poorly supported in current products; indeed, they seem to be underappreciated at best, if not completely misunderstood. The emphasis in practice always seems to be on performance, performance, performance; other objectives, such as ease of use, physical data independence, and in particular integrity, seem so often to be sacrificed to-or at best to take a back seat to-that overriding goal. ${ }^{14}$

Now, I don't want you to misunderstand me here. Of course performance is important too. Functionally speaking, a system that doesn't deliver at least adequate performance isn't a system (not a usable one, at any rate). But what's the point of a system performing well if we can't be sure the results we're getting from it are correct? Frankly, I don't care how fast a system runs if I don't feel I can trust it to give me the right answers to my queries.

## EXERCISES

8.1 Define the terms type constraint and database constraint. When are such constraints checked? What happens if the check fails?
8.2 State The Golden Rule. Is it true that this rule can be violated if and only if some individually declared single relvar constraint is violated?
8.3 What do you understand by the following terms?-assertion; attribute constraint; base table constraint; column constraint; multirelvar constraint; referential constraint; relvar constraint; row constraint; single relvar constraint; state constraint; "the" (total) database constraint; "the" (total) relvar constraint; transition constraint; tuple constraint. Which of these categories if any do (a) key constraints, (b) foreign key constraints, fall into?

### 8.4 Distinguish between possible and physical representations.

8.5 With the Tutorial D definition of type QTY as given in the body of the chapter, what do the following expressions return?
a. THE_Q ( QTY ( 345 ) )
b. QTY ( THE_Q ( QTY ) )

[^99]8.6 Explain as carefully as you can (a) what a selector is; (b) what a THE_operator is. Note: This exercise essentially repeats ones in previous chapters, but now you should be able to be more specific in your answers.

### 8.7 Suppose the only legal CITY values are London, Paris, Rome, Athens, Oslo, Stockholm, Madrid, and

 Amsterdam. Define a Tutorial D type called CITY that satisfies this constraint.8.8 Following on from the previous exercise, show how you could impose the corresponding constraint in SQL on the CITY columns in base tables S and P. Give at least two solutions. Compare and contrast those solutions with each other and with your answer to the previous exercise.
8.9 Define supplier numbers as a Tutorial D user defined type. You can assume the only legal supplier numbers are ones that can be represented by a character string of at least two characters, of which the first is an " S " and the remainder are numerals denoting a decimal integer in the range 1 to 9999 . State any assumptions you make regarding the availability of operators to help with your definition.
8.10 A line segment is a straight line connecting two points in the euclidean plane. Give a corresponding Tutorial D type definition.
8.11 Can you think of a type for which we might want to specify two different possreps? Does it make sense for two or more possreps for the same type each to include a type constraint?
8.12 Can you think of a type for which different possreps might have different numbers of components?
8.13 Which operations might cause constraints CX1-CX9 from the body of the chapter to be violated?
8.14 Does Tutorial D have anything directly analogous to SQL's base table constraints?
8.15 In SQL, what is it exactly (i.e., formally) that makes base table constraints a little easier to state than their CREATE ASSERTION counterparts? Note: I haven't covered enough in this book yet to enable you to answer this question. Nevertheless, you might want to think about it now, or possibly use it as a basis for group discussion.
8.16 Following on from the previous question, a base table constraint is automatically regarded as satisfied in SQL if the pertinent base table is empty. Why exactly do you think this is so (I mean, what's the formal reason)? Does
Tutorial D display any analogous behavior?
8.17 In the body of the chapter, I gave a version of constraint CX5 as a base table constraint on table SP. However, I pointed out that it could alternatively have been formulated as such a constraint on base table S , or base table P, or in fact any base table in the database. Give such alternative formulations.
8.18 Constraint CX1 (for example) had the property that it could be checked for a given tuple by examining just that tuple in isolation; constraint CX5 (for example) did not. What is it, formally, that accounts for this difference? What's the pragmatic significance, if any, of this difference?
8.19 Can you give either a Tutorial D database constraint or an SQL assertion that's exactly equivalent to the specification $\mathrm{KEY}\{\mathrm{SNO}$ \} for relvar S ?
8.20 Give an SQL formulation of constraint CX8 from the body of the chapter.
8.21 Using Tutorial D and/or SQL, write constraints for the suppliers-and-parts database to express the following requirements:
a. All red parts must weigh less than 50 pounds.
b. Every London supplier must supply part P2.
c. No two suppliers can be located in the same city.
d. At most one supplier can be located in Athens at any one time.
e. There must be at least one London supplier.
f. At least one red part must weigh less than 50 pounds.
g. The average supplier status must be at least 10 .
h. No shipment can have a quantity more than double the average of all such quantities.
i. No supplier with maximum status can be located in the same city as any supplier with minimum status.
j. Every part must be located in a city in which there is at least one supplier.
k. Every part must be located in a city in which there is at least one supplier of that part.

1. Suppliers in London must supply more different kinds of parts than suppliers in Paris.
m . The total quantity of parts supplied by suppliers in London must be greater than the corresponding total for suppliers in Paris.
n. No shipment can have a total weight (part weight times shipment quantity) greater than 20,000 pounds.

In each case, state which operations might cause the constraint to be violated.
8.22 Suppose there's a constraint in effect that says if two suppliers are in the same city, they must have the same status; in other words, suppose relvar S is subject to the functional dependency \{CITY\} $\rightarrow$ \{STATUS\} (I mentioned this possibility in the discussion of constraint CX4 in the body of the chapter). Do either of the following Tutorial D CONSTRAINT statements accurately represent this constraint?

```
CONSTRAINT CX22a
    COUNT ( S { CITY } ) = COUNT ( S { CITY , STATUS } ) ;
CONSTRAINT CX22b
    S = JOIN { S { ALL BUT STATUS } , S { CITY , STATUS } } ;
```

8.23 In the body of the chapter, I defined the total database constraint to be a boolean expression of this form:

```
( RC1 ) AND ( RC2 ) AND ... AND ( RCn ) AND TRUE
```

What's the significance of that "AND TRUE"?
8.24 In a footnote in the section "Constraints and Predicates," I said that if the values S1 and London appeared together in some tuple, then it might mean (among many other possible interpretations) that supplier S1 doesn't have an office in London. Actually, this particular interpretation is extremely unlikely. Why? Hint: Remember The Closed World Assumption.
8.25 Suppose no cascade delete rule is stated for suppliers and shipments. Write a Tutorial D statement that will delete some specified supplier and all shipments for that supplier in a single operation (i.e., without raising the possibility of a referential integrity violation).
8.26 Using the syntax sketched for transition constraints in the section "Miscellaneous Issues," write transition constraints to express the following requirements:
a. The total shipment quantity for a given part can never decrease.
b. Suppliers in Athens can move only to London or Paris, and suppliers in London can move only to Paris.
c. The total shipment quantity for a given supplier cannot be reduced in a single update to less than half its current value. (What do you think the qualification "in a single update" means here? Why is it important? Is it important?)
8.27 Investigate any SQL product that might be available to you. What semantic optimization does it support, if any?
8.28 Why do you think SQL fails to support type constraints? What are the consequences of this state of affairs?
8.29 The discussion in this chapter of types in general, and type constraints in particular, tacitly assumed that types were all (a) scalar and (b) user defined. To what extent do the concepts discussed apply to nonscalar types and/or system defined types?
8.30 Show that any arbitrary UPDATE can be expressed in terms of DELETE and INSERT.

## Chapter 9

## SQL and Views

They're concerned by adverse publicity and that I have to move more into public eye.
Problem is to define first the exact view we want to project.
-H. R. Haldeman: The Haldeman Diaries: Inside the Nixon White House (1994)

Intuitively, there are several different ways of looking at what a view is, all of which are valid and all of which can be helpful in the right circumstances:

- A view is a virtual relvar; in other words, it's a relvar that "looks and feels" just like a base relvar but (unlike a base relvar) doesn't exist independently of other relvars-rather, it's defined in terms of such other relvars.
- A view is a derived relvar; in other words, it's a relvar that's explicitly derived (and known to be derived, at least by some people) from certain other relvars. Note: If you're wondering what the difference is between a derived relvar and a virtual one (see the previous bullet item), I should explain that all virtual relvars are derived but some derived ones aren't virtual. See the section "Views and Snapshots" later in this chapter.
- A view is a "window into" the relvars from which it's derived; thus, operations on the view are to be understood as "really" being operations on those underlying relvars.

■ A view is what some writers call a "canned query" (more precisely, it's a named relational expression).
As usual, in what follows I'll discuss these ideas in both relational and SQL terms. Regarding SQL specifically, however, let me remind you of something I said in Chapter 1: A view is a table!-or, as I would prefer to say, a relvar. SQL documentation often uses expressions like "tables and views," thereby suggesting that tables and views are different things-but they're not; in many ways, in fact, it's the whole point about a view that it is a table (just as, in mathematics, the whole point about, e.g., the union or intersection of two sets is that the result is a set). So don't fall into the common trap of thinking the term table means a base table specifically. People who fall into that trap aren't thinking relationally, and they're likely to make mistakes as a consequence; in fact, several such mistakes can be found in the design of SQL itself. Indeed, it could be argued that the very names of the operators CREATE TABLE and CREATE VIEW in SQL are at least a psychological mistake, in that they tend to reinforce both (a) the idea that the term table means a base table specifically and (b) the idea that views and tables are different things. Be on the lookout for confusion in this area.

One last preliminary point: On the question of whether the database should "always" be accessed through views, see the section "SQL Column Naming" in Chapter 3 or the section "The Reliance on Attribute Names" in Chapter 6.

## VIEWS ARE RELVARS

Of those informal characterizations listed above of what a view is, the following definition might appear to favor one over the rest-but those informal characterizations are all equivalent anyway, loosely speaking:

Definition: A view $V$ is a relvar whose value at time $t$ is the result of evaluating a certain relational expression at that time $t$. The expression in question (the view defining expression) is specified when $V$ is defined and must mention at least one relvar.

The following examples ("London suppliers" and "non London suppliers") are repeated from Chapter 8, except that I now give SQL definitions as well:

```
VAR LS VIRTUAL
    ( S WHERE CITY = 'London' ) ;
CREATE VIEW LS AS
    ( SELECT *
        FROM S
        WHERE CITY = 'London' )
        WITH CHECK OPTION ;
VAR NLS VIRTUAL
CREATE VIEW NLS AS
    ( S WHERE CITY # 'London' ) ;
    ( SELECT *
        FROM S
        WHERE CITY <> 'London' )
        WITH CHECK OPTION ;
```

Note that these are restriction views-their value at any given time is a certain restriction of the value at that time of relvar S . Some syntax issues:

The parentheses in the SQL examples are unnecessary but not wrong; I include them for clarity. The parentheses in the Tutorial D examples are required.

CREATE VIEW in SQL allows a parenthesized commalist of view column names to appear following the view name, as in this example:

```
CREATE VIEW SDS ( SNAME , DOUBLE STATUS )
    AS ( SELECT DISTINCT SNAME , 2 }\mp@subsup{}{}{*
        FROM S ) ;
```

Recommendation: Don't do this-follow the recommendations given in Chapter 3 under "Column Naming in SQL" instead. For example, the foregoing view can equally well (in fact, better) be defined like this:

```
CREATE VIEW SDS
    AS ( SELECT DISTINCT SNAME , 2 * STATUS AS DOUBLE_STATUS
```

Note in particular that this latter style means we're telling the system once instead of twice that one of the view columns is called SNAME.

- CREATE VIEW in SQL also allows WITH CHECK OPTION to be specified if (but only if!) it regards the view as updatable. Recommendation: Always specify this option if possible. See the section "Update Operations" for further discussion.


## The Principle of Interchangeability

Since views are relvars, essentially everything I said in previous chapters regarding relvars in general applies to views in particular. Subsequent sections discuss specific aspects of this observation in detail. First, however, there's a more fundamental point I need to explain.

Consider the example of London vs. non London suppliers again. In that example, S is a base relvar and LS and NLS are views. But it could have been the other way around-that is, we could have made LS and NLS base relvars and S a view, like this (Tutorial D only, for simplicity):

```
VAR LS BASE RELATION
    { SNO CHAR , SNAME CHAR , STATUS INTEGER , CITY CHAR }
    KEY { SNO } ;
VAR NLS BASE RELATION
    { SNO CHAR , SNAME CHAR , STATUS INTEGER , CITY CHAR }
    KEY { SNO } ;
VAR S VIRTUAL ( LS UNION NLS ) ;
```

Note: In order to guarantee that this design is logically equivalent to the original one, we would also have to state and enforce certain additional constraints-including in particular constraints to the effect that every CITY value in LS is London and no CITY value in NLS is-but I omit such details here. See the sections "Views and Constraints" and "Update Operations" later for further consideration of such matters.

Anyway, the message of the example is that, in general, which relvars are base ones and which virtual is arbitrary (at least from a formal point of view). In the example, we could design the database in at least two different ways: ways, that is, that are logically distinct but information equivalent. (By information equivalent here, I mean the two designs represent the same information; i.e., for any query on one, there's a logically equivalent query on the other.) And The Principle of Interchangeability follows logically from such considerations:

Definition: The Principle of Interchangeability (of base and virtual relvars) states that there must be no arbitrary and unnecessary distinctions between base and virtual relvars; i.e., virtual relvars should "look and feel" just like base relvars so far as users are concerned.

Here are some implications of this principle:

- As I've already suggested, views are subject to integrity constraints, just like base relvars. (We usually think of integrity constraints as applying to base relvars specifically, but The Principle of Interchangeability shows this position isn't really tenable.) See the section "Views and Constraints," later.

In particular, views have keys (and so I should perhaps have included some key specifications in my examples of views prior to this point; Tutorial D permits such specifications but SQL doesn't). They might also have foreign keys, and foreign keys might refer to them. Again, see the section "Views and Constraints," later.

I didn't mention this point in Chapter 1, but the "entity integrity" rule is supposed to apply specifically to base relvars, not views. It thereby violates The Principle of Interchangeability. Of course, I reject that rule anyway, because it has to do with nulls (I also reject it because it has to do with primary keys specifically instead of keys in general, but let that pass).

■ Many SQL products, and the SQL standard, provide some kind of "row ID" feature. ${ }^{1}$ If that feature is available for base tables but not for views-which in practice is quite likely-then it violates The Principle of Interchangeability. (It probably violates The Information Principle, too. See Appendix A.) Now, row IDs as such aren't part of the relational model, but that fact in itself doesn't mean they're prohibited. But I observe as an important aside that if those row IDs are regarded-as they are, most unfortunately, in the SQL standard, as well as in at least some of the major SQL products-as some kind of object ID in the object oriented sense, then they are prohibited, very definitely! Object IDs are effectively pointers, and (to repeat from Chapter 2) the relational model explicitly prohibits pointers.

- The distinction discussed in the previous chapter between single relvar and multirelvar constraints is more apparent than real (and the terminology is therefore deprecated, somewhat, for that very reason). Indeed, an example in that chapter-essentially the same London vs. non London suppliers example, in fact-showed that the very same constraint could be a "single relvar" constraint with one design for the database and a "multirelvar" constraint with another.
- Perhaps most important of all, we must be able to update views-because if not, then that fact in itself would constitute the clearest possible violation of The Principle of Interchangeability. Again, see the section "Update Operations," later.


## Relation Constants

You might have noticed that, in the formal definition I gave for what a view was at the beginning of the present section, I said the defining expression had to mention at least one relvar. Why? Because if it didn't, the "virtual relvar" wouldn't be a relvar at all!-I mean, it wouldn't be a variable, and it wouldn't be updatable. For example, the following is a valid CREATE VIEW statement in SQL:

```
CREATE VIEW S_CONST AS
    ( SELECT }\mp@subsup{}{}{-}\mathrm{ TEMP.*
        FROM ( VALUES ( 'S1' , 'Smith' , 20 , 'London' ) ,
            ( 'S2' , 'Jones' , 10 , 'Paris' ) ,
            ( 'S3' , 'Blake' , 30 , 'Paris' ) ,
            ( 'S4', 'Clark' , 20 , 'London' ) ,
            ( 'S5' ', 'Adams' ', 30 ', 'Athens' ) )
                AS TEMP ( SNO , SNAME , STATUS , CITY ) ) ;
```

But this view certainly can't be updated. In other words, it's not a variable at all, let alone a virtual one; rather, it's what might be called a named relation constant. Let me elaborate:

- First of all, I regard the terms constant and value as synonymous. Note, therefore, that there's a logical difference between a constant and a literal; a literal isn't a constant but is, rather, a symbol-sometimes referred to as a self-defining symbol-that denotes a constant (as in fact we already know from Chapter 2).
- Strictly speaking, there's also a logical difference between a constant and a named constant; a constant is a value, but a named constant is like a variable, except that its value can't be changed. That said, however, for the remainder of this brief discussion I'll take the term constant to mean a named constant specifically, for brevity.

[^100]Constants can be of any type we like, naturally, but relation constants (i.e., constants of some relation type) are our major focus here. Now, Tutorial D doesn't currently support relation constants, but if it did, a relation constant (or "relcon") definition would probably look something like this example:

```
CONST PERIODIC TABLE INIT ( RELATION
    { TUPLE { ELEMENT 'Hydrogen' , SYMBOL 'H', ATOMICNO 1 } ,
        TUPLE { ELEMENT 'Helium' , SYMBOL 'He', ATOMICNO 2 } ,
        TUPLE { ELEMENT 'Uranium' , SYMBOL 'U' , ATOMICNO 92 } } ) ;
```

Now, I do believe it would be desirable to provide some kind of relation constant or "relcon" functionality along the lines sketched above. In fact, Tutorial $\mathbf{D}$ already provides two system defined relcons: namely, TABLE_DUM and TABLE_DEE, both of which are extremely important, as we know. Apart from these two, however, neither Tutorial D nor SQL currently provides any direct support for relcons. It's true that (as we've seen) such support can be simulated by means of the conventional view mechanism; however, there's a serious logical difference involved here, and I don't think it helps the cause of understanding to pretend that constants are variables.

## VIEWS AND PREDICATES

The Principle of Interchangeability implies that a view (just like a base relvar) has a relvar predicate, in which the parameters correspond one to one to the attributes of the relvar-i.e., the view-in question. However, the predicate that applies to a view $V$ is a derived predicate: It's derived from the predicates for the relvars in terms of which $V$ is defined, in accordance with the semantics of the relational operations involved in the view defining expression. In fact, you already know this: In Chapter 6, I explained that every relational expression has a corresponding predicate, and of course a view has exactly the predicate that corresponds to its defining expression. For example, consider view LS ("London suppliers") once again, as defined near the beginning of the section "Views Are Relvars." That view is a restriction of relvar S , and its predicate is therefore the logical AND of the predicate for S and the restriction condition:

Supplier SNO is under contract, is named SNAME, has status STATUS, and is located in city CITY
AND
city CITY is London.
Or more colloquially:
Supplier SNO is under contract, is named SNAME, has status STATUS, and is located in London.

Note, however, that this more colloquial form obscures the fact that CITY is a parameter. Indeed it is a parameter, but the corresponding argument is always the constant 'London'. (Precisely for this reason, in fact, a more realistic version of view LS would probably project away the CITY attribute. I prefer not to do this here, in order to keep the example as simple as possible.)

Similarly, the predicate for view NLS is:

Supplier SNO is under contract, is named SNAME, has status STATUS, and is located in city CITY, which isn't London.

## RETRIEVAL OPERATIONS

The Principle of Interchangeability implies that (a) users should be able to operate on views as if they were base relvars and (b) the DBMS should be able to map those user operations into suitable operations on the base relvars in terms of which the views are ultimately defined. I say "ultimately defined" here because if views really do behave just like base relvars, then one thing we can do is define further views on top of them, as in this SQL example:

```
CREATE VIEW LS STATUS
    AS ( SELECT \overline{SNO, STATUS}
```

In this section, I limit my attention to the mapping of read-only or "retrieval" operations, for simplicity (I remind you that the operations of the relational algebra are indeed all read-only). In fact, the process of mapping a read-only operation on a view to operations on the underlying relvars is in principle quite straightforward. For example, suppose we issue this SQL query on the London suppliers view LS (I deliberately show all name qualifications explicitly):

```
SELECT LS.SNO
FROM LS
WHERE LS.STATUS > 10
```

First, then, the DBMS replaces the reference to the view in the FROM clause by the expression that defines that view, yielding:

```
SELECT LS.SNO
FROM ( SELECT S.*
        FROM S
        WHERE S.CITY = 'London' ) AS LS
WHERE LS.STATUS > 10
```

This expression can now be directly evaluated. However-and for performance reasons perhaps more significantly-it can first be simplified to:

```
SELECT S.SNO
FROM S
WHERE S.CITY = 'London'
AND S.STATUS > 10
```

In all likelihood, this latter expression is the one that will actually be evaluated.
Now, it's important to understand that the reason the foregoing procedure works is precisely because of the relational closure property. Closure implies among other things that wherever we're allowed to use the name of a variable to denote the value of the variable in question-for example, in a query-we can always replace that name by a more general expression (just so long as that expression denotes a value of the appropriate type, of course). In the FROM clause, for example, we can have an SQL table name; thus we can also have a more general SQL table expression, and that's why we're allowed to substitute the expression that defines the view LS for the name LS in the example.

For obvious reasons, the foregoing procedure for implementing read-only operations on views is known as the substitution procedure. Incidentally, it's worth noting that the procedure didn't always work in early versions of SQL-to be specific, in versions prior to 1992-and the reason was that those early versions didn't fully support the closure property. As a result, certain apparently innocuous queries against certain apparently innocuous tables (actually views) failed, and failed, moreover, in ways that were hard to explain. Here's a simple example. First the view definition:

```
CREATE VIEW V
    AS ( SELECT CITY , SUM ( STATUS ) AS SST
        FROM S
        GROUP BY CITY ) ;
```

Now a query:

```
SELECT CITY
FROM V
WHERE SST > 25
```

This query failed in the SQL standard, prior to 1992, because simple substitution yielded something like the following syntactically invalid expression:

```
SELECT CITY
FROM S
WHERE SUM ( STATUS ) > 25 /* warning: invalid !!! */
GROUP BY CITY
```

(This expression is invalid because SQL doesn't allow "set function" invocations like SUM(STATUS) to be used in the WHERE clause in this manner.)

Now, the standard has been fixed in this regard, as you probably know; ${ }^{2}$ however, it doesn't follow that the products have!-and indeed, the last time I looked, there was at least one major product that hadn't. Indeed, precisely because of problems like the foregoing among others, the product in question actually implements certain view retrievals by materialization instead of substitution; that is, it actually evaluates the view defining expression, builds a table to hold the result of that evaluation, and then executes the requested retrieval against that materialized table. And while such an implementation might be argued to conform to the letter of the relational model, as it were, I don't think it can be said to conform to the spirit. (It probably won't perform very well, either.)

## VIEWS AND CONSTRAINTS

The Principle of Interchangeability implies that views not only have relvar predicates like base relvars, they also have relvar constraints like base relvars-by which I mean they have both individual relvar constraints and what in Chapter 8 I called a total relvar constraint (for the relvar in question). As with predicates, however, the constraints that apply to a view $V$ are derived: They're derived from the constraints for the relvars in terms of which $V$ is defined, in accordance with the semantics of the relational operations involved in the defining expression. By way of example, consider view LS once again. That view is a restriction of relvar $\mathrm{S}-\mathrm{i} . \mathrm{e}$., its defining expression specifies a restriction operation on relvar $S$-and so its (total) relvar constraint is the logical AND of the (total) relvar constraint for $S$ and the specified restriction condition. Let's suppose for the sake of the example that the only

[^101]constraint that applies to base relvar $S$ is the constraint that $\{\mathrm{SNO}\}$ is a key. Then the total relvar constraint for view LS is the AND of that key constraint and the constraint that the city is London, and view LS is required to satisfy that constraint at all times. (In other words, The Golden Rule applies to views just as it does to base relvars.)

For simplicity, from this point forward I'll use the term view constraint to refer to any constraint that applies to some view. Now, just because view constraints are always derived in the sense explained above, it doesn't follow that there's no need to declare them explicitly. For one thing, the system might not be "intelligent" enough to carry out the inferences needed to determine for itself the constraints that apply to some view; for another, such explicit declarations can at least serve documentation purposes (i.e., they can help explain the semantics of the view in question to users, if not to the system); and there's another reason too, which I'll get to in a little while.

I claim, then, that it should be possible to declare explicit constraints for views. In particular, it should be possible (a) to include explicit KEY and FOREIGN KEY specifications in view definitions and (b) to allow the target relvar in a FOREIGN KEY specification to be a view. Here's an example to illustrate possibility (a):

```
VAR LS VIRTUAL ( S WHERE CITY = 'London' )
    KEY { SNO } ;
```

Tutorial D does permit such specifications; SQL doesn't. Recommendation: In SQL, include such specifications in the form of comments. For example:

```
CREATE VIEW LS
    AS ( SELECT S.*
        FROM S
        WHERE S.CITY = 'London' )
        /* UNIQUE ( SNO ) */
            WITH CHECK OPTION ;
```

Note: As I've said, SQL doesn't permit view constraints to be formulated explicitly as part of the view definition; however, logically equivalent constraints can always be formulated by means of CREATE ASSERTION (if it's supported, that is). More generally, in fact, CREATE ASSERTION allows us to formulate constraints of any kind we like for any table that could be a view if we chose to define it as such-in other words, for any table that can be defined by some arbitrarily complex table expression (which is to say, any table at all). ${ }^{3}$ I'll have more to say about this possibility in a few moments.

Now, having said that it should be possible to declare explicit constraints on views, I should now add that sometimes it might be a good idea not to, because it could lead to redundant checking. For example, as I've said, the specification $\operatorname{KEY}\{\operatorname{SNO}\}$ clearly applies to view LS-but that's because it applies to base relvar S as well, ${ }^{4}$ and declaring it explicitly for view LS might simply lead to the same constraint being checked twice. (But it should still be stated as part of the view documentation, somehow, because it's part of the semantics of the view.)

Perhaps more to the point, there certainly are situations where declaring view constraints explicitly could be a good idea. Here's an example, expressed in SQL for definiteness. We're given two base tables that look like this (in outline):

[^102]```
CREATE TABLE FDH
    ( FLIGHT ...
        DESTINATION ... ,
        HOUR ... ,
        UNIQUE ( FLIGHT ) ) ;
CREATE TABLE DFGP
    ( DAY ... ,
        FLIGHT ... ,
        GATE ... ,
        PILOT ... ,
        UNIQUE ( DAY , FLIGHT ) ) ;
```

The tables have predicates as follows: ${ }^{5}$

- FDH: Flight FLIGHT leaves at hour HOUR for destination DESTINATION.
- DFGP: On day DAY, flight FLIGHT with pilot PILOT leaves from gate GATE.

They're subject to the following constraints (expressed here in a kind of pseudo logical style):

```
IF (fl,n1,h ) , ( f2,n2,h ) \in FDH AND
    (d,f1,g,p1) , ( d,f2,g,p2 ) \in DFGP
THEN f1 = f2 AND p1 = p2 /* and n1 = n2, incidentally */
IF (fl,n1,h) , ( f2,n2,h ) \in FDH AND
    (d,f1,g1,p ) , ( d,f2,g2,p ) \in DFGP
THEN f1 = f2 AND g1 = g2 /* and n1 = n2, incidentally */
```

Explanation: The first of these constraints says:
a. If two rows of FDH , one for flight $f 1$ (with destination $n 1$ ) and one for flight $f 2$ (with destination $n 2$ ), have the same HOUR $h$, and
b. Two rows of DFGP, one each for the FLIGHTs $f 1$ and $f 2$ from the two FDH rows, have the same DAY $d$ and GATE $g$, then
c. The two FDH rows must be the same and the two DFGP rows must be the same. In other words, if we know the HOUR, DAY, and GATE, then the FLIGHT and PILOT (and DESTINATION) are determined.

The second constraint is analogous:
a. If two rows of FDH, one for flight $f 1$ (with destination $n 1$ ) and one for flight $f 2$ (with destination $n 2$ ), have the same HOUR $h$, and

[^103]b. Two rows of DFGP, one each for the FLIGHTs $f 1$ and $f 2$ from the two FDH rows, have the same DAY $d$ and PILOT $p$, then
c. The two FDH rows must be the same and the two DFGP rows must be the same-in other words, if we know the HOUR, DAY, and PILOT, then the FLIGHT and GATE (and DESTINATION) are determined.

Now, stating these constraints directly in terms of the two base tables is fairly nontrivial:

```
CREATE ASSERTION BTCX1 CHECK
    ( NOT ( EXISTS ( SELECT * FROM FDH AS FX WHERE
                EXISTS ( SELECT * FROM FDH AS FY WHERE
                EXISTS ( SELECT * FROM DFGP AS DX WHERE
                EXISTS ( SELECT * FROM DFGP AS DY WHERE
                        FY.HOUR = FX.HOUR AND
                        DX.FLIGHT = FX.FLIGHT AND
                        DY.FLIGHT = FY.FLIGHT AND
                        DY.DAY = DX.DAY AND
                        DY.GATE = DX.GATE AND
                            ( FX.FLIGHT <> FY.FLIGHT OR
                            DX.PILOT <> DY.PILOT ) ) ) ) ) ( ) ;
CREATE ASSERTION BTCX2 CHECK
    ( NOT ( EXISTS ( SELECT * FROM FDH AS FX WHERE
        EXISTS ( SELECT * FROM FDH AS FY WHERE
        EXISTS ( SELECT * FROM DFGP AS DX WHERE
        EXISTS ( SELECT * FROM DFGP AS DY WHERE
        FY.HOUR = FX.HOUR AND
                                DX.FLIGHT = FX.FLIGHT AND
                                DY.FLIGHT = FY.FLIGHT AND
                        DY.DAY = DX.DAY AND
                        DY.PILOT = DX.PILOT AND
            ( FX.FLIGHT <> FY.FLIGHT OR
                DX.GATE <> DY.GATE ) ) ) ( ) ) ) ;
```

But stating them in the form of key constraints on a view definition, if that were permitted, would take care of matters nicely:

```
CREATE VIEW V AS
    ( SELECT * FROM FDH NATURAL JOIN DFGP ,
        UNIQUE ( DAY , HOUR , GATE ) , /* hypothetical */
        UNIQUE ( DAY , HOUR , PILOT ) ) ; /* syntax !!!!! */
```

Since this solution isn't available, we should at least specify those hypothetical view constraints in terms of suitable assertions:

```
CREATE VIEW V AS ( SELECT * FROM FDH NATURAL JOIN DFGP ) ;
CREATE ASSERTION vCX1
    CHECK ( UNIQUE ( SELECT DAY , HOUR , GATE FROM V ) ) ;
CREATE ASSERTION VCX2
    CHECK ( UNIQUE ( SELECT DAY , HOUR , PILOT FROM V ) ) ;
```

In fact, of course, we don't actually have to define the view V in order to define these constraints-we could simply replace the references to view V in the UNIQUE expressions in the constraints by the defining expression for V, like this: ${ }^{6}$

```
CREATE ASSERTION VCX1
    CHECK ( UNIQUE ( SELECT DAY , HOUR , GATE
        FROM FDH NATURAL JOIN DFGP ) ) ;
CREATE ASSERTION VCX2
    CHECK ( UNIQUE ( SELECT DAY , HOUR , PILOT
        FROM FDH NATURAL JOIN DFGP ) ) ;
```

Note: I didn't mention the point in Chapter 8, but Tutorial D does provide direct support for saying the relation denoted by some relational expression is required to satisfy some key constraint. By way of illustration, here are Tutorial D analogs of assertions VCX1 and VCX2:

```
CONSTRAINT VCX1 ( FDH JOIN DFGP ) KEY { DAY , HOUR , GATE } ;
CONSTRAINT VCX2 ( FDH JOIN DFGP ) KEY { DAY , HOUR , PILOT } ;
```


## UPDATE OPERATIONS

I claimed earlier that The Principle of Interchangeability implies that views must be updatable (i.e., assignable to). Now, I can hear some readers objecting right away: Surely some views just can't be updated, can they? For example, consider a view defined as the join-a many to many join, observe-of relvars S and P on \{CITY\}; surely we can't insert a tuple into, or delete a tuple from, that view, can we? Note: I apologize for the sloppy manner of speaking here; as we know from Chapter 5, there's no such thing as "inserting or deleting a tuple" in the relational model. But to be too pedantic about such matters in the present discussion would get in the way of understanding, probably.

Well, even if it's true-which it might or might not be-that we can't insert a tuple into or delete a tuple from S JOIN P, let me point out that certain updates on certain base relvars can't be done, either. For example, inserting a tuple into relvar SP will fail if the SNO value in that tuple doesn't currently exist in relvar S. Thus, updates on base relvars can always fail on integrity constraint violations-and the same is true for updates on views. In other words, it isn't that some views are inherently nonupdatable; rather, it's that some updates on some views will fail on integrity constraint violations (i.e., violations of The Golden Rule). Note: Actually, updates, on both base relvars and views, can fail on violations of The Assignment Principle too, as we'll quickly see.

To illustrate the point (albeit briefly), let's take a slightly closer look at the foregoing many to many join example (S JOIN P). Here's a Tutorial D definition:

VAR SCP VIRTUAL ( S JOIN P ) KEY \{ SNO , PNO \} ;

Now, as a basis for discussing updates on this or any other view, it's helpful to think of the view in question as if it were another base relvar, living alongside (as it were) the base relvars in terms of which it's defined. Let's

[^104]see what happens if we adopt this mode of thinking in the case at hand. First of all, note that-by definition-relvar SCP is subject to the constraint that it's equal to the join of S and P on $\{\mathrm{CITY}\}$ :

```
CONSTRAINT VCX SCP = S JOIN P ;
```

Given our usual sample values, then, the following INSERT will succeed:

```
INSERT SCP RELATION { TUPLE { SNO 'S6' , ... , CITY 'Madrid' ,
    PNO 'P7' , ... } } ;
```

(I'm making the obvious assumption here that inserting a tuple into SCP causes appropriate subtuples of that tuple to be inserted into S and P .) By contrast, consider the following INSERT:

```
INSERT SCP RELATION { TUPLE { SNO 'S6' , ... , CITY 'London' ,
    PNO 'P7' , ... } } ;
```

Here there are two possibilities:

- Appropriate subtuples are inserted into S and P , but the only tuple inserted into SCP is the one specified in the INSERT statement. Net effect: The Golden Rule is violated (to be specific, constraint VCX is violated), so the INSERT fails.
- Appropriate subtuples are inserted into S and P and certain additional tuples, over and above the one specified, are inserted into SCP in order to ensure constraint VCX remains satisfied. Net effect: The Assignment Principle is violated, so the INSERT fails. Note: We could-and in my opinion we should-avoid this failure, however, by specifying some appropriate compensatory actions. See the subsection "London vs. Non London Suppliers Revisited," later.

From these examples, I hope you can see that it's not that INSERT operations are intrinsically impossible on view SCP; rather, it's that some INSERTs (not all) on that view fail on a violation of either The Golden Rule or The Assignment Principle. I'll go into more detail regarding such matters later; for now, let me just say that a much more detailed discussion of the foregoing example and others like it can be found in the paper "How to Update Views" (see Appendix G).

So let $V$ be a view; in order to support updates on $V$ properly, then, the system needs to know the total constraint, $V C$ say, for $V$. In other words, it needs to be able to perform constraint inference, so that, given the constraints that apply to the relvars in terms of which $V$ is defined, it can determine $V C$. As I'm sure you realize, however, SQL products today do, or are capable of doing, very little in the way of such constraint inference. As a result, SQL's support for view updating is quite weak (and this is true of the standard as well as the major products). Typically, in fact, SQL products don't allow updating on views any more complex than simple restrictions and/or projections of a single underlying base table (and even here there are problems). By way of example, consider view LS once again. That view is just a restriction of base table S , and we can certainly perform the following DELETE on it:

```
DELETE
FROM LS
WHERE STATUS > 15 ;
```

This DELETE maps to:

```
DELETE
FROM
S
WHERE CITY = 'London'
AND STATUS > 15 ;
```

But what about INSERTs on this view? Here there are at least two problems:

1. The new row might violate the requirement that the city must be London.
2. The new row might have the same supplier number as some non London supplier.

I'll address the first of these issues in the subsection "The CHECK Option" below and the second in the subsection "London vs. Non London Suppliers Revisited" later. As for SQL view updatability in general, I'll have a little more to say on that topic in the next subsection but one ("More on SQL").

## The CHECK Option

Consider the following SQL INSERT on view LS:

```
INSERT INTO LS ( SNO , SNAME , STATUS , CITY )
    VALUES ( 'S6', ............ , 'Madrid' ) ;
```

This INSERT maps to:

```
INSERT INTO S ( SNO , SNAME , STATUS , CITY )
    VALUES ( 'S6', ............. , 'Madrid' ) ;
```

(The change is just in the target table name.) Observe now that the new row violates the constraint for view LS, because the city isn't London. So what happens? By default, SQL will insert that row into base table S; precisely because it doesn't satisfy the defining expression for view LS, however, it won't be visible through that view. From the perspective of that view, in other words, the new row just drops out of sight (alternatively, we might say the INSERT is a "no op"-again, from the perspective of the view). Actually, however, what's happened from the perspective of view LS is that The Assignment Principle has been violated.

Now, I hope it goes without saying that the foregoing behavior is logically incorrect. It wouldn't be tolerated in Tutorial D. As for SQL, the CHECK option is provided to address the problem: If (but only if) WITH CASCADED CHECK OPTION is specified for a given view, then updates to that view are required to conform to the defining expression for that view. Recommendation: Specify WITH CASCADED CHECK OPTION on view definitions whenever possible. Be aware, however, that SQL permits such a specification only if it regards the view as updatable, ${ }^{7}$ and (as previously noted) not all logically updatable views are regarded as such in SQL. Note: The alternative to CASCADED is LOCAL, but don't use it. (The reason I say this is that the semantics of LOCAL are bizarre in the extreme-so bizarre, in fact, that (a) I don't want to waste time and space and energy attempting to explain them here, and in any case (b) it's hard to see why anyone would ever want such semantics. Indeed, it's hard to resist the suspicion that LOCAL was included in the standard originally for no other reason than to allow

[^105]certain flawed implementations, extant at the time, to be able to claim conformance.) It's all right to specify neither CASCADED nor LOCAL, however, because CASCADED is the default.

## More on SQL

As we've seen, SQL's support for view updating is limited. It's also extremely hard to understand!-in fact, the standard is even more impenetrable in this area than it usually is. The following extract from the 2003 version of the standard gives some idea of the complexities involved:
[The] <query expression> QE1 is updatable if and only if for every <query expression> or <query specification> QE2 that is simply contained in QE1:
a) $Q E 1$ contains $Q E 2$ without an intervening <query expression body> that specifies UNION DISTINCT, EXCEPT ALL, or EXCEPT DISTINCT.
b) If $Q E 1$ simply contains a <query expression body> $Q E B$ that specifies UNION ALL, then:
i) $Q E B$ immediately contains a <query expression> $L O$ and a <query term> $R O$ such that no leaf generally underlying table of $L O$ is also a leaf generally underlying table of $R O$.
ii) For every column of $Q E B$, the underlying columns in the tables identified by $L O$ and $R O$, respectively, are either both updatable or not updatable.
c) $Q E 1$ contains $Q E 2$ without an intervening $<$ query term> that specifies INTERSECT.
d) $\quad Q E 2$ is updatable.

Here's my own gloss on the foregoing extract:

- The extract doesn't seem to make sense, at least on the face of it. For one thing, the opening sentence says, in effect, that four conditions a), b), c), and d) have to be satisfied "for every ... $Q E 2$ that is simply contained in $Q E 1$ "-yet item b) in particular doesn't even mention $Q E 2$ (?). For another, a careful reading appears to indicate that the following <query expression> is updatable, which surely can't be correct (I'm assuming here, not unreasonably, that columns X and Y are "both ... not updatable"):

```
SELECT 2 * STATUS AS X
FROM S
UNION ALL
SELECT 3 * QTY AS Y
FROM SP
```

- Next, even if I'm wrong and the extract does make sense, note that it (i.e., the extract) states just one of the many rules that have to be taken in combination in order to determine whether a given view is updatable in SQL.
- The rules in question aren't given all in one place but are scattered over many different portions of the standard.
- All of those rules rely on a variety of additional concepts and constructs-e.g., updatable columns, leaf generally underlying tables, <query term>s-that are in turn defined in still further portions of the standard.

Because of such considerations, I won't even attempt a precise characterization here of just which views SQL regards as updatable. Loosely speaking, however, they do at least include the following:

1. Views defined as a restriction and/or projection of a single base table
2. Views defined as a one to one or one to many join of two base tables (in the one to many case, only the many side is updatable)
3. Views defined as a UNION ALL or INTERSECT of two distinct base tables
4. Certain combinations of Cases 1-3 above

But even these limited cases are treated incorrectly, thanks to SQL's lack of understanding of (a) constraint inference, (b) The Golden Rule, and (c) The Assignment Principle, and thanks also to the fact that SQL permits nulls and duplicate rows. And the picture is complicated still further by the fact that SQL identifies four distinct cases: A view in SQL can be updatable, potentially updatable, simply updatable, or insertable into. Now, the standard defines these terms formally but gives no insight into their intuitive meaning or why they were given those names. However, I can at least say that "updatable" refers to UPDATE and DELETE and "insertable into" refers to INSERT, and a view can't be insertable into unless it's updatable. ${ }^{8}$ But note the suggestion that some views might permit some updates but not others (e.g., DELETEs but not INSERTs), and the further suggestion that it's therefore possible that DELETE and INSERT might not be inverses of each other. Both of these facts, if facts they are, I regard as further violations of The Principle of Interchangeability.

Regarding Case 1 above, however, I can be a little more precise. To be specific, an SQL view is certainly updatable if all of the following conditions are satisfied:

The defining expression is either (a) a simple SELECT expression (not a UNION, INTERSECT, or EXCEPT involving two such expressions) or (b) an "explicit table" (see Chapter 12) that's logically equivalent to such an expression. Note: I'll assume for simplicity in what follows that Case (b) is automatically converted to Case (a).

The SELECT clause in that SELECT expression specifies ALL, not DISTINCT.

- After expansion of any "asterisk style" items, every item in the SELECT item commalist is a simple column name (possibly qualified, and possibly with an AS specification), and no such item appears more than once.
- The FROM clause in that SELECT expression takes the form FROM $T$ [AS ...], where $T$ is the name of an updatable table (either a base table or an updatable view).

[^106]- The WHERE clause, if any, in that SELECT expression contains no subquery in which the FROM clause references $T$.

The SELECT expression has no GROUP BY or HAVING clause.
Recommendation: Lobby the SQL vendors to improve their support for view updating as soon as possible.

## London vs. Non London Suppliers Revisited

Consider the case of London vs. non London suppliers once again (relvars LS and NLS). By The Principle of Interchangeability, the behavior of these relvars, and indeed that of relvar S also, shouldn't depend on which relvars if any are base ones and which if any are views. Until further notice, therefore, let's suppose all three are base relvars:

```
VAR S BASE RELATION { ... } KEY { SNO } ;
VAR LS BASE RELATION { ... } KEY { SNO } ;
VAR NLS BASE RELATION { ... } KEY { SNO } ;
```

As the definitions show, $\{\mathrm{SNO}\}$ is a key for each of these relvars. The relvars are also clearly subject to the following constraints:

```
CONSTRAINT ... LS = S WHERE CITY = 'London' ;
CONSTRAINT ... NLS = S WHERE CITY \not= 'London' ;
```

What's more, these constraints taken singly or together imply certain additional ones, as follows:

```
CONSTRAINT ... IS EMPTY ( LS WHERE CITY # 'London' ) ;
CONSTRAINT ... IS_EMPTY ( NLS WHERE CITY = 'London' ) ;
CONSTRAINT ... S = UNION { LS , NLS } ;
CONSTRAINT ... IS_EMPTY ( JOIN { LS { SNO } , NLS { SNO } } ) ;
```

The first two of these additional constraints are self-explanatory; the third says every supplier is represented in either LS or NLS, and the fourth says no supplier is represented in both. (In other words, the union in the third constraint is actually a disjoint union, and the join in the fourth constraint is actually an intersection.)

Now, in order to ensure these constraints remain satisfied when updates are done, certain compensatory actions need to be in effect. In general, a compensatory action is an additional update (over and above some update that's requested by the user) that's performed automatically by the DBMS, precisely in order to avoid some integrity violation that might otherwise occur. Cascade delete is a typical example (see Chapter 5). ${ }^{9}$ In the case at hand, in fact, it should be clear that "cascading" is exactly what we need to deal with DELETE operations in particular. To be specific, deleting a tuple from either LS or NLS clearly needs to "cascade" to cause that same tuple to be deleted from S. So we might imagine a couple of compensatory actions-actually cascade delete rules-that look something like this (hypothetical syntax):

[^107]```
ON DELETE ls FROM LS : DELETE Is FROM S ;
ON DELETE nls FROM LS : DELETE nls FROM S ;
```

Likewise, deleting a tuple from S clearly needs to "cascade" to cause that same tuple to be deleted from whichever of LS or NLS it appears in:

```
ON DELETE S FROM S : DELETE ( s WHERE CITY = 'London' ) FROM LS
    DELETE ( s WHERE CITY f= 'London' ) FROM NLS ;
```

As an aside, I remark that, given that an attempt (via DELETE, as opposed to I_DELETE) to delete a nonexistent tuple has no effect-see Chapter 5-we could replace each of the expressions in parentheses here by just $s$. However, the expressions in parentheses are perhaps preferable, inasmuch as they're clearly more specific.

Analogously, we'll need some compensatory actions ("cascade insert rules") for INSERT operations:

```
ON INSERT ls INTO LS : INSERT ls INTO S ;
ON INSERT nls INTO LS : INSERT nls INTO S ;
ON INSERT s INTO S : INSERT ( s WHERE CITY = 'London' ) INTO LS ,
    INSERT ( s WHERE CITY # 'London' ) INTO NLS ;
```

As for UPDATE operations, they can be regarded, at least in the case at hand, as a DELETE followed by an INSERT; in other words, the necessary compensatory actions are just a combination of the corresponding delete and insert rules, loosely speaking. For example, consider the following UPDATE on relvar S:

```
UPDATE S WHERE SNO = 'S1' : { CITY := 'Oslo' } ;
```

What happens here is this:

1. The existing tuple for supplier $S 1$ is deleted from relvar $S$ and (thanks to the cascade delete rule from $S$ to LS) from relvar LS also.
2. Another tuple for supplier S1, with CITY value Oslo, is inserted into relvar S and (thanks to the cascade insert rule from S to NLS) into relvar NLS also. In other words, the tuple for supplier S1 has moved from relvar LS to relvar NLS!-now speaking very loosely, of course.

Suppose now that the original UPDATE had been directed at relvar LS rather than relvar S:

```
UPDATE LS WHERE SNO = 'S1' : { CITY := 'Oslo' } ;
```

Now what happens is this:

1. The existing tuple for supplier S1 is deleted from relvar LS and (thanks to the cascade delete rule from LS to $S$ ) from relvar S also.
2. An attempt is made to insert another tuple for supplier S1, with CITY value Oslo, into relvar LS. This attempt fails, however, because it violates the constraint on that relvar that the CITY value must always be

## 210 Chapter 9 / SQL and Views

London. So the update fails overall; the first step (viz., deleting the original tuple for supplier S1 from LS and $S$ ) is undone, and the net effect is that the database remains unchanged.

The foregoing examples notwithstanding, sometimes an UPDATE compensatory action will have to be specified explicitly. In the case of our usual suppliers and shipments relvars, for example, there might be a cascade delete rule from S to SP but (for obvious reasons) no corresponding cascade insert rule, and so we might want to specify an explicit update rule along the following lines:

```
ON UPDATE s { SNO := sno } IN S :
    UPDATE ( SP MATCHING s ) { SNO := sno } IN SP ;
```

Anyway, now I come to my real point: Everything I've said in this subsection so far applies pretty much unchanged if some or all of the relvars concerned are views. For example, suppose as we originally did that S is a base relvar and LS and NLS are views:

```
VAR S BASE RELATION { ................ } KEY { SNO } ;
VAR LS VIRTUAL ( S WHERE CITY = 'London' ) KEY { SNO } ;
VAR NLS VIRTUAL ( S WHERE CITY # 'London' ) KEY { SNO } ;
```

Now consider a user who sees only views LS and NLS, but wants to be able to behave as if those views were actually base relvars. Of course, that user will be aware of the corresponding relvar predicates, which as we saw earlier are essentially as follows:

## LS: Supplier SNO is under contract, is named SNAME, has status STATUS, and is located in city CITY (which is London).

NLS:Supplier SNO is under contract, is named SNAME, has status STATUS, and is located in city CITY (which isn't London).

That same user will also be aware of the following constraints (as well as the fact that $\{\mathrm{SNO}\}$ is a key for both relvars):

```
CONSTRAINT ... IS_EMPTY ( LS WHERE CITY # 'London' ) ;
CONSTRAINT ... IS EMPTY ( NLS WHERE CITY = 'London' ) ;
CONSTRAINT ... IS_EMPTY ( JOIN { LS { SNO } , NLS { SNO } } ) ;
```

However, the user won't be aware of any of the compensatory actions as such, precisely because that user isn't aware that LS and NLS are actually views of relvar S; in fact, the user won't even be aware of the existence of relvar $S$ (which is why the user is also unaware of the constraint that the union of LS and NLS is equal to S). But updates by that user on relvars LS and NLS will all work correctly, just as if LS and NLS really were base relvars.

What about a user who sees only view LS, say (i.e., not view NLS and not base relvar S), but still wants to behave as if LS were a base relvar? Well, that user will certainly be aware of the pertinent relvar predicate and the following constraint:

```
CONSTRAINT ... IS_EMPTY ( LS WHERE CITY f 'London' ) ;
```

Clearly, this user mustn't be allowed to insert tuples into this relvar, nor to update supplier numbers within this relvar, because such operations have the potential to violate constraints of which this user is unaware (and must be unaware). Again, however, there are parallels with base relvars as such: With base relvars in general, it'll be the
case that certain users will be prohibited from performing certain updates on the relvar in question (e.g., consider a user who sees only base relvar SP and not base relvar S). So this state of affairs doesn't in and of itself constitute a violation of The Principle of Interchangeability, either.

One last point: Please understand that I'm not suggesting that the DBA should have to specify, explicitly, all of the various constraints and compensatory actions that apply in connection with any given view. Au contraire: In many cases if not all, I believe the DBMS should be able to determine those constraints and actions for itself, automatically, from the pertinent view definitions. Again, see the paper "How to Update Views" for further explanation.

## WHAT ARE VIEWS FOR?

So far in this chapter, I've been tacitly assuming you already know what views are for-but now I'd like to say something about that topic nonetheless. In fact, views serve two rather different purposes:

- The user who actually defines view $V$ is, obviously, aware of the corresponding defining expression exp. Thus, that user can use the name $V$ wherever the expression $\exp$ is intended; however, such uses are basically just shorthand, and are explicitly understood to be just shorthand by the user in question. (What's more, the user in question is unlikely to request any updates on $V$-though if such updates are requested, they must perform as expected, of course.)

By contrast, a user who's merely informed that $V$ exists and is available for use is supposed (at least ideally) not to be aware of the expression exp; to that user, in fact, $V$ is supposed to look and feel just like a base relvar, as I've already explained at length. And it's this second use of views that's the really important one, and the one I've been concentrating on, tacitly, throughout this chapter prior to this point.

## Logical Data Independence

The second of the foregoing purposes is intimately related to the question of logical data independence. Recall from Chapter 1 that physical data independence means we can change the way the data is physically stored and accessed without having to make corresponding changes in the way the data is perceived by the user. Reasonably enough, then, logical data independence means we can change the way the data is logically stored and accessed without having to make corresponding changes in the way the data is perceived by the user. And it's views that are supposed to provide that logical data independence.

By way of example, suppose that for some reason (the precise reason isn't important for present purposes) we wish to replace base relvar S by base relvars LS and NLS, as follows:

```
VAR LS BASE RELATION /* London suppliers */
    { SNO CHAR , SNAME CHAR , STATUS INTEGER , CITY CHAR }
    KEY { SNO } ;
VAR NLS BASE RELATION /* non London suppliers */
    { SNO CHAR , SNAME CHAR , STATUS INTEGER , CITY CHAR }
    KEY { SNO } ;
```

As we saw earlier, the old relvar $S$ is the disjoint union of the two new relvars LS and NLS (and LS and NLS are both restrictions of that old relvar S). So we can define a view that's exactly that union, and name it S:

```
VAR S VIRTUAL ( LS D_UNION NLS ) KEY { SNO } ;
```

(Note that now I've specified D_UNION instead of UNION, for explicitness.) Any expression that previously referred to base relvar $S$ will now refer to view $S$ instead. Assuming the system supports operations on views correctly-unfortunately a rather large assumption, given the state of today's products-users will be immune to this particular change in the logical structure of the database.

I note in passing that replacing the original suppliers relvar $S$ by its two restrictions LS and NLS isn't a totally trivial matter. In particular, something might have to be done about the shipments relvar SP, since that relvar has a foreign key that references the original suppliers relvar S. See Exercise 9.8 at the end of the chapter.

## VIEWS AND SNAPSHOTS

Throughout this chapter, I've been using the term view in its original sense-the sense, that is, in which (in the relational context, at least) it was originally defined. Unfortunately, however, some terminological confusion has arisen in recent years: certainly in the academic world, and to some extent in the commercial world also. Recall that a view can be thought of as a derived relvar. Well, there's another kind of derived relvar too, called a snapshot. As the name might tend to suggest, a snapshot, although it's derived, is real, not virtual-meaning it's represented not just by its definition in terms of other relvars, but also (at least conceptually) by its own separate copy of the data. For example (to invent some syntax on the fly):

```
VAR LSS SNAPSHOT ( S WHERE CITY = 'London' )
    KEY { SNO }
    REFRESH EVERY DAY ;
```

Defining a snapshot is just like executing a query, except that:

- The result of the query is saved in the database under the specified name (LSS in the example) as a "readonly relvar" (read-only, that is, apart from the periodic refresh-see the bullet item immediately following).
- Periodically (EVERY DAY in the example) the snapshot is refreshed, meaning its current value is discarded, the query is executed again, and the result of that new execution becomes the new snapshot value. (Of course, other REFRESH options are possible: for example, EVERY MONDAY, EVERY 5 MINUTES, EVERY MONTH, and so on.)

In the example, therefore, snapshot LSS represents the data as it was at most 24 hours ago.
Snapshots are important in data warehouses, distributed systems, and many other contexts. In all such cases, the rationale is that applications can often tolerate-in some cases even require-data "as of" some particular point in time. Reporting and accounting applications are a case in point; such applications typically require the data to be frozen at an appropriate moment (for example, at the end of an accounting period), and snapshots allow such freezing to occur without locking out other applications.

So far, so good. The problem is, snapshots have come to be known (at least in some circles) not as snapshots at all but as materialized views. But they aren't views! Views aren't supposed to be materialized at all; ${ }^{10}$ as we've seen, operations on views are supposed to be implemented by mapping them into suitable operations on the underlying relvars. Thus, "materialized view" is simply a contradiction in terms. Worse yet, the unqualified term view is now often taken to mean a "materialized view" specifically-again, at least in some circles-and so we're in danger of no longer having a good term to mean a view in the original sense. In this book I do use the term view in

[^108]its original sense, but be warned that it doesn't always have that meaning elsewhere. Recommendations: Never use the term view, unqualified, to mean a snapshot; never use the term materialized view; and watch out for violations of these recommendations on the part of others!

## EXERCISES

9.1 Define a view consisting of supplier-number / part-number pairs for suppliers and parts that aren't colocated. Give both Tutorial D and SQL definitions.
9.2 Let view LSSP be defined as follows (SQL):

```
CREATE VIEW LSSP AS ( SELECT SNO , SNAME , STATUS , PNO , QTY
    FROM S NATURAL JOIN SP
    WHERE CITY = 'London' ) ;
```

Here's a query on this view:

```
SELECT DISTINCT STATUS , QTY FROM LSSP
WHERE PNO IN ( SELECT PNO FROM P WHERE CITY <> 'London' )
```

What might the query that's actually executed on the underlying base tables look like?
9.3 What key(s) does view LSSP from Exercise 9.2 have? What's the predicate for that view?
9.4 Given the following Tutorial D view definition-

VAR HP VIRTUAL ( P WHERE WEIGHT > 14.0 ) KEY \{ PNO \} ;
-show the converted form after the substitution procedure has been applied for each of the following expressions and statements:
a. HP WHERE COLOR = 'Green'
b. ( EXTEND HP : \{ $W$ : $=W E I G H T+5.3$ \} ) \{ PNO , W \}
c. INSERT HP RELATION \{ TUPLE \{ PNO 'P9' , PNAME 'Screw' , WEIGHT 15.0, COLOR 'Purple' , CITY 'Rome' \} \} ;
d. DELETE HP WHERE WEIGHT < 9.0 ;
e. UPDATE HP WHERE WEIGHT = 18.0 : \{ COLOR := 'White' \} ;

### 9.5 Give SQL solutions to Exercise 9.4.

9.6 Give as many reasons as you can think of for wanting to be able to declare keys for a view.

## 214 Chapter 9 / SQL and Views

9.7 Using either the suppliers-and-parts database or any other database you happen to be familiar with, give some further examples (over and above the London vs. non London suppliers example, that is) to illustrate the point that which relvars are base and which virtual is largely arbitrary.
9.8 In the body of the chapter, in the discussion of logical data independence, I discussed the possibility of restructuring-i.e., changing the logical structure of-the suppliers-and-parts database by replacing base relvar $S$ by two of its restrictions (LS and NLS). However, I also observed that such a replacement wasn't a completely trivial matter. Why not?
9.9 Investigate any SQL product available to you:
a. Are there any apparently legitimate queries on views that fail in that product? If so, state as precisely as you can which ones they are. What justification does the vendor offer for failing to provide full support?
b. What updates on what views does that product support? Be as precise as you can in your answer. Are the view updating rules in that product identical to those in the SQL standard?
c. More generally, in what ways-there will be some!-does that product violate The Principle of Interchangeability?
9.10 Distinguish between views and snapshots. Does SQL support snapshots? Does any product that you're aware of?
9.11 What's a "materialized view"? Why is the term deprecated?
9.12 Consider the suppliers-and-parts database, but ignore the parts relvar for simplicity. Here in outline are two possible designs for suppliers and shipments:

```
a. S { SNO , SNAME , STATUS , CITY }
    SP { SNO , PNO , QTY }
b. SSP { SNO , SNAME , STATUS , CITY , PNO , QTY }
    XSS { SNO , SNAME , STATUS , CITY }
```

Design a. is as usual. In Design b., by contrast, relvar SSP contains a tuple for every shipment, giving the applicable part number and quantity and full supplier details, and relvar XSS contains supplier details for suppliers who supply no parts at all. (Are these designs information equivalent?) Write view definitions to express Design b. as views over Design a. and vice versa. Also, show the applicable constraints for each design. Does either design have any obvious advantages over the other? If so, what are they?
9.13 Following on from the previous exercise: In the body of the chapter, I said two database designs were information equivalent if they represented the same information (meaning that for every query on one, there's a logically equivalent query on the other). But can you pin down this notion more precisely?
9.14 Views are supposed to provide logical data independence. But didn't I say in Chapter 6 that a hypothetical mechanism called "public tables" was supposed to perform that task? How do you account for the discrepancy?

## Chapter 10

## SQLand Logic

Logic takes care of itself; all we have to do is look and see how it does it.<br>-Ludwig Wittgenstein: Tractatus Logico-Philosophicus (1922)

As I mentioned in Chapter 1, there's an alternative to the relational algebra called the relational calculus. What this means is that queries, constraints, view definitions, and so forth can all be formulated in calculus terms as well as algebraic ones; sometimes, in fact, it's easier to come up with a calculus formulation than an algebraic one, though the opposite can also be true.

What is the relational calculus? Essentially, it's an applied form of predicate calculus (also known as predicate logic), tailored to the needs of relational databases. So the aims of this chapter are to introduce the relevant features of predicate logic (hereinafter abbreviated to just logic); to show how those features are realized in concrete form in the relational calculus; and, of course, to consider the relevant features of SQL as we go.

Incidentally, it follows from the above that a relational language can be based on either the algebra or the calculus. For example, Tutorial $\mathbf{D}$ is explicitly based on the algebra (which is why there aren't many references to Tutorial D in this chapter), and both QUEL and Query-By-Example (see Appendix G) are explicitly based on the calculus. So which is SQL based on? The answer, regrettably, is partly both and partly neither ... When it was first designed, SQL was specifically intended to be different from both the algebra and the calculus (the latter explicitly, the former perhaps a little less so); indeed, such a goal was the prime motivation for the introduction of the SQL "IN subquery" construct. (The name SQL originally stood for Structured Query Language; the idea behind that name was precisely that SQL queries typically involved one SELECT - FROM - WHERE expression nested inside the WHERE clause of another such expression, and so on, recursively.) As time went on, however, it turned out that certain features of both the algebra and the calculus were needed after all, and the language grew to accommodate them. The consequence today is that some aspects of SQL are "algebra like," some are "calculus like," and some are neither-with the further consequence that, as I mentioned in passing in Chapter 6, most queries, constraints, and so on that can be expressed in SQL at all can in fact be expressed in numerous different ways.

Aside: The goal mentioned in the previous paragraph-the goal, that is, of making SQL different from both the algebra and the calculus-was based on what I regard as a fundamental misconception: namely, the idea that the algebra and calculus were both somewhat "user hostile." But that perception was, I believe, founded on a confusion over syntax vs. semantics. Certainly the syntax in Codd's early papers was a little daunting, based as it was on formal mathematical notation. But semantics is another matter; the algebra and the calculus both have (I would argue) very simple semantics, and it's fairly easy, as many writers have demonstrated, to wrap that semantics up in syntax that's very user friendly indeed. End of aside.

## 216

 Chapter 10 / SQL and Logic
## WHY DO WE NEED LOGIC?

Logic is useful because (among other things) it's a great aid to clear and precise thinking. By contrast, everyone would surely agree that natural language is often vague and ambiguous. The following piece by Robert Graves and Alan Hodge illustrates the point delightfully: ${ }^{1}$

From the Minutes of a Borough Council Meeting:

Councillor Trafford took exception to the proposed notice at the entrance of South Park: "No dogs must be brought to this Park except on a lead." He pointed out that this order would not prevent an owner from releasing his pets, or pet, from a lead when once safely inside the Park.

The Chairman (Colonel Vine): What alternative wording would you propose, Councillor?

Councillor Trafford: "Dogs are not allowed in this Park without leads."
Councillor Hogg: Mr. Chairman, I object. The order should be addressed to the owners, not to the dogs.

Councillor Trafford: That is a nice point. Very well then:"Owners of dogs are not allowed in this Park unless they keep them on leads."

Councillor Hogg: Mr. Chairman, I object. Strictly speaking, this would keep me as a dog-owner from leaving my dog in the back-garden at home and walking with Mrs. Hogg across the Park.

Councillor Trafford: Mr. Chairman, I suggest that our legalistic friend be asked to redraft the notice himself.

Councillor Hogg: Mr. Chairman, since Councillor Trafford finds it so difficult to improve on my original wording, I accept. "Nobody without his dog on a lead is allowed in this Park."

Councillor Trafford: Mr. Chairman, I object. Strictly speaking, this notice would prevent me, as a citizen, who owns no dog, from walking in the Park without first acquiring one.

Councillor Hogg (with some warmth): Very simply, then: "Dogs must be led in this Park."

Councillor Trafford: Mr. Chairman, I object: This reads as if it were a general injunction to the Borough to lead their dogs into the Park.

Councillor Hogg interposed a remark for which he was called to order; upon his withdrawing it, it was directed to be expunged from the Minutes.

The Chairman: Councillor Trafford, Councillor Hogg has had three tries; you have had only two ...
Councillor Trafford: "All dogs must be kept on leads in this Park."

[^109]The Chairman: I see Councillor Hogg rising quite rightly to raise another objection. May I anticipate him with another amendment: "All dogs in this Park must be kept on the lead."

This draft was put to the vote and carried unanimously, with two abstentions.
Note: I can't resist pointing out that the final draft is still ambiguous-it could logically be interpreted to mean that all dogs in the park must be kept on the same lead. But enough of dogs ... Let's move on.

## SIMPLE AND COMPOUND PROPOSITIONS

Recall from Chapter 5 that, in logic, a proposition is something that evaluates unequivocally to either TRUE or FALSE. Here are some examples:

1. $2+3=5$
2. $2+3>7$
3. Jupiter is a star
4. Mars has two moons
5. Venus is between Earth and Mercury

Of these, Nos. 1, 4, and 5 are true and Nos. 2 and 3 are false-though we do need to be rather careful in the case of No. 5 over what exactly we mean by "between"! (To be a little more precise about the matter, what I mean by it is this: If we denote the distances of Mercury, Venus, and Earth from the sun by $m, v$, and $e$, respectively, then $m<v<e$.) Be that as it may, a good informal test for whether something, $p$ say, is a valid proposition is to ask whether "Is it true that $p$ ?" is a sensible question. For example, "Is it true that $2+3>7$ ?" is certainly a sensible question, even though the answer is no. To check your understanding of this point, which of the following do you think are legal propositions? (You might want to check the answers in Appendix F before continuing with this chapter.)

Bach is the greatest musician who ever lived.
What's the time?

- Supplier S2 is located in some city, $x$.
- Some countries have a female president.
- All politicians are corrupt.
- Supplier S1 is located in Paris.
- We both have the same favorite author, $x$.


## 218 Chapter 10 / SQL and Logic

- Nothing is heavier than lead.
- It will rain tomorrow.

Supplier S6's city is unknown.
By the way, there's a very fine point here (which I'm mostly going to ignore; I mention it only to head off at the pass, as it were, certain criticisms that persons trained in formal logic might be tempted to level at this chapter): A proposition isn't really a declarative sentence as such; rather, it's the assertion made by that sentence. For example, "It's hot" and "Il fait chaud" are distinct sentences, but they both assert the same proposition. That said, I'll continue to assume from this point forward for simplicity that a proposition is indeed just a declarative sentence. Analogous remarks apply to predicates also (see later).

## Connectives

Given some set of propositions, we can combine propositions from that set to form further propositions, using various connectives. The connectives most commonly encountered in practice are NOT, AND, OR, IF ... THEN ... (also known as IMPLIES or " $\Rightarrow$ "), and IF AND ONLY IF (also known as IFF, or BI-IMPLIES, or IS EQUIVALENT TO, or " $\Leftrightarrow$ ", or " $\equiv$ "). Here are a few examples of propositions that can be formed from Nos. 3, 4, and 5 from the foregoing list:

```
6. ( Jupiter is a star ) OR ( Mars has two moons )
7. ( Jupiter is a star ) AND ( Jupiter is a star )
8. ( Venus is between Earth and Mercury AND NOT ( Jupiter is a star )
9. IF ( Mars has two moons ) THEN ( Venus is between Earth and Mercury )
10. IF ( Jupiter is a star ) THEN ( Mars has two moons )
```

Note: I've used parentheses to make the scope of the connectives clear in these examples; in practice, we adopt certain precedence rules that allow us to omit many of the parentheses that might otherwise be required. Of course, it's never wrong to include them, even when they're logically unnecessary, and sometimes they can improve clarity.

In general, the connectives can be regarded as logical operators-they take one or more propositions as their input and return another proposition as their output. NOT is a monadic operator, the other four are dyadic. A proposition that involves no connectives is called a simple proposition; a proposition that isn't simple is called compound, or composite. And the truth value of a compound proposition can be determined from the truth values of its constituent simple propositions in accordance with the following truth tables (in which, for space reasons, I've abbreviated TRUE and FALSE to just T and F, respectively):

| $p$ | NOT $p$ |
| :---: | :---: |
| $T$ | $F$ |
| $F$ | $T$ |


| $p$ | $q$ | $p$ AND $q$ | $p$ OR $q$ | IF $p$ THEN $q$ | $p$ IFF $q$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $T$ | $T$ | $T$ | $T$ | $T$ | $T$ |
| $T$ | F | F | T | F | F |
| F | T | F | T | T | F |
| F | F | F | F | T | T |

By the way, truth tables can also be drawn in the following slightly different style (and here I've abbreviated IF ... THEN ... to just IF, again for space reasons):

| NOT |  | AND | T | F | OR | T | F | IF | T | F | IFF |  | F |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| T | F | T | T |  | T | T | T | T | T | F | T | T | F |
| F | T | F | F | F | F | T | F | F | T | T | F | F | T |

Neither style is more correct than the other; it's just that sometimes one is more convenient, sometimes the other is. Anyway, let's take a closer look at one of the foregoing compound propositions (number 9, to be specific). Here it is again:

```
9. IF ( Mars has two moons ) THEN ( Venus is between Earth and Mercury )
```

This proposition is of the form IF $p$ THEN $q$ (equivalently, $p$ IMPLIES $q$ ), where $p$ is the antecedent and $q$ is the consequent. Since the antecedent and the consequent both evaluate to TRUE, the overall proposition evaluates to TRUE also, as you can see from the truth table. But whether Venus is between Earth and Mercury obviously has nothing to do with whether Mars has two moons! So what exactly is going on here?

The foregoing example highlights a problem that people with no training in formal logic often experience: namely, that implication is notoriously difficult to come to grips with. So I'd like to offer the following argument, or rationale, in an attempt to clarify the matter:

- First of all, observe that there are exactly 16 dyadic connectives altogether, corresponding to the 16 possible dyadic truth tables (just four of which are shown above). Note: Exercise 10.1 asks you to draw all of those truth tables, and it might be worth having a go at that exercise right now.

Of those 16 dyadic connectives, some but not all are given common names such as AND and OR. But those names are really nothing more than a mnemonic device; they don't have any intrinsic meaning, they're chosen simply because the connectives so named have behavior that's similar (not necessarily identical) to that of their natural language counterparts. Indeed, it's easy to see that even AND doesn't mean quite the same thing as "and" in natural language. In logic, $p$ AND $q$ and $q$ AND $p$ are equivalent-but their natural language counterparts might not be. Here's an illustration: The natural language statements
"I was seriously disappointed and I voted for a change in leadership"
and
"I voted for a change in leadership and I was seriously disappointed"
are most certainly not equivalent! In other words, AND is a kind of logical distillate of "and" in natural language; very importantly-and unlike "and" in natural language-its meaning is context independent. Similar remarks apply to all of the other connectives.

Aside: The foregoing example (concerning AND) is perhaps a little misleading, in that it could be argued that "I was seriously disappointed" means different things in the two statements quoted. In the first, it means "I was seriously disappointed in the status quo"; in the second, it means "I was seriously disappointed in the outcome of the vote." If this analysis is correct, it would be strictly incorrect to symbolize the two statements as $p$ AND $q$ and $q$ AND $p$, respectively; although the two $q$ 's are the same, the two $p$ 's aren't. But the
example does at least show-not for the first time, perhaps-that we have to be rather careful in mapping natural language utterances to their symbolic logic counterparts. End of aside.

- Of the 16 available dyadic connectives, the one called IMPLIES has behavior that most closely resembles that of implication as understood in natural language. For example, "if Mars has two moons, then it certainly has at least one moon" is a valid implication, both in logic and in natural language. But nobody would or should claim that logical implication and natural language implication are the same thing. In fact, logical implication, like all of the connectives, is (of necessity) formally defined-i.e., it's defined purely in terms of the truth values, not the meanings, of its operands-whereas the same obviously can't be said of its natural language counterpart.
- Let's look at another example (number 10 from the foregoing list):

```
IF ( Jupiter is a star ) THEN ( Mars has two moons )
```

Perhaps even more counterintuitively, this one evaluates to TRUE also (check the truth table), because the antecedent is false; yet whether Mars has two moons, again obviously, has nothing to do with whether Jupiter is a star. Again, part of the justification-for the fact that the implication evaluates to TRUE, that is-is just that IMPLIES is formally defined. In this case, however, there's another argument (a database example, in fact) that you might find a little more satisfying. Suppose the suppliers-and-parts database is subject to the constraint that red parts must be stored in London (I deliberately state that constraint here in somewhat simplified form):

IF ( COLOR $=$ 'Red' ) THEN ( CITY = 'London' )
Clearly we don't want this constraint to be violated by a part that isn't red. It follows, therefore, that we want the proposition overall (which is a logical implication) to evaluate to TRUE if the antecedent evaluates to FALSE.

It follows from all of the above that the proposition $p$ IMPLIES $q$ (equivalently, IF $p$ THEN $q$ ) is logically equivalent to the proposition (NOT $p$ ) OR $q$-it evaluates to FALSE if and only if $p$ evaluates to TRUE and $q$ to FALSE, as you can see from the truth table. And, just incidentally, this equivalence serves to illustrate the point that the connectives NOT, AND, OR, IMPLIES, and IF AND ONLY IF aren't all primitive; some of them can be expressed in terms of others. As a matter of fact, all possible monadic and dyadic connectives can be expressed in terms of suitable combinations of NOT and either AND or OR. ${ }^{2}$ (Exercise: Check this claim.) Perhaps even more remarkably, all such connectives can in fact be expressed in terms of just one primitive. Can you find it?

## A Remark on Commutativity

The connectives AND and OR are commutative; that is, the compound propositions $p$ AND $q$ and $q$ AND $p$ are logically equivalent, and so are the compound propositions $p \mathrm{OR} q$ and $q \mathrm{OR} p$. As a consequence, you should never write code involving such propositions that assumes that $p$ will be evaluated before $q$ or the other way around.

[^110]For example, let the function SQRT ("nonnegative square root") be defined in such a way that an exception is raised if its argument is negative, and consider the following SQL expression:

```
SELECT ...
FROM ...
WHERE X >= 0 AND SQRT ( X ) <= 100 ...
```

This expression isn't guaranteed to avoid raising the exception, because the SQRT function might be invoked before the test to ensure X is nonnegative is done.

## Another Example

Consider again the database constraint discussed above in connection with logical implication:

```
IF ( COLOR = 'Red' ) THEN ( CITY = 'London' )
```

Let me now point out that this expression is logically equivalent to the following one:

```
IF NOT ( CITY = 'London' ) THEN NOT ( COLOR = 'Red' )
```

This latter expression is the contrapositive of the original one. In general, in fact, we have the following equivalence:

```
IF p THEN q \equiv IF NOT q THEN NOT p
```

So here's a question for you: How many of the following expressions are logically distinct?
■ IF ( WEIGHT > 17.0 ) THEN ( CITY $\neq$ 'Paris' )

- IF ( CITY = 'Paris' ) THEN ( WEIGHT $\leq 17.0$ )

■ ( WEIGHT $\leq 17.0$ ) OR ( CITY $\neq$ 'Paris' )

■ NOT ( ( CITY = 'Paris' ) AND ( WEIGHT > 17.0 ) )

Well, I hope you can see that all four of these expressions in fact say the same thing. However, I think you'll agree also that this fact isn't immediately obvious! Let's take a closer look. Let's use $p$ and $q$ to denote the subexpressions (WEIGHT > 17.0) and (CITY $\neq$ 'Paris'), respectively. The four expressions become:

- IF $p$ THEN $q$
- IF NOT q THEN NOT $p$
- NOT $p$ OR $q$
- NOT ( ( NOT q ) AND p )

Now I think it's easier to see that the four are all equivalent to one another. ${ }^{3}$ So the example demonstrates two things: First (to repeat), the equivalences aren't always obvious; second, introducing symbols like $p$ and $q$ allows us to manipulate the expressions in a purely formal manner and makes it easier to see what's really going on (easier to see the forest as well as the trees, one might say). I'll have more to say about such matters in the next chapter.

## SIMPLE AND COMPOUND PREDICATES

Consider the following statements: ${ }^{4}$

```
11. }x\mathrm{ is a star
12. x has two moons
13. }x\mathrm{ has m moons
14. }x\mathrm{ is between Earth and y
15. }x\mathrm{ is between }y\mathrm{ and }
```

Here $x, y, z$, and $m$ are parameters or placeholders. As a consequence, the statements aren't propositions (i.e., they aren't unequivocally either true or false), precisely because they do involve such parameters. For example, the statement " $x$ is a star" involves the parameter $x$, and we can't say whether it's true or false unless and until we're told what that $x$ stands for-at which point we're no longer dealing with the given statement anyway but a different one instead, as the paragraph immediately following makes clear.

Now, we can substitute arguments for the parameters and thereby obtain propositions from those parameterized statements. For example, if we substitute the argument the sun for the parameter $x$ in " $x$ is a star," we obtain "the sun is a star." And this statement is indeed a proposition, because it's unequivocally either true or false (in fact, of course, it's true). But the original statement as such (" $x$ is a star") is, to say it again, not itself a proposition. Rather, it's a predicate, which-as you'll recall from Chapter 5-is a truth valued function; that is to say, it's a function that, when invoked, returns a truth value. Like all functions, a predicate has a set of parameters; when it's invoked, arguments are substituted for the parameters; substituting arguments for the parameters effectively converts the predicate into a proposition; and we say the arguments satisfy the predicate if and only if that proposition is true. For example, the argument the sun satisfies the predicate " $x$ is a star," while the argument the moon does not.

As an aside, I remind you from Chapter 5 that logicians speak not of invoking a predicate but rather of instantiating it (in fact, for reasons that needn't concern us here, their concept of instantiation is slightly more general than that of the familiar notion of function invocation). However, I'll favor the terminology of invocation in

[^111]this chapter. Also, Exercise 5.18 in Chapter 5 showed that a proposition can be regarded as a degenerate predicate; to be precise, it's a predicate for which the set of parameters is empty (and the truth valued function that is that predicate thus always returns the same result, either TRUE or FALSE, every time it's invoked). In other words, all propositions are predicates, but most predicates aren't propositions.

Now consider the predicate " $x$ has $m$ moons," which involves two parameters, $x$ and $m$. (For example, substituting the arguments Mars for $x$ and 2 for $m$ yields a true proposition; substituting the arguments Earth for $x$ and 2 for $m$ yields a false one.) In general, in fact, predicates can conveniently be classified according to the cardinality of their set of parameters. Thus we speak of an $n$-place predicate, meaning a predicate with exactly $n$ parameters; for example, " $x$ is between $y$ and $z$ " is a 3-place predicate, while " $x$ has $m$ moons" is a 2-place predicate. A proposition is a 0 -place predicate. Note: An $n$-place predicate is also called an $n$-adic predicate. If $n=1$, the predicate is monadic; if $n=2$, it's dyadic. And a proposition is a niladic predicate.

Next, given a set of predicates, we can combine predicates from that set to form further predicates using the logical connectives already discussed (NOT, AND, OR, and so forth); in other words, the connectives are logical operators that operate on predicates in general, not just on the special predicates that happen to be propositions. A predicate that involves no connectives is called simple; a predicate that isn't simple is called compound, or composite. Here's an example of a compound predicate:

```
16. ( }x\mathrm{ is a star ) OR ( }x\mathrm{ is between Earth and y )
```

This predicate is dyadic-not because it involves two simple predicates, but because it involves two parameters, $x$ and $y$ (one of which is referenced twice and the other once only).

## Rules of Inference

It's a bit of a digression from my main purpose in this chapter, but as an aside I can now give a (somewhat loose) definition of predicate logic. Logic in general can be defined as the science or scientific study of the methods and principles used in valid reasoning. Predicate logic in particular can be defined as a formal system involving predicates and connectives and the inferences that can be made using such predicates and connectives. Observe, therefore, that predicate logic involves certain rules of inference-i.e., rules by which additional truths can be proved from established truths. The additional truths are called theorems, and the established truths are either axioms or theorems that have previously been proved.

One important inference rule is called modus ponens. This rule states that if we know that $p$ is true, and if we also know that IF $p$ THEN $q$ is true, then we can infer that $q$ is true. For example, given the truth of both "I have no money" and "If I have no money, then I will have to wash dishes," we can infer the truth of "I will have to wash dishes."

Another important inference rule is modus tollens, which says that if we know that IF $p$ THEN $q$ is true, but we also know that $q$ is false, then we can infer that $p$ is false. This rule is relevant to the process of database integrity checking. Conceptually, what happens is this: When an update is requested, the proposed new database value is checked against known integrity constraints; if the proposition expressed by some constraint-see Chapter 8-now evaluates to FALSE, that proposed new value must also represent falsehood, and so the update must be rejected.

## QUANTIFICATION

I showed in the previous section that one way to get a proposition from a predicate is to invoke it with an appropriate set of arguments. But there's another way, too, and that's by means of quantification. Let $p(x)$ be a monadic predicate (I show the single parameter $x$ explicitly for clarity). Then:

- The expression

EXISTS $x(p(x))$
is a proposition, and it means: "There exists at least one possible argument value $a$ that can be substituted for the parameter $x$ such that $p(a)$ evaluates to TRUE" (in other words, the argument value $a$ satisfies predicate $p$ ). For example, if $p$ is the predicate " $x$ is a logician," then

EXISTS x ( $x$ is a logician )
is a proposition-one that evaluates to TRUE, as it happens (for example, take $a$ to be Bertrand Russell).

- The expression

FORALL $x(p(x))$
is a proposition, and it means: "All possible argument values $a$ that can be substituted for the parameter $x$ are such that $p(a)$ evaluates to TRUE" (in other words, all such argument values $a$ satisfy predicate $p$ ). For example, if again $p$ is the predicate " $x$ is a logician," then

FORALL $x$ ( $x$ is a logician )
is a proposition-one that evaluates to FALSE, as it happens (for example, take $a$ to be George W. Bush).
Observe that it's sufficient to produce a single example to show the truth of the EXISTS proposition and a single counterexample to show the falsity of the FORALL proposition. Observe too in both cases that the parameter must be constrained to "range over" some set of permissible values (the set of all persons, in the example). I'll come back to this latter point in the section "Relational Calculus," later.

The term used in logic for constructs like EXISTS $x$ and FORALL $x$ is quantifiers (the term derives from the verb to quantify, which simply means to express as a quantity-that is, to say how much of something there is or how many somethings there are). Quantifiers of the form EXISTS ... are said to be existential; quantifiers of the form FORALL ... are said to be universal. And in logic texts, EXISTS is usually represented by a backward E ("ق") and FORALL by an upside down A ("V"). I use the keywords EXISTS and FORALL here for readability.

Aside: At this point, one reviewer asked whether a quantifier is just another connective. No, it isn't. Let $p(x)$ and $q(x)$ be predicates, each with a single parameter $x$. Then $p(x)$ and $q(x)$ can be combined in various ways by means of connectives (as in, e.g., $p(x)$ AND $q(x)$ ), but the result is always just another predicate with that same single parameter $x$. By contrast, quantifying over $x$-that is, forming an expression such as EXISTS $x(p(x))$ or FORALL $x(q(x))$-has the effect of converting the predicate concerned into a proposition. Thus, there's a clear logical difference between the two concepts. (Though I should add that in the database context, at least, the quantifiers can be defined in terms of certain connectives. I'll explain this point later, in the section "More on Quantification.") End of aside.

By way of another example, consider the dyadic predicate " $x$ is taller than $y$." If we quantify existentially over $x$, we obtain:

```
EXISTS x ( x is taller than y )
```

This statement isn't a proposition, because it isn't unequivocally either true or false; in fact, it's a monadic predicate-it has a single parameter, $y$. Suppose we invoke this predicate with argument Steve. We obtain:

```
EXISTS x ( x is taller than Steve )
```

This statement is a proposition (and if there exists at least one person-Arnold, say-who's taller than Steve, then it evaluates to TRUE). But another way to obtain a proposition from the original predicate is to quantify over both parameters. For example:

```
EXISTS x ( EXISTS y ( x is taller than y ) )
```

This statement is indeed a proposition; it evaluates to FALSE only if nobody is taller than anybody and to TRUE otherwise (think about it!).

There are several lessons to be learned from this example:

- To obtain a proposition from an $n$-adic predicate by quantification alone, it's necessary to quantify over every parameter. More generally, if we quantify over $m$ parameters $(m \leq n)$, we obtain a $k$-adic predicate, where $k$ $n-m$.
- Let's focus on existential quantification only for the moment. Then there are apparently two different propositions we can obtain in the example by "quantifying over everything":

```
EXISTS x ( EXISTS }y ( x is taller than y ) 
EXISTS y ( EXISTS x ( x is taller than }y\mathrm{ ) )
```

It should be clear, however, that these two propositions both say the same thing: "There exist persons $x$ and $y$ such that $x$ is taller than $y$." More generally, in fact, it's easy to see that a series of like quantifiers (all existential or all universal) can be written in any sequence we choose without changing the overall meaning. By contrast, with unlike quantifiers, the sequence matters (see the point immediately following).

When we "quantify over everything," each individual quantifier can be either existential or universal. In the example, therefore, there are six distinct propositions that can be obtained by fully quantifying, and I've listed them below. (Actually there are eight, but two of them can be ignored by virtue of the previous point.) I've also shown a precise natural language interpretation in each case. Note that those interpretations are all logically different!-in particular, some of them evaluate to TRUE and some to FALSE. Please note, however, that I've had to assume in connection with certain of those evaluations that there does exist at least one person "in the universe," as it were. I'll come back to this assumption in the section "More on Quantification," later.

```
EXISTS x ( EXISTS y ( x is taller than y ) )
```

Meaning: Somebody is taller than somebody; TRUE, unless everybody is the same height.

```
EXISTS x ( FORALL y ( x is taller than y ) )
```

Meaning: Somebody is taller than everybody (that particular somebody included!); clearly FALSE.
FORALL $x$ ( EXISTS $y(x$ is taller than $y)$
Meaning: Everybody is taller than somebody; clearly FALSE.
EXISTS $y$ ( FORALL $x$ ( $x$ is taller than $y$ )
Meaning: Somebody is shorter than everybody (that particular somebody included); clearly FALSE. Note: Actually I'm cheating a little bit here, because I haven't said what I mean by "shorter." But I could have done-i.e., I could have stated explicitly, somehow, that the predicates " $x$ is taller than $y$ " and " $y$ is shorter than $x$ " are logically equivalent-and I'll assume for the rest of this section that I've done so.

FORALL $y$ ( EXISTS $x$ ( $x$ is taller than $y$ ) )
Meaning: Everybody is shorter than somebody; clearly FALSE.
FORALL $x$ ( FORALL $y$ ( $x$ is taller than $y$ )
Meaning: Everybody is taller than everybody; clearly FALSE.
Last (I apologize for the repetition, but the point is important): Even though five out of six of the foregoing propositions do all evaluate to the same truth value, FALSE, it doesn't follow that they all mean the same thing, and indeed they don't; in fact, no two of them do.

## Free and Bound Variables

What I've so far been calling parameters are more usually known in logic as free variables-and quantifying over a free variable, using either EXISTS or FORALL, converts that free variable into what's called a bound variable. For example, consider again the 2-place predicate from the previous section:

```
x is taller than }
```

Here $x$ and $y$ are free variables. If we now quantify existentially over $x,{ }^{5}$ we obtain:

```
EXISTS x ( x is taller than }y\mathrm{ )
```

Now $y$ is free (still) but $x$ is bound. And if we now quantify existentially over $y$ as well, we obtain:

```
EXISTS x EXISTS y ( x is taller than y )
```

Now $x$ and $y$ are both bound, and there are no free variables at all (the predicate has degenerated to a proposition).
Now, we already know that free variables correspond to parameters, in conventional programming terms. Bound variables, by contrast, don't have an exact counterpart in conventional programming terms; instead, they're just a kind of dummy-they serve only to link the predicate inside the parentheses to the quantifier outside. For example, consider the simple predicate (actually a proposition):

[^112]```
EXISTS x ( x > 3 )
```

This proposition merely asserts that there exists some integer greater than three. (I'm assuming here that $x$ is constrained to "range over" the set of integers. Again, I'll come back to this question of "ranges" later.) Note, therefore, that the meaning of the proposition would remain totally unchanged if the two $x$ 's were both replaced by some other variable $y$. In other words, the proposition

```
EXISTS y ( y > 3 )
```

is semantically identical to the one just shown.
Now consider the predicate:

```
EXISTS x ( x > 3 ) AND x < 0
```

Here there are three $x$ 's-but they don't all mean the same thing. The first two are bound, and can be replaced by (say) $y$ without changing the overall meaning; but the third is free and can't be replaced with impunity. Thus, of the following two predicates, the first is equivalent to the one just shown and the second isn't:

```
EXISTS y ( y > 3 ) AND x < 0
EXISTS y ( y > 3 ) AND y<0
```

As this example demonstrates, the terminology of free vs. bound "variables" doesn't really refer to variables per se, but rather to variable occurrences-occurrences of references to variables within some predicate, to be precise. In the predicate EXISTS $y(y>3)$ AND $y<0$, for example, it's the first two occurrences of the reference to $y$ that are bound, and the third such occurrence that's free. Despite this state of affairs, it's usual (perhaps regrettably) to talk about free and bound variables as such, ${ }^{6}$ even though such talk is really quite sloppy. Be on your guard for confusion in this area!

To close this section, I remark that we can now (re)define a proposition to be a predicate in which all of the variables are bound: equivalently, one that involves no free variables.

## RELATIONAL CALCULUS

Essentially everything I've discussed in this chapter so far maps very directly into the relational calculus. Let's look at a simple example-a relational calculus representation of the query "Get supplier number and status for suppliers in Paris who supply part P2." Here first for comparison purposes is an algebraic formulation:

```
( S WHERE CITY = 'Paris' ) { SNO , STATUS }
    MATCHING ( SP WHERE PNO = 'P2' )
```

And here's a relational calculus equivalent:

[^113]```
RANGEVAR SX RANGES OVER S ;
RANGEVAR SPX RANGES OVER SP ;
{ SX.SNO , SX.STATUS }
    WHERE SX.CITY = 'Paris' AND
    EXISTS SPX ( SPX.SNO = SX.SNO AND SPX.PNO = 'P2' )
```


## Explanation:

- The first two lines are definitions, defining SX and SPX to be range variables that range over S and SP, respectively. What those definitions mean is that, at any given time, permitted values of SX are tuples in the relation that's the value of relvar $S$ at that time; likewise, permitted values of SPX are tuples in the relation that's the value of relvar SP at that time.
- The remaining lines are the actual query. Observe that they take the following generic form:

```
proto tuple WHERE predicate
```

This expression overall is the relational calculus version of a relational expression (i.e., an expression that denotes a relation), and it evaluates to a relation containing every possible value of the proto tuple for which the predicate evaluates to TRUE, and no other tuples. (The term proto tuple, standing for "prototype tuple," is apt but nonstandard; in fact, a standard term for the concept doesn't seem to exist.) In the example, therefore, the result is a relation of degree two, containing every (SNO,STATUS) pair from relvar S such that (a) the corresponding city is Paris and (b) there exists a shipment in relvar SP with the same supplier number as the one in that (SNO,STATUS) pair and with part number P2.

Note the use of dot qualified names in this example (in both the proto tuple and the predicate); I won't go into details, because dot qualified names will be familiar to you from SQL. Indeed, SQL has a formulation of the query under discussion that's very similar in general terms to the foregoing relational calculus formulation:

```
SELECT SX.SNO, SX.STATUS
FROM S AS SX
WHERE SX.CITY = 'Paris'
AND EXISTS
    ( SELECT *
        FROM SP AS SPX
        WHERE SPX.SNO = SX.SNO
        AND SPX.PNO = 'P2' )
```

As this example indicates:
■ First, SQL does support range variables (though it doesn't usually use that term): The specifications S AS SX and SP AS SPX serve to define such variables, and those variables are then explicitly referenced elsewhere in the overall expression by means of dot qualified names such as SX.SNO, SPX.SO, and so on. Note: In practice, such AS specifications and such explicit range variable references are often omitted, at least in simple queries. I'll explain exactly how such omissions are possible in Chapter 12, when I discuss range variables in SQL in more detail.

Second, and perhaps more important, SQL also supports EXISTS. However, that support is somewhat indirect. To be specific, let $s q$ be a subquery; then EXISTS $s q$ is a boolean expression (and so represents a
predicate), and it evaluates to FALSE if the table denoted by $s q$ is empty and TRUE otherwise. ${ }^{7}$ Note: The table expression $t x$ in parentheses that constitutes $s q$ will usually, though not invariably, be of the form SELECT * FROM ... WHERE ..., and the WHERE clause will usually, though not invariably, include some reference to some "outer" table, meaning $s q$ will typically be a correlated subquery specifically. In the foregoing example, S is that outer table, and it's referenced by means of the range variable SX. Again, see Chapter 12 for further explanation.

Aside: There's a certain irony here, though. As we saw in Chapter 4, SQL, because it supports nulls, is based on what's called three-valued logic, 3VL (instead of the conventional two-valued logic I'm discussing in this chapter, which is what the relational model is based on). In 3VL, the existential quantifier can return three different results: TRUE, FALSE, and UNKNOWN (where UNKNOWN is "the third truth value"; again, see Chapter 4). But SQL's EXISTS operator always returns TRUE or FALSE, never UNKNOWN. For example, EXISTS $(t x)$ will return TRUE, not UNKNOWN, if $t x$ evaluates to a table containing nothing but nulls (I'm speaking a trifle loosely here); yet UNKNOWN is the logically correct result. ${ }^{8}$ As a consequence, (a) SQL's EXISTS isn't a faithful implementation of the existential quantifier of 3VL, and (b) once again, therefore, SQL queries sometimes return the wrong answer. Example 3 in the next chapter is a case in point. End of aside.

Let's look at another example-the query "Get supplier names for suppliers who supply all parts." I'll assume we have the same range variables SX and SPX as before, but I'll also define another one (PX) ranging over P:

```
RANGEVAR PX RANGES OVER P ;
{ SX.SNAME } WHERE FORALL PX ( EXISTS SPX ( SPX.SNO = SX.SNO AND
    SPX.PNO = PX.PNO ) )
```

In somewhat stilted natural language: "Get names of suppliers such that, for all parts, there exists a shipment with the same supplier number as the supplier and the same part number as the part." Note: As you probably know, SQL has no direct support for FORALL. For that reason, I won't show an SQL analog of this example here-I'll come back to it later, in the section "More on Quantification." I will point out, however, that there's a logical difference between the foregoing calculus expression and this one, where the quantifiers have been switched:
$\begin{aligned}\{\text { SX.SNAME \} WHERE EXISTS SPX ( FORALL PX ( SPX.SNO } & =\text { SX.SNO AND } \\ \text { SPX.PNO } & =\text { PX.PNO ) })\end{aligned}$
Exercise: What does this latter expression mean? And do you think the query is a "sensible" one?
One more example ("Get supplier names for suppliers who supply at least one red part"):
\{ SX. SNAME \} WHERE EXISTS PX ( PX.COLOR = 'Red' AND
EXISTS SPX ( SPX.SNO $=$ SX.SNO AND SPX.PNO = PX.PNO ) )

[^114]I'm assuming here that we have the same range variables available to us as we had in the earlier examples; in fact, I'll continue to make that same assumption for the rest of this chapter.

By the way, here's another possible formulation of the foregoing query:

```
{ SX.SNAME } WHERE EXISTS PX ( EXISTS SPX ( PX.COLOR = 'Red' AND
    SPX.SNO = SX.SNO AND
    SPX.PNO = PX.PNO ) )
```

In this latter formulation, the predicate in the WHERE clause is in what's called "prenex normal form," meaning, loosely, that the quantifiers all appear at the beginning. Here's a precise definition of this concept:

Definition: A predicate is in prenex normal form (PNF) if and only if (a) it's quantifier free (i.e., it contains no quantifiers at all) or (b) it's of the form EXISTS $x(p)$ or FORALL $x(p)$, where $p$ is in PNF in turn. In other words, a PNF predicate takes the form

```
Q1 x1 ( Q2 x2 ( ... ( Qn xn ( q ) ) ... ) )
```

where (a) $n \geq 0$; (b) each $Q i(i=1,2, \ldots, n)$ is either EXISTS or FORALL; and (c) the predicate $q$-which is sometimes called the matrix-is quantifier free.

Prenex normal form isn't more or less correct than any other form, but with a little practice it does tend to become the most natural formulation, and the easiest to write, in many cases (not all).

## More on Range Variables

From what I've said in this section so far, it should be clear that range variables in the relational calculus serve as the free and bound variables that are required by formal logic. As I mentioned earlier, those variables always have to range over some set of permissible values; in the relational calculus context specifically, that set is always the body of some relation (usually but not necessarily the relation that's the current value of some relvar). Note: It follows that a given range variable always denotes some tuple. For that reason, the relational calculus is sometimes known more specifically as the tuple calculus, and the variables themselves as tuple variables. This latter usage can be confusing, however, since the term tuple variable already has a somewhat different (and more conventional) meaning-see Chapter 2-and I won't adopt it in this book. ${ }^{9}$

Now I can say a little more about the syntax of relational calculus expressions:
First of all, a proto tuple is a commalist of items enclosed in braces, in which each item is either a range attribute reference-possibly with an associated AS clause to introduce a new attribute name-or a range variable reference. (There are other possibilities too, but I'll limit my attention to just these cases until further notice. See Example 5 below.) Note: It's usual to omit the braces if the commalist contains just a single item, but I'll generally show them even when they're not actually required, for clarity.

- A range attribute reference is an expression of the form $R . A$, where $A$ is an attribute of the relation that range variable $R$ ranges over; SX.SNO is an example. And a range variable reference is just a range variable

[^115]name, like SX, and it's shorthand for a commalist of range attribute references, one for each attribute of the relation the range variable ranges over.

Let some range attribute reference involving range variable $R$ appear, explicitly or implicitly, within some proto tuple. Then the predicate in the corresponding WHERE clause can, and usually will, contain at least one free range attribute reference involving $R$-where by "free range attribute reference involving $R$ " I mean a range attribute reference of the form $R . A$ that's not within the scope of any quantifier in which $R$ is the bound variable.

- The WHERE clause is optional; omitting it is equivalent to specifying WHERE TRUE.


## More Sample Queries

I'll give a few more examples of relational calculus queries, in order to illustrate a few more points; however, I'm not trying to be exhaustive in my treatment. For simplicity, I'll omit the RANGEVAR definitions that would be needed in practice and will just assume that SX, SY, etc., have been defined as range variables over S; PX, PY, etc., have been defined as range variables over P; and SPX, SPY, etc., have been defined as range variables over SP. Please note that the formulations shown aren't the only ones possible, in general. I'll leave it as an exercise for you to show equivalent SQL formulations in each case.

Example 1: Get all pairs of supplier numbers such that the suppliers concerned are colocated.

$$
\begin{array}{r}
\{\mathrm{SA}:=\mathrm{SX} . \mathrm{SNO}, \mathrm{SB}:=\text { SY.SNO \} WHERE SX.CITY = SY.CITY } \\
\text { AND SX.SNO }<\text { SY.SNO }
\end{array}
$$

Note the introduction of result attribute names SA and SB in this example. Incidentally, this example provides a good illustration of the point that some queries are more easily formulated in the calculus than they are in the algebra (an algebraic formulation for this query-rather more complicated than the calculus formulation just shown-was given if you recall in the section "Formulating Expressions One Step at a Time" in Chapter 6).

Example 2: Get supplier names for suppliers who supply at least one Paris part.

$$
\begin{array}{r}
\{\text { SX.SNAME \} WHERE EXISTS SPX ( EXISTS PX ( SX.SNO }=\text { SPX.SNO AND } \\
\text { SPX.PNO }=\text { PX.PNO AND } \\
\text { PX.CITY }=\text { 'Paris' ) })
\end{array}
$$

Example 3: Get supplier names for suppliers who supply at least one part supplied by supplier S2.

$$
\begin{aligned}
\{\text { SX.SNAME \} WHERE EXISTS SPX ( EXISTS SPY ( SX.SNO }=\text { SPX.SNO AND } \\
\text { SPX.PNO }=\text { SPY.PNO AND } \\
\text { SPY.SNO }=' \text { 'S2' ) ) }
\end{aligned}
$$

Example 4: Get supplier names for suppliers who don't supply part P2.

$$
\begin{array}{r}
\{\text { SX.SNAME }\} \text { WHERE NOT ( EXISTS SPX ( SPX.SNO }=\text { SX.SNO AND } \\
\text { SPX.PNO }=\text { 'P2' ) ) }
\end{array}
$$

The outer parentheses in this example (i.e., the ones enclosing the expression following NOT) might not be needed in practice; indeed, I'll often omit such parentheses in later examples.

Incidentally, the predicate in the foregoing formulation isn't in prenex normal form. It would be possible to replace it by one that is, like this-

```
{ SX.SNAME } WHERE FORALL SPX ( SPX.SNO f SX.SNO OR SPX.PNO # `P2' )
```

-but I don't think this alternative formulation is as "natural" as the non PNF version; that is, I think this example illustrates the point that a PNF formulation isn't always the one that comes most readily to mind.

Example 5: For each shipment, get full shipment details, including total shipment weight.

```
{ SPX , SHIPWT := PX.WEIGHT * SPX.QTY } WHERE PX.PNO = SPX.PNO
```

Note the use of a computational expression in the proto tuple here. An algebraic version of this example would involve EXTEND, and probably image relations also.

Example 6: For each part, get the part number and the total shipment quantity.

```
{ PX.PNO , TOTQ := SUM ( SPX WHERE SPX.PNO = PX.PNO , QTY ) }
```

This example illustrates the use of an aggregate operator invocation within the proto tuple (it's also the first example to omit the WHERE clause). Incidentally, note that the following expression, though syntactically legal, would not be a correct formulation of the query (why not?):

```
{ PX.PNO , TOTQ := SUM ( SPX.QTY WHERE SPX.PNO = PX.PNO ) }
```

Answer: Because duplicate quantities would be eliminated before the sum is computed.
Example 7: Get part cities that store more than five red parts.

```
{ PX.CITY }
    WHERE COUNT ( PY WHERE PY.CITY = PX.CITY AND PY.COLOR = `Red' ) > 5
```


## Sample Constraints

Now I'd like to give some examples of the use of relational calculus to formulate constraints. The first eight are based on, and use the same numbering as, the examples in Chapter 8. I'll assume the availability of range variables as in the previous subsection. Please note again that the formulations shown aren't the only ones possible, in general.

Example 1: Status values must be in the range 1 to 100 inclusive.

```
CONSTRAINT CX1 FORALL SX ( SX.STATUS \geq 1 AND SX.STATUS \leq 100 ) ;
```

Note: SQL allows a constraint like this one to be simplified by (in effect) eliding both the explicit range variable and, more important, the explicit universal quantification. To be specific, we can specify a base table constraint-see Chapter 8-as part of the definition of base table S that looks like this:

```
CONSTRAINT CX1 CHECK ( STATUS >= 1 AND STATUS <= 100 )
```

Similar remarks apply to subsequent examples also.
Example 2: Suppliers in London must have status 20.

```
CONSTRAINT CX2 FORALL SX ( IF SX.CITY = 'London'
    THEN SX.STATUS = 20 ) ;
```

Example 3: No two tuples in relvar S have the same supplier number (i.e., $\{\mathrm{SNO}\}$ is a key, or rather a superkey, for relvar S).

```
CONSTRAINT CX3 FORALL SX ( FORALL SY ( IF SX.SNO = SY.SNO THEN
    SX.SNAME = SY.SNAME AND
    SX.STATUS = SY.STATUS AND
    SX.CITY = SY.CITY ) ) ;
```

This formulation isn't very elegant, to say the least! I'll come back to this example and give a better formulation of it in the next section.

Example 4: Whenever two tuples in relvar S have the same supplier number, they also have the same city (in other words, the functional dependency $\{\mathrm{SNO}\} \rightarrow\{$ CITY $\}$ holds in relvar S).

```
CONSTRAINT CX4 FORALL SX ( FORALL SY ( IF SX.SNO = SY.SNO
    THEN SX.CITY = SY.CITY ) ) ;
```

As noted in Chapter 8, this constraint is actually a logical consequence of the fact that $\{\mathrm{SNO}\}$ is a superkey for relvar S. If this latter constraint is stated, therefore, constraint CX4 needn't be.

Example 5: No supplier with status less than 20 can supply part P6.

```
CONSTRAINT CX5 FORALL SX ( IF SX.STATUS < 20 THEN
    NOT EXISTS SPX ( SPX.SNO = SX.SNO AND
    SPX.PNO = 'P6' ) ) ;
```

Example 6: Every supplier number in relvar SP must appear in relvar S.

```
CONSTRAINT CX6 FORALL SPX ( EXISTS SX ( SX.SNO = SPX.SNO ) ) ;
```

As with Example 3, I'll have more to say about this example in the next section.
Example 7: No supplier number appears in both relvar LS and relvar NLS.

```
CONSTRAINT CX7 FORALL LX ( FORALL NX ( LX.SNO # NX.SNO ) ) ;
```

LX and NX range over LS and NLS, respectively.

Example 8: Supplier S 1 and part P1 must never be in different cities.

```
CONSTRAINT CX8 NOT EXISTS SX ( EXISTS PX ( SX.SNO = 'S1' AND
    PX.PNO = 'P1' AND
    SX.CITY f PX.CITY ) ) ;
```

By the way, is this constraint satisfied if there's no tuple for supplier S1 in relvar S and/or no tuple for part P1 in relvar P? (Answer: Yes, it is.)

Example 9: There must always be at least one supplier. (There's no counterpart to this example in Chapter 8.)

```
CONSTRAINT CX9 EXISTS SX ( TRUE ) ;
```

The expression EXISTS SX (TRUE) evaluates to FALSE if and only if SX ranges over an empty relation. (By contrast, the expression EXISTS SX (FALSE) always evaluates to FALSE. Conversely, the expression FORALL SX (FALSE) evaluates to TRUE if and only if SX ranges over an empty relation-see the discussion of empty ranges in the next section-while the expression FORALL SX (TRUE) always evaluates to TRUE.)

## MORE ON QUANTIFICATION

There are a number of further issues I need to discuss regarding quantification in particular.

## We Don't Need Both Quantifiers

It's easy to see that any predicate that can be expressed in terms of EXISTS can be expressed in terms of FORALL instead and vice versa. By way of example, consider the following predicate once again:

```
EXISTS x ( x is taller than Steve )
```

("Somebody is taller than Steve"; of course, this predicate is in fact a simple proposition). Another way to say the same thing is:

```
NOT ( FORALL x ( NOT ( x is taller than Steve ) ) )
```

("It is not the case that nobody is taller than Steve"). More generally, in fact, the predicate

```
EXISTS x ( p ( x ) )
```

is logically equivalent to the predicate

```
NOT ( FORALL x ( NOT ( p ( x ) ) ) )
```

(where the predicate $p$ might legitimately involve other parameters in addition to $x$ ). Likewise, the predicate

```
FORALL x ( p ( x ) )
```

is logically equivalent to the predicate

```
NOT ( EXISTS x ( NOT ( p ( x ) ) ) )
```

(where, again, the predicate $p$ might legitimately involve other parameters in addition to $x$ ).
It follows from all of the above that a formal language doesn't need to support both EXISTS and FORALL explicitly. But it's very desirable to support them both in practice. The reason is that some problems are "more naturally" formulated in terms of EXISTS, while others are "more naturally" formulated in terms of FORALL instead. For example, SQL supports EXISTS but not FORALL, as we know; as a consequence, certain queries are quite awkward to formulate in SQL. Consider again the query "Get suppliers who supply all parts," which can be expressed in relational calculus quite simply as follows:

```
{ SX } WHERE FORALL PX ( EXISTS SPX
    ( SPX.SNO = SX.SNO AND SPX.PNO = PX.PNO ) )
```

In SQL, by contrast, the query has to look something like this:

```
SELECT SX.*
FROM S AS SX
WHERE NOT EXISTS
    ( SELECT *
        FROM P AS PX
        WHERE NOT EXISTS
            ( SELECT *
                FROM SP AS SPX
                WHERE SX.SNO = SPX.SNO
                    AND SPX.PNO = PX.PNO ) )
```

("Get suppliers SX such that there does not exist a part PX such that there does not exist a shipment SPX linking that supplier SX to that part PX"). Well, single negation is bad enough (users often have trouble with it); double negation, as in this example, is much worse.

## Empty Ranges

Consider again the fact that the predicates

```
EXISTS x ( p ( x ) )
```

and

```
NOT ( FORALL x ( NOT ( p ( x ) ) ) )
```

are logically equivalent. As we know, the bound variable $x$ in each of these predicates must range over some set of permissible values. Suppose now that the set in question is empty; it might, for example, be the set of persons over fifty feet tall (or in the database context, more realistically, it might be the set of tuples in a relvar that's currently empty). Then:

- The expression EXISTS $x(p(x))$ evaluates to FALSE, because "there is no $x$ "-i.e., there's no value available to be substituted for $x$ in order to make the expression true. Note carefully that these remarks are valid
regardless of what $p(x)$ happens to be. For example, "There exists a person over fifty feet tall who works for IBM" evaluates to FALSE (unsurprisingly).
- It follows that the negation NOT EXISTS $x(p(x))$ evaluates to TRUE-again, regardless of what $p(x)$ happens to be. For example, "There doesn't exist a person over fifty feet tall who works for IBM"-more colloquially, "No person over fifty feet tall works for IBM"-evaluates to TRUE (again unsurprisingly).
- But NOT EXISTS $x(p(x)$ ) is equivalent to FORALL $x$ (NOT $(p(x))$ ), and so this latter expression also evaluates to TRUE-once again, regardless of what $p(x)$ happens to be.
- But if the predicate $p(x)$ is arbitrary, then so is the predicate NOT $(p(x))$. And so we have the following possibly surprising result: The expression FORALL $x$ (...) evaluates to TRUE if there are no $x$ 's, regardless of what appears inside the parentheses. For example, "All persons over fifty feet tall do work for IBM" also evaluates to TRUE-because, to say it again, there aren't any persons over fifty feet tall.

One implication of the foregoing state of affairs is that certain queries produce a result that you might not expect (if you don't know logic, that is). For example, the query discussed earlier-

```
{ SX } WHERE FORALL PX ( EXISTS SPX ( SPX.SNO = SX.SNO AND
    SPX.PNO = PX.PNO ) )
```

("Get suppliers who supply all parts")-will return all suppliers if there aren't any parts.
Incidentally, the foregoing example serves as a good illustration of the point that while logic is certainly necessary as a foundation for database systems, it might not be sufficient. For example, how do you think an only child should respond to the question "Are your siblings all boys?" The logically correct answer is, of course, yes (though I observe that yes is the logically correct answer to the question "Are your siblings all girls?" as well). In practice, however, we would surely expect some more informative response, along the lines of "Well, actually I don't have any siblings." In other words-and now reverting to database systems as such-it might be nice if the system, as well as simply giving its responses as such, could also explain those responses if it's asked to do so.

## Defining EXISTS and FORALL

As you might have realized, EXISTS and FORALL can be defined as an iterated $O R$ and an iterated $A N D$, respectively. I'll consider EXISTS first. Let $p(x)$ be a predicate with a parameter $x$ and let $x$ range over the set $X=$ $\{x 1, x 2, \ldots, x n\}$. Then

```
EXISTS x ( p ( x ) )
```

is a predicate, and it's defined to be equivalent to (and hence shorthand for) the predicate

```
p (x1 ) OR p ( x2 ) OR ... OR p ( xn ) OR FALSE
```

Observe in particular that this expression evaluates to FALSE if $X$ is empty (equivalently, if $n=0$ ), as we already know. By way of example, let $p(x)$ be " $x$ has a moon" and let $X$ be the set \{Mercury, Venus, Earth, Mars\}. Then the predicate EXISTS $x(p(x))$ becomes "EXISTS $x$ ( $x$ has a moon)," and it's shorthand for

```
( Mercury has a moon ) OR (Venus has a moon ) OR
( Earth has a moon ) OR ( Mars has a moon ) OR FALSE
```

which evaluates to TRUE because, e.g., "Mars has a moon" is true. Similarly,

```
FORALL X ( P ( }x\mathrm{ ) )
```

is a predicate, and it's defined to be equivalent to (and hence shorthand for) the predicate

```
p ( x1 ) AND p ( x2 ) AND ... AND p ( xn ) AND TRUE
```

And this expression evaluates to TRUE if $X$ is empty (again, as we already know). By way of example, let $p(x)$ and $X$ be as in the EXISTS example above. Then the predicate FORALL $x(p(x))$ becomes "FORALL $x$ ( $x$ has a moon)," and it's shorthand for
( Mercury has a moon ) AND ( Venus has a moon ) AND
( Earth has a moon ) AND ( Mars has a moon ) AND TRUE
which evaluates to FALSE because, e.g., "Venus has a moon" is false.
As an aside, let me remark that, as the examples demonstrate, defining EXISTS and FORALL as iterated OR and AND, respectively, means that every predicate that involves quantification is equivalent to one that doesn't. Thus, you might be wondering, not without some justification, just what this business of quantification is really all about ... Why all the fuss? The answer is as follows: We can define EXISTS and FORALL as iterated OR and AND only because the sets we have to deal with are-thankfully-always finite (because we're operating in the realm of computers and computers are finite in turn). In pure logic, where there's no such restriction, those definitions aren't valid. ${ }^{10}$

Perhaps I should add that, even though we're always dealing with finite sets and EXISTS and FORALL are thus merely shorthand, they're extremely useful shorthand! For my part, I certainly wouldn't want to have to formulate queries and the like purely in terms of AND and OR, without being able to use the quantifiers. Much more to the point, the quantifiers allow us to formulate queries without having to know the precise content of the database at any given time (which wouldn't be the case if we always had to use the explicit iterated OR and AND equivalents). ${ }^{11}$

## Other Kinds of Quantifiers

While it's certainly true that EXISTS and FORALL are the most important quantifiers in practice, they aren't the only ones possible. There's no a priori reason, for example, why we shouldn't allow quantifiers of the form

there exist at least three $x$ 's such that

[^116]or
a majority of $x$ 's are such that
or
an odd number of $x$ 's are such that
(and so on). One fairly important special case is there exists exactly one $x$ such that. I'll use the keyword UNIQUE for this one. Here are some examples:

```
UNIQUE x ( x is taller than Arnold )
```

Meaning: Exactly one person is taller than Arnold; probably FALSE.

```
UNIQUE x ( }x\mathrm{ has social security number }y\mathrm{ )
```

Meaning: Exactly one person has social security number $y$ ( $y$ is a parameter). We can't assign a truth value to this example because it's a (monadic) predicate and not a proposition.

```
FORALL y ( UNIQUE x ( }x\mathrm{ has social security number y ) )
```

Meaning: Everybody has a unique social security number (I'm assuming here that $y$ ranges over the set of all social security numbers actually assigned, not all possible ones). Exercise: Does this predicate-which is in fact a proposition-evaluate to TRUE?

As another exercise, what does the following predicate mean?

```
FORALL x ( UNIQUE }y\mathrm{ ( }x\mathrm{ has social security number }y\mathrm{ ) )
```

Here's how UNIQUE might be used in the formulation of constraints. Recall the formulation I gave earlier for constraint CX3 ("every supplier has a unique supplier number"):

```
CONSTRAINT CX3 FORALL SX ( FORALL SY ( IF SX.SNO = SY.SNO THEN
    SX.SNAME = SY.SNAME AND
    SX.STATUS = SY.STATUS AND
    SX.CITY = SY.CITY ) ) ;
```

A much better formulation would clearly be as follows:

```
CONSTRAINT CX3 FORALL SX ( UNIQUE SY ( SX.SNO = SY.SNO ) ) ;
```

("For all suppliers SX, there's exactly one supplier SY with the same supplier number.") For example, if SX denotes the tuple for supplier S4, say, then SY must also denote the tuple for supplier S4-in other words, SX and SY must denote the very same tuple-in order for the constraint to be satisfied.

By way of another example, recall the following constraint: "Every supplier number in relvar SP must appear in relvar S." Here's the formulation I gave previously:

```
CONSTRAINT CX6 FORALL SPX ( EXISTS SX ( SX.SNO = SPX.SNO ) ) ;
```

However, I hope you can see a more accurate formulation is:

```
CONSTRAINT CX6 FORALL SPX ( UNIQUE SX ( SX.SNO = SPX.SNO ) ) ;
```

In other words, for a given tuple in relvar SP, we want there to be not at least one (EXISTS), but exactly one (UNIQUE), corresponding tuple in relvar S. The previous formulation "works" because there's an additional constraint in effect: viz., that $\{\mathrm{SNO}\}$ is a key for relvar S . But the revised formulation is closer to what we really want to say.

Now, SQL does support UNIQUE (sort of), though its support is even more indirect than its support for EXISTS is. To be specific, let $s q$ be a subquery; then UNIQUE $s q$ is a boolean expression, and it evaluates to FALSE if the table denoted by $s q$ contains any duplicate rows and TRUE otherwise. Note that it follows from this definition that the operator certainly returns TRUE if its argument table has either just one row or no rows at all. ${ }^{12}$ And it further follows that, whereas the logic expression

```
UNIQUE x ( p ( x ) )
```

means "There exists exactly one argument value $a$ corresponding to the parameter $x$ such that $p(a)$ evaluates to TRUE," the (very approximate!) SQL analog-

```
UNIQUE ( SELECT k FROM T AS ... WHERE p ( x ) )
```

-where $k$ denotes an arbitrary constant value, say the integer 0-means "Given an argument value $a$ corresponding to the parameter $x$, there exists at most one row in the pertinent table $T$ such that $p(a)$ evaluates to TRUE." For example, given our usual sample value for relvar $S$, the SQL expression

UNIQUE ( SELECT 0 FROM S AS SX WHERE SX.CITY = 'Athens' )
returns TRUE, while the SQL expression
UNIQUE ( SELECT 0 FROM S AS SX WHERE SX.CITY = 'Paris' )
returns FALSE. ${ }^{13}$
All of that being said, I won't attempt to give an SQL formulation here for constraint CX6 that uses UNIQUE-I'll leave it to Chapter 11 (see Example 10 in that chapter).

As you can see, the foregoing examples, in which the only item in the SELECT clause is a simple literal, are designed to exploit the fact that SQL retains duplicates in the result of a SELECT expression if DISTINCT isn't specified. Of course, I've suggested elsewhere in this book-in Chapter 4, to be specific-that DISTINCT should "always" be specified. In contexts like the one under discussion, however, DISTINCT must definitely not be specified (right?).

[^117]
## 240

Now, I don't mean to suggest that the argument expression in an SQL UNIQUE invocation must always be of the form "SELECT $k$ FROM ..." where $k$ denotes some constant. By way of a counterexample, here repeated from Chapter 8 is one possible SQL formulation of the constraint that distinct suppliers must have distinct supplier numbers:

```
CREATE ASSERTION CX3 CHECK ( UNIQUE ( SELECT SNO FROM S ) ) ;
```

Recall now that SQL also uses the keyword UNIQUE in key constraints. For example, the CREATE TABLE for table S includes the following specification:

```
UNIQUE ( SNO )
```

You can think of this specification as shorthand for the following (which could be part of a more general base table constraint or a CREATE ASSERTION statement): ${ }^{14}$

```
CHECK ( UNIQUE ( SELECT SNO FROM S ) )
```

SQL also uses the keyword UNIQUE in MATCH expressions. Here's an example ("Get suppliers who supply exactly one part"): ${ }^{15}$

```
SELECT SX.*
FROM S AS SX
WHERE SX.SNO MATCH UNIQUE
    ( SELECT SPX.SNO
        FROM SP AS SPX )
```

But this usage too is basically just shorthand. For example, the example just shown is equivalent to the following:

```
SELECT SX.*
FROM S AS SX
WHERE UNIQUE ( SELECT SPX.SNO /* i.e., there's AT */
    FROM SP AS SPX /* MOST one shipment */
    WHERE SPX.SNO = SX.SNO ) /* for supplier SX */
AND EXISTS ( SELECT SPX.SNO /* ... and there's */
    FROM SP AS SPX /* also AT LEAST one */
    WHERE SPX.SNO = SX.SNO )
```

Incidentally, note that the UNIQUE invocation here is indeed of the form "UNIQUE (SELECT $k$ FROM ...)" where $k$ denotes a constant value, thanks to the boolean expression in the inner WHERE clause ("SPX.SNO $=$ SX.SNO")-constant, that is, with respect to "the current row" of table S, and hence with respect to each evaluation

[^118]of the outer WHERE clause. Of course, it's a distinct constant value with respect to distinct evaluations of that clause.

## SOME EQUIVALENCES

I'll finish up this chapter with a few remarks regarding certain equivalences that might have already occurred to you (indeed, I've touched on some of them myself from time to time at earlier points). First of all, recall the IS_EMPTY operator, which I introduced in Chapter 3 and made heavy use of in Chapter 8. If the system supports that operator, then there's no logical need for it to support the quantifiers, thanks to the following equivalences:

```
    EXISTS x ( p ) \equiv NOT ( IS_EMPTY ( X WHERE p ) )
    FORALL x ( p ) \equiv IS_EMPTY ( X WHERE NOT ( p ) )
```

(I'm assuming here that the variable $x$ ranges over the set $X$.)
In fact, SQL's support for EXISTS-and FORALL, such as it is-is based on exactly the foregoing equivalences. The fact is, SQL's EXISTS isn't really a quantifier, as such, at all, because it doesn't involve any bound variables. Instead, it's an operator, in the conventional sense of that term: a monadic operator of type BOOLEAN, to be precise. Like any monadic operator invocation, an invocation of the SQL EXISTS operator is evaluated by first evaluating the expression that denotes its sole argument, and then applying the operator per se-in this case EXISTS-to the result of that evaluation. Thus, given the expression EXISTS $(t x)$, where $t x$ is a table expression, the system first evaluates $t x$ to obtain a table $t$; then it applies EXISTS to $t$, returning TRUE if $t$ is nonempty and FALSE otherwise. (At least, that's the conceptual algorithm; numerous optimizations are possible, but they're irrelevant to the present discussion.)

And now I can explain why SQL doesn't support FORALL. The reason is that representing the universal quantifier by means of an operator with syntax of the form FORALL $(t x)$-where $t x$ is again a table expression-couldn't possibly make any sense. For example, consider the hypothetical expression FORALL (SELECT * FROM S WHERE CITY = 'Paris'). What could such an expression possibly mean? It certainly couldn't mean anything like "All suppliers are in Paris," because-loosely speaking-the argument to which the hypothetical operator is applied isn't all suppliers, it's all suppliers in Paris.

In fact, however, we don't need the quantifiers anyway if the system supports the aggregate operator COUNT, thanks to the following equivalences:

EXISTS $x(p) \equiv \operatorname{COUNT}(X$ WHERE $p)>0$
FORALL $x(p) \equiv \operatorname{COUNT}(X$ WHERE $p)=\operatorname{COUNT}(X)$

UNIQUE $x(p) \equiv \operatorname{COUNT}(X$ WHERE $p)=1$

Now, I'm certainly not a fan of the idea of replacing quantified expressions by expressions involving COUNT invocations-though sometimes we have to, if we're in an algebraic context ${ }^{16}$-but it would be wrong of me not to mention the possibility.

Aside: Although this book generally has little to say on performance, I should at least point out that the foregoing equivalences (the ones involving COUNT, I mean) could lead to performance problems. For example, consider the following expression, which is an SQL formulation of the query "Get suppliers who supply at least one part":

```
SELECT *
FROM S
WHERE EXISTS
    ( SELECT *
        FROM SP
        WHERE SP.SNO = S.SNO )
```

Now, here's another formulation that's logically equivalent to the foregoing:

```
SELECT *
FROM S
WHERE ( SELECT COUNT ( * )
    FROM SP
    WHERE SP.SNO = S.SNO ) > 0
```

But we don't really want the system to perform the complete count that's apparently being requested here and then check to see whether that count is greater than zero; rather, we want it to stop counting, for any given supplier, as soon as it finds the first shipment for that supplier. In other words, we'd really like some optimization to be done. Writing code that effectively requires a certain optimization to be done is usually not a good idea! Recommendation: Be careful over the use of COUNT, therefore; in particular, don't use it where EXISTS would be more logically correct. End of aside.

## Relational Completeness

Every operator of the relational algebra has a precise definition in terms of logic. (I didn't state this fact explicitly before, but it's easy to see the definitions I gave in Chapters 6 and 7 for join and the rest can be reformulated in terms of logic as described in the present chapter.) It follows as a direct consequence that, for every expression of the relational algebra, there's an expression of the relational calculus that's logically equivalent to-i.e., has the same semantics as-that algebraic expression. In other words, the relational calculus is at least as "powerful" (better: expressive) as the relational algebra: Anything that can be expressed in the algebra can be expressed in the calculus.

Now, it might not be obvious, but actually the opposite is true too; that is, for every expression of the relational calculus, there's an expression of the relational algebra that's logically equivalent to that calculus expression. Thus, the algebra is at least as expressive as the calculus, and so the two formalisms are logically

[^119]equivalent: Both are what's called relationally complete. ${ }^{17}$ Relational completeness is a basic measure of the expressive capability of a language; if a language is relationally complete, it means (among other things, and speaking a trifle loosely) that queries of arbitrary complexity can be formulated without having to resort to iterative loops or branching. In other words, it's relational completeness that allows end users-at least in principle, though possibly not in practice-to access the database directly, without having to go through the potential bottleneck of the IT department.

## The Importance of Consistency

I have a small piece of unfinished business to attend to. Recall my claim in Chapter 8 that any proposition whatsoever (even obviously false ones like $1=0$ ) can be shown to be "true" in an inconsistent system. Now I can elaborate on that claim.

I'll start with a really simple example. Suppose (a) relvar S is currently nonempty; (b) there's a constraint to the effect that there must always be at least one part; but (c) relvar P is in fact currently empty (there's the inconsistency). Now consider the relational calculus query:
\{ SX \} WHERE EXISTS PX ( TRUE )
Or if you prefer SQL:

```
SELECT *
FROM S
WHERE EXISTS
    ( SELECT *
        FROM P )
```

Now, if this expression is evaluated directly, the result will be empty. Alternatively, if the system (or the user) observes that there's a constraint that says that EXISTS PX (TRUE) must evaluate to TRUE-or, in SQL, that SELECT * FROM P must return a nonempty result-the WHERE clause can be reduced to one saying simply WHERE TRUE, and the result will then be all suppliers. At least one of these results must be wrong! In a sense, in fact, they're both wrong; given an inconsistent database, there simply isn't-there can't be-any well defined notion of correctness, and any answer is as good (or bad) as any other. Indeed, this state of affairs should be self-evident: If I tell you some proposition $p$ is both true and false, and then ask you whether some proposition that relies on $p$ in some way is true, there's simply no right answer you can give me.

In case you're still not convinced, consider the following slightly more realistic SQL example (under the same assumptions as before):

```
SELECT DISTINCT
    CASE WHEN EXISTS ( SELECT * FROM P ) THEN x ELSE y END
FROM
    S
```

This expression will return either $x$ or $y$-more precisely, it will return a table containing (a row containing) either $x$ or $y$-depending, in effect, on whether or not the EXISTS invocation is replaced by just TRUE. Now consider that $x$ and $y$ can each be essentially anything at all ... For example, $x$ might be an SQL expression denoting the total weight of all parts, while $y$ might be the literal 0 -in which case executing the query could easily lead to the erroneous conclusion that the total part weight is null instead of zero.

[^120]
## CONCLUDING REMARKS

It's my strong belief that database professionals in general, and SQL practitioners in particular, should have some familiarity with the basic concepts of predicate logic (or relational calculus-it comes to the same thing). I'd like to conclude by trying to justify this position.

My basic point is simply that a knowledge of logic helps you think precisely (and in our field, the importance of thinking precisely is surely paramount). In particular, it forces you to appreciate the significance of proper quantification. Natural language is so often imprecise; however, careful consideration of what quantifiers are needed allows you to pin down the meaning of what can otherwise be very imprecise natural language statements. By way of example, you might like to meditate on exactly what Abraham Lincoln meant-or might have meant, or thought he might have meant, or might have thought he meant-when he famously said: "You can fool some of the people some of the time, and some of the people all the time, but you cannot fool all the people all of the time."

Now, I'm well aware there are many who disagree with me here; that is, there are many who feel ordinary mortals shouldn't have to grapple with a subject as abstruse as logic seems to be. In effect, they claim that logic is just too difficult for most people to deal with. Now, that claim might be true in general (logic is a big subject). But you don't need to understand the whole of logic for the purpose at hand; in fact, I doubt whether you need much more than what I've covered in this chapter. And the benefits are so huge! I made essentially the same point in another book-Logic and Databases: The Roots of Relational Theory (Trafford, 2007)-and I'd like to quote the concluding remarks from that earlier discussion here:

Surely it's worth investing a little effort up front in becoming familiar with [basic logic] in order to avoid the problems associated with ambiguous business rules. Ambiguity in business rules leads to implementation delays at best or implementation errors at worst (possibly both). And such delays and errors certainly have costs associated with them, costs that are likely to outweigh those initial learning costs many times over. In other words, framing business rules properly is a serious matter, and it requires a certain level of technical competence.

As you can see, these remarks are set in the context of business rules specifically, but I think they're of wider applicability-as I'll try to demonstrate in the next chapter.

## EXERCISES

10.1 As noted in the body of the chapter, there are exactly 16 dyadic connectives. Show the corresponding truth tables. How many monadic connectives are there?
10.2 Let $p$ and $q$ stand for arbitrary propositions. Prove that

```
NOT (pAND q ) \equiv(NOT p ) OR (NOT q )
```

10.3 Again let $p$ and $q$ denote arbitrary propositions. Prove that

```
( ( NOT p ) AND ( p OR q ) ) IMPLIES q
```

is a tautology. (I remind you from Chapter 4 that a tautology in logic is an expression that's guaranteed to evaluate to TRUE, regardless of the values of any variables involved. Likewise, a contradiction is an expression that's guaranteed to evaluate to FALSE, regardless of the values of any variables involved.)
10.4 (Repeated from the body of the chapter, but reworded here.) (a) Prove that all of the monadic and dyadic connectives can be expressed in terms of suitable combinations of NOT and either AND or OR; (b) prove also that they can all be expressed in terms of just a single connective.
10.5 Consider the predicate " $x$ is a star." (a) First, if the argument the sun is substituted for $x$, does the predicate become a proposition? If not, why not? And what about the argument the moon? (b) Second, if the argument the sun is substituted for $x$, is the predicate satisfied? If not, why not? And what about the argument the moon?
10.6 Consider the predicate " $x$ has two moons." If the argument Jupiter is substituted for $x$, is the predicate satisfied? Justify your answer.
10.7 Here's constraint CX1 once again from Chapter 8:

```
CONSTRAINT CX1 IS_EMPTY ( S WHERE STATUS < 1 OR STATUS > 100 ) ;
```

The expression IS_EMPTY (...) here is clearly a predicate. Now, in Chapter 8, I said the relvar name " S " in that predicate was acting as a designator. But isn't it actually a parameter? If not, what's the difference?
10.8 (Repeated from the body of the chapter.) What query does the following expression represent? And do you think that query is a "sensible" one?

```
{ SX.SNAME } WHERE EXISTS SPX ( FORALL PX ( SPX.SNO = SX.SNO AND
    SPX.PNO = PX.PNO ) )
```

10.9 (Repeated from the body of the chapter.) Give SQL analogs of the relational calculus expressions in the subsection "More Sample Queries" in the body of the chapter.
10.10 Prove that AND and OR are associative.
10.11 Let $p(x)$ and $q$ be predicates in which $x$ does and does not appear, respectively, as a free variable. Which of the following statements are valid $?^{18}$ I remind you that the symbol " $\Rightarrow$ " means implies; the symbol " $\equiv$ " means is equivalent to. Note too that $A \Rightarrow B$ and $B \Rightarrow A$ are together the same as $A \equiv B$ (in other words, $(A \Rightarrow B$ AND $B \Rightarrow A)$ $\equiv(A \equiv B)$ is a tautology $)$.
a. EXISTS $x(q) \equiv q$
b. FORALL $x(q) \equiv q$
c. EXISTS $x(p(x)$ AND $q) \equiv \operatorname{EXISTS} x(p(x))$ AND $q$
d. FORALL $x(p(x)$ AND $q) \equiv$ FORALL $x(p(x))$ AND $q$

[^121]```
e. FORALL x ( p(x) ) = EXISTS x ( p(x) )
f. EXISTS x ( TRUE ) \equiv TRUE
g. FORALL x ( FALSE ) \equiv FALSE
h. UNIQUE x ( p(x) ) = EXISTS x ( p(x) )
i. UNIQUE }x(p(x))=>\mathrm{ FORALL }x(p(x) 
j. FORALL x ( p(x) ) AND EXISTS x ( p(x) ) => UNIQUE x ( p(x) )
k. FORALL x ( p(x) ) AND UNIQUE x ( p(x) ) = EXISTS x ( p(x) )
```

10.12 Let $p(x, y)$ be a predicate with free variables $x$ and $y$. Which of the following statements are valid?

```
a. EXISTS x EXISTS y ( p(x,y) ) \equiv EXISTS y EXISTS x ( p(x,y) )
```

b. FORALL $x$ FORALL $y(p(x, y)) \equiv$ FORALL $y$ FORALL $x(p(x, y))$
c. FORALL $x(p(x, y)) \equiv$ NOT EXISTS $x(\operatorname{NOT} p(x, y))$
d. EXISTS $x(p(x, y)) \equiv$ NOT FORALL $x(\operatorname{NOT} p(x, y))$
e. EXISTS $x$ FORALL $y(p(x, y) \equiv$ FORALL $y \operatorname{EXISTS} x(p(x, y))$
f. EXISTS $y$ FORALL $x(p(x, y)) \Rightarrow$ FORALL $x \operatorname{EXISTS} y(p(x, y))$
10.13 Where possible and reasonable, give relational calculus solutions to exercises from Chapters 6-9.
10.14 Consider this query: "Get cities in which either a supplier or a part is located." Can this query be expressed in the relational calculus? If not, why not?
10.15 Is SQL relationally complete? Note: To prove it is, you need to show that for every expression of the relational algebra, there exists a semantically equivalent expression in SQL. Alternatively, to prove it isn't, you need to show there exists at least one expression of the relational algebra for which no such SQL equivalent exists.
10.16 Here's a lightly edited excerpt from Chapter 8: "If the database contains only true propositions, then it's consistent, but the converse isn't necessarily so; if the database is inconsistent, then it contains at least one false proposition, but the converse isn't necessarily so." Are these two statements logically equivalent? In other words, is there some duplication here?
10.17 Is prenex normal form always achievable?

## Chapter 11

## Using Logic

## to Formulate SQLExpressions

There is science, logic, reason; there is thought verified by experience.
And then there is California. ${ }^{1}$
-Edward Abbey: A Voice Crying in the Wilderness (1989)

In Chapter 6, I described the process of expression transformation as it applied to expressions of the relational algebra; to be specific, I showed how one such expression could be transformed into another logically equivalent one, using various transformation laws. The laws I considered included such things as:
a. Restriction distributes over union, intersection, and difference
b. Projection distributes over union but not over intersection or difference
and several others. (As you might expect, analogous laws apply to expressions of the relational calculus also, though I didn't say much about such laws in Chapter 10.)

Now, the purpose of such transformations, as I discussed them earlier, was essentially optimization; the aim was to come up with an expression with the same semantics as the original one but better performance characteristics. However, the concept of expression transformation-or query rewrite, as it's sometimes (not very appropriately) known-has application in other areas, too. In particular, and very importantly, it can be used to transform precise logical expressions representing queries and the like into SQL equivalents. And that's what this chapter is all about: It shows how to take the logical or relational calculus formulation of some query or constraint (for example) and map it systematically into an SQL equivalent. And while the SQL formulation so obtained can sometimes be hard to understand, we know it's correct, because of the systematic manner in which it's been obtained. Hence the subtitle of this book: How to Write Accurate SQL Code.

## SOME TRANSFORMATION LAWS

Laws of transformation like the ones mentioned above are also known variously as:
■ Equivalences, because they take the general form $\exp 1 \equiv \exp 2$ (recall from Chapter 10 and elsewhere that the symbol " $\equiv$ " means "is equivalent to").

[^122]- Identities, because a law of the form $\exp 1 \equiv \exp 2$ can be read as saying that $\exp 1$ and $\exp 2$ are "identically equal," meaning they have identical semantics
- Rewrite rules, because a law of the form exp $1 \equiv \exp 2$ implies that an expression containing an occurrence of $\exp 1$ can be rewritten as one containing an occurrence of exp 2 instead without changing the meaning

I'd like to expand on this last point, because it's crucial to what we're going to be doing in the present chapter. Let $X 1$ be an expression containing an occurrence of $x 1$ as a subexpression; let $x 2$ be equivalent to $x l$; and let $X 2$ be the expression obtained from $X 1$ by substituting an occurrence of $x 2$ for the occurrence of $x 1$ in question. Then $X 1$ and $X 2$ are logically and semantically equivalent; hence, $X 1$ can be rewritten as $X 2$. By way of a simple example, consider the following SQL expression:

```
SELECT SNO
FROM S
WHERE ( STATUS > 10 AND CITY = 'London' )
OR ( STATUS > 10 AND CITY = 'Athens' )
```

The boolean expression in the WHERE clause here is clearly equivalent (thanks to the distributivity of AND over $\mathrm{OR}-$ see later) to the following:

```
STATUS > 10 AND ( CITY = 'London' OR CITY = 'Athens' )
```

Hence the overall expression can be rewritten as:

```
SELECT SNO
FROM S
WHERE STATUS > 10
AND ( CITY = 'London' OR CITY = 'Athens' )
```

Here then are some of the transformation laws we'll be using in this chapter:

- The implication law:

```
IF p THEN q \equiv(NOT p ) OR q
```

I did state this law in Chapter 10, but I didn't have much to say about its use there. Take a moment (if you need to) to check the truth tables and convince yourself the law is valid. Note: The symbols $p$ and $q$ stand for arbitrary boolean expressions or predicates. In this chapter, I'll favor the term boolean expression over predicate, since the emphasis throughout is on such expressions-i.e., on pieces of program text, in effect-rather than on logic per se. Logic in general, and predicates in particular, are more abstract than pieces of program text (or an argument can be made to that effect, at least). You can think of a boolean expression as a concrete representation of some predicate, if you like.

- The double negation law (also known as the involution law):

NOT (NOT $p$ ) $\equiv p$
This law is obvious (but it's important).

De Morgan's laws:

```
NOT ( p AND q ) \equiv(NOT p ) OR (NOT q )
NOT ( p OR q ) \equiv(NOT p ) AND ( NOT q )
```

I didn't discuss these laws in the previous chapter, but they make good intuitive sense. For example, the first one says, loosely, that if it's not the case that $p$ and $q$ are both true, then it must be the case that either $p$ isn't true or $q$ isn't true (or both). Be that as it may, the validity of both laws follows immediately from the truth tables. Here, for example, is the truth table corresponding to the first law:

| $p$ | $q$ | $p$ AND $q$ | NOT ( $p$ AND $q$ ) | (NOT $p$ ) | OR (NOT $q$ ) |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $T$ | $T$ | $T$ | $F$ | F |  |
| T | F | F | T |  |  |
| F | T | F | T | T |  |
| F | F | F | T | T |  |

Since the columns for NOT ( $p$ AND $q$ ) and (NOT $p$ ) OR (NOT $q$ ) are identical, the validity of the first law follows. Proof of the validity of the second is analogous (exercise for the reader).

The distributive laws:

```
pAND ( q OR r ) \equiv ( p AND q ) OR ( p AND r )
pOR (qAND r ) \equiv( p OR q ) AND ( p OR r )
```

I'll leave the proof of these two to you. Note, however, that (as in fact I did mention in passing) I was using the first of these laws in the SQL example near the beginning of this section. You might also note that these distributive laws are a little more general, in a sense, than the ones we saw in Chapter 6. In that chapter we saw examples of a monadic operator, such as restriction, distributing over a dyadic operator, such as union; here, by contrast, we see dyadic operators (AND and OR) each distributing over the other.

## - The quantification law:

```
FORALL x ( p ( x ) ) \equiv NOT EXISTS x (NOT p ( x ) )
```

I discussed this one in the previous chapter. In fact, I hope you can see it's really just an application of De Morgan's laws to EXISTS and FORALL expressions specifically (recall from the previous chapter that EXISTS and FORALL are basically just iterated OR and iterated AND, respectively).

One further remark on these laws: Because De Morgan's laws in particular will often be applied to the result of a prior application of the implication law, it's convenient to restate the first of them, at least, in the following form (in which $q$ is replaced by NOT $q$ and the double negation law has been tacitly applied):

```
NOT ( p AND NOT q ) \equiv(NOT p ) OR q
```

Or rather (but it's the same thing, logically):

```
( NOT p ) OR q \equiv NOT ( p AND NOT q )
```

Equivalently:

```
IF p THEN q \equiv NOT ( p AND NOT q )
```

Most of the references to one of De Morgan's laws in what follows will be to this restated formulation.
The remainder of this chapter offers practical guidelines on the use of these laws to help in the formulation of "complex" SQL expressions. I'll start with some very simple examples and build up gradually to ones that are quite complicated.

## EXAMPLE 1: LOGICAL IMPLICATION

Consider again the constraint from the previous chapter to the effect that all red parts must be stored in London. For a given part, this constraint corresponds to a business rule that might be stated more or less formally like this:

```
IF COLOR = 'Red' THEN CITY = 'London'
```

In other words, it's a logical implication. Now, SQL doesn't support logical implication as such, but the implication law tells us that the foregoing expression can be transformed into this one:

```
( NOT ( COLOR = 'Red' ) ) OR ( CITY = 'London' )
```

(I've added some parentheses for clarity.) And this expression involves only operators that SQL does support, so it can be formulated directly as a base table constraint:

```
CONSTRAINT BTCX1 CHECK ( NOT ( COLOR = 'Red' ) OR ( CITY = 'London' ) )
```

Or perhaps a little more naturally, making use of the fact that NOT $(a=b)$ can be transformed into $a \neq b-$ in SQL, $a<>b$-and dropping unnecessary parentheses (in other words, applying some further simple transformations):

```
CONSTRAINT BTCX1 CHECK ( COLOR <> 'Red' OR CITY = 'London' )
```

Note: I've said that SQL doesn't support logical implication (IF ... THEN ...) as such. That's true. But it does support CASE expressions, and so this first example might alternatively be formulated in SQL as follows:

```
CONSTRAINT BTCX1 CHECK ( CASE
                        WHEN COLOR = 'Red' THEN CITY = 'London'
                                ELSE TRUE
    END ) ;
```

In general, the logical implication IF $p$ THEN $q$ can be mapped into the SQL CASE expression CASE WHEN $p^{\prime}$ THEN $q^{\prime}$ ELSE TRUE END, where $p^{\prime}$ and $q^{\prime}$ are SQL analogs of $p$ and $q$, respectively. ${ }^{2}$ For simplicity, however, I'll ignore this possibility in future examples.

## EXAMPLE 2: UNIVERSAL QUANTIFICATION

Now, I was practicing a tiny deception in Example 1, inasmuch as I was pretending that the specific part to which the constraint applied was understood. But that's effectively just what happens with base table constraints in SQL; they're tacitly understood to apply to each and every row of the base table whose definition they're part of. However, suppose we wanted to be more explicit-i.e., suppose we wanted to state explicitly that the constraint applies to every part that happens to be represented in table P. In other words, for all such parts PX, if the color of part PX is red, then the city for part PX is London:

```
FORALL PX ( IF PX.COLOR = 'Red' THEN PX.CITY = 'London' )
```

Note: The name PX and others like it in this chapter are deliberately chosen to be reminiscent of the range variables used in examples in the previous chapter. In fact, I'm going to assume from this point forward that names of the form PX, PY, etc., denote variables that range over the current value of table P; names of the form SX, SY, etc., denote variables that range over the current value of table $S$; and so on. ${ }^{3}$ Details of how such variables are defined-in logic, I mean, not in SQL-aren't important for present purposes and are therefore omitted. In SQL, they're defined by means of AS clauses, which I'll show when we get to the SQL formulations as such.

Now, SQL doesn't support FORALL, but the quantification law tells us that the foregoing expression can be transformed into this one:

```
NOT EXISTS PX ( NOT ( IF PX.COLOR = 'Red' THEN PX.CITY = 'London' ) )
```

(Again I've added some parentheses for clarity. From this point forward, in fact, I'll feel free to introduce or drop parentheses as and when I feel it's desirable to do so, without further comment.) Now applying the implication law:

```
NOT EXISTS PX ( NOT ( NOT ( PX.COLOR = `Red' ) OR PX.CITY = `London' ) )
```

This expression could now be mapped directly into SQL, but it's probably worth tidying it up a little first. Applying De Morgan:

```
NOT EXISTS PX ( NOT ( NOT ( ( PX.COLOR = 'Red' )
    AND NOT ( PX.CITY = 'London' ) ) ) )
```

Applying the double negation law and dropping some parentheses:

```
NOT EXISTS PX ( PX.COLOR = 'Red' AND NOT ( PX.CITY = 'London' ) )
```

[^123]Finally:

```
NOT EXISTS PX ( PX.COLOR = 'Red' AND PX.CITY # `London' )
```

Now, the transformations so far have all been very simple; you might even have found them rather tedious. But mapping this final logical expression into SQL isn't quite so straightforward. Here are the details of that mapping:

- First of all, NOT maps to NOT (unsurprisingly).

■ Second, "EXISTS PX $(b x)$ " maps to "EXISTS (SELECT * FROM P AS PX WHERE ( $b x$ ) )," where $b x^{\prime}$ is the SQL analog of the boolean expression $b x$. Note: Of course, mapping $b x$ to $b x^{\prime}$ might require further (recursive) application of these rules.

- Third, the parentheses surrounding $s b x$ can be dropped, though they don't have to be.
- Last, the entire expression needs to be wrapped up inside some suitable CREATE ASSERTION syntax.

So here's the final version:

```
CREATE ASSERTION . .. CHECK
    ( NOT EXISTS ( SELECT *
                FROM P AS PX
                WHERE PX.COLOR = 'Red'
                        AND PX.CITY <> 'London' ) ) ;
```


## EXAMPLE 3: IMPLICATION AND UNIVERSAL QUANTIFICATION

A query example this time-"Get part names for parts whose weight is different from that of every part in Paris." Here's a straightforward logical (i.e., relational calculus) formulation:

```
{ PX.PNAME } WHERE FORALL PY ( IF PY.CITY = 'Paris'
    THEN PY.WEIGHT # PX.WEIGHT )
```

This expression can be interpreted as follows: "Get PNAME values from parts PX such that, for all parts PY, if PY is in Paris, then PY and PX have different weights." Note that I use the terms where and such that interchangeably-whichever seems to read best in the case at hand-when I'm giving natural language interpretations like the one under discussion.

As a first transformation, let's apply the quantification law:

```
{ PX.PNAME } WHERE NOT EXISTS PY ( NOT ( IF PY.CITY = 'Paris'
    THEN PY.WEIGHT f PX.WEIGHT ) )
```

Next, apply the implication law:

```
{ PX.PNAME } WHERE
    NOT EXISTS PY ( NOT ( NOT ( PY.CITY = 'Paris' )
    OR ( PY.WEIGHT f PX.WEIGHT ) ) )
```

Apply De Morgan:

```
{ PX.PNAME } WHERE
    NOT EXISTS PY ( NOT ( NOT ( ( PY.CITY = 'Paris' )
    AND NOT ( PY.WEIGHT f PX.WEIGHT ) ) ) )
```

Tidy up, using the double negation law, plus the fact that NOT $(a \neq b)$ is equivalent to $a=b$ :

```
{ PX.PNAME } WHERE NOT EXISTS PY ( PY.CITY = 'Paris' AND
    PY.WEIGHT = PX.WEIGHT )
```

Map to SQL:

```
SELECT DISTINCT PX.PNAME
FROM P AS PX
WHERE NOT EXISTS
    ( SELECT *
        FROM P AS PY
        WHERE PY.CITY = 'Paris'
        AND PY.WEIGHT = PX.WEIGHT )
```

Incidentally, that DISTINCT is really needed in the opening SELECT clause here! Here's the result: ${ }^{4}$

| PNAME |
| :--- |
| Screw <br> Cog |

Unfortunately, there's a fly in the ointment in this example. Suppose there's at least one part in Paris, but all such parts have a null weight. Then we simply don't know-we can't possibly say-whether there are any parts whose weight is different from that of every part in Paris; the query is unanswerable. But SQL gives us an answer anyway ... To be specific, the subquery following the keyword EXISTS evaluates to an empty table for every part PX represented in P; the NOT EXISTS therefore evaluates to TRUE for every such part PX; and the expression overall therefore incorrectly returns all part names in table P.

Aside: As explained in Chapter 4, this is the biggest practical problem with nulls-they lead to wrong answers. What's more, of course, we don't know in general which answers are right and which wrong! For further elaboration of such matters, refer to the paper "Why Three- and Four-Valued Logic Don't Work" (see Appendix G). End of aside.

What's more, not only is the foregoing SQL result incorrect, but any definite result would represent, in effect, a lie on the part of the system. To say it again, the only logically correct result is "I don't know"-or, to be more precise and a little more honest about the matter, "The system doesn't have enough information to give a definitive response to this query."

What makes matters even worse is that under the same conditions as before (i.e., if there's at least one part in Paris and those parts all have a null weight), the SQL expression

[^124]```
SELECT DISTINCT PX.PNAME
FROM P AS PX
WHERE PX.WEIGHT NOT IN
    ( SELECT PY.WEIGHT
        FROM P AS PY
        WHERE PY.CITY = 'Paris' )
```

-which looks as if it ought to be logically equivalent to the one shown previously (and indeed is so, in the absence of nulls)-will return an empty result: a different, though equally incorrect, result.

The moral is obvious: Avoid nulls!-and then the transformations all work properly.

## EXAMPLE 4: CORRELATED SUBQUERIES

Consider the query "Get names of suppliers who supply both part P1 and part P2." Here's a logical formulation:

```
{ SX.SNAME } WHERE EXISTS SPX ( SPX.SNO = SX.SNO AND SPX.PNO = 'P1' )
    AND EXISTS SPX ( SPX.SNO = SX.SNO AND SPX.PNO = 'P2' )
```

An equivalent SQL formulation is straightforward:

```
SELECT DISTINCT SX.SNAME
FROM S AS SX
WHERE EXISTS ( SELECT *
    FROM SP AS SPX
    WHERE SPX.SNO = SX.SNO
    AND SPX.PNO = 'P1' )
AND EXISTS ( SELECT *
    FROM SP AS SPX
    WHERE SPX.SNO = SX.SNO
    AND SPX.PNO = 'P2' )
```

Here's the result:

```
SNAME
    Smith
    Jones
```

As you can see, however, this SQL expression involves two correlated subqueries. (In fact, Example 3 involved a correlated subquery also. See Chapter 12 for further discussion.) But correlated subqueries are often contraindicated from a performance point of view, because-conceptually, at any rate-they have to be evaluated repeatedly, once for each row in the outer table, instead of just once and for all. The possibility of eliminating them thus seems worth investigating. Now, in the case at hand (where the correlated subqueries appear within EXISTS invocations), there's a simple transformation that can be used to achieve precisely that effect. The resulting expression is:

```
SELECT DISTINCT SX.SNAME
FROM S AS SX
```

```
WHERE SX.SNO IN ( SELECT SPX.SNO
    FROM SP AS SPX
    WHERE SPX.PNO = 'P1' )
AND SX.SNO IN ( SELECT SPX.SNO
    FROM SP AS SPX
    WHERE SPX.PNO = 'P2' )
```

More generally, the SQL expression

```
SELECT sic /* "SELECT item commalist" */
FROM
    T1
WHERE [ NOT ] EXISTS ( SELECT *
    FROM T2
    WHERE T2.C = T1.C
    AND bx )
```

can be transformed into

```
SELECT sic
FROM T1
WHERE T1.C [ NOT ] IN ( SELECT T2.C
    FROM T2
    WHERE bx )
```

In practice, this transformation is probably worth applying whenever it can be. (Of course, it would be better if the optimizer could perform the transformation automatically; unfortunately, however, we can't always count on the optimizer to do what's best.) But there are many situations where the transformation simply doesn't apply. As Example 3 showed, nulls can be one reason it doesn't apply-by the way, are nulls a consideration in Example 4?-but there are cases where it doesn't apply even if nulls are avoided. As an exercise, you might like to try deciding which of the remaining examples in this chapter it does apply to.

## EXAMPLE 5: NAMING SUBEXPRESSIONS

Another query: "Get full supplier details for suppliers who supply all purple parts." Note: This query, or one very like it, is often used to demonstrate a flaw in the relational divide operator as originally defined. See the further remarks on this topic at the end of the present section.

Here first is a logical formulation:

```
{ SX } WHERE FORALL PX ( IF PX.COLOR = 'Purple' THEN
EXISTS SPX ( SPX.SNO = SX.SNO AND SPX.PNO = PX.PNO ) )
```

("names of suppliers SX such that, for all parts PX, if PX is purple, there exists a shipment SPX with SNO equal to the supplier number for supplier SX and PNO equal to the part number for part PX"). First we apply the implication law:

```
{ SX } WHERE FORALL PX ( NOT ( PX.COLOR = 'Purple' ) OR
    EXISTS SPX ( SPX.SNO = SX.SNO AND SPX.PNO = PX.PNO ) )
```

Next De Morgan:

```
{ SX } WHERE
    FORALL PX ( NOT ( ( PX.COLOR = 'Purple' ) AND
    NOT EXISTS SPX (SPX.SNO = SX.SNO AND SPX.PNO = PX.PNO ) ) )
```

Apply the quantification law:

```
{ SX } WHERE
    NOT EXISTS PX ( NOT ( NOT ( ( PX.COLOR = 'Purple' ) AND
    NOT EXISTS SPX ( SPX.SNO = SX.SNO AND SPX.PNO = PX.PNO ) ) ) )
```

Double negation:

```
{ SX } WHERE
    NOT EXISTS PX ( ( PX.COLOR = 'Purple' ) AND
    NOT EXISTS SPX ( SPX.SNO = SX.SNO AND SPX.PNO = PX.PNO ) )
```

Drop some parentheses and map to SQL:

```
SELECT *
FROM S AS SX
WHERE NOT EXISTS
    ( SELECT *
        FROM P AS PX
        WHERE PX.COLOR = 'Purple'
        AND NOT EXISTS
            ( SELECT *
                FROM SP AS SPX
                WHERE SPX.SNO = SX.SNO
                AND SPX.PNO = PX.PNO ) )
```

Recall now from Chapter 7 that if there aren't any purple parts, every supplier supplies all of them-even supplier S5, who supplies no parts at all (see the discussion of empty ranges in Chapter 10 for further explanation). So the result is the entire suppliers relation:

| SNO | SNAME | STATUS | CITY |
| :---: | :---: | ---: | :--- |
| S1 | Smith | 20 | London |
| S2 | Jones | 10 | Paris |
| S3 | Blake | 30 | Paris |
| S4 | Clark | 20 | London |
| S5 | Adams | 30 | Athens |

Now, you might have had some difficulty in following the transformations in the foregoing example, and you might also be having some difficulty in understanding the final SQL formulation. Well, a useful technique, when the expressions start getting a little complicated as in this example, is to abstract a little by introducing symbolic names for subexpressions (I did briefly mention this point in the previous chapter, but now I want to get more specific). Let's use expl to denote the subexpression

```
PX.COLOR = 'Purple'
```

and $\exp 2$ to denote the subexpression

```
EXISTS SPX ( SPX.SNO = SX.SNO AND SPX.PNO = PX.PNO )
```

(note that both of these subexpressions can be directly represented, more or less, in SQL). Then the original relational calculus expression becomes:

```
{ SX } WHERE FORALL PX ( IF exp1 THEN exp2 )
```

As I said in the previous chapter, now we can see the forest as well as the trees (as it were), and we can start to apply our usual transformations-though now it seems to make more sense to apply them in a different sequence, precisely because we do now have a better grasp of the big picture. First, then, the quantification law:

```
{ SX } WHERE NOT EXISTS PX ( NOT ( IF exp1 THEN exp2 ) )
```

Implication law:

```
{ SX } WHERE NOT EXISTS PX ( NOT ( NOT ( exp1 ) OR exp2 ) )
```

De Morgan:

```
{ SX } WHERE NOT EXISTS PX ( NOT ( NOT ( exp1 AND NOT ( exp2 ) ) ) )
```

Double negation:
\{ SX \} WHERE NOT EXISTS PX ( exp1 AND NOT ( exp2 ) )

Finally, expand $\exp 1$ and $\exp 2$ and map to SQL:

```
SELECT *
FROM S AS SX
WHERE NOT EXISTS
    ( SELECT *
        FROM P AS PX
        WHERE PX.COLOR = 'Purple'
        AND NOT EXISTS
            ( SELECT *
                FROM SP AS SPX
                WHERE SPX.SNO = SX.SNO
                AND SPX.PNO = PX.PNO ) )
```

As I think this example demonstrates, SQL expressions obtained by the techniques under discussion are often quite hard to understand directly; as I said earlier, however, we know they're correct, because of the systematic manner in which they've been derived. ${ }^{5}$

As an aside, I can't resist showing a Tutorial D version of the example by way of comparison:

[^125]```
S WHERE ( !!SP ) { PNO } ` ( P WHERE COLOR = 'Purple' ) { PNO }
```

Now let me explain the remark I made at the beginning of this section, regarding divide. Let's denote the restriction P WHERE COLOR = 'Purple' by the symbol PP. Also, let's simplify the query at hand-"Get full supplier details for suppliers who supply all purple parts"-such that it asks for supplier numbers only, instead of full supplier details. Then it might be thought that the query could be represented by the following algebraic expression:

```
SP { SNO , PNO } DIVIDEBY PP { PNO }
```

Note: DIVIDEBY here represents the divide operator as originally defined. See Chapter 7 if you need an explanation of this point.

With our usual sample data values, however, relation PP, and hence the projection of relation PP on $\{\mathrm{PNO}\}$, are both empty (because there aren't any purple parts), and the foregoing expression therefore returns the supplier numbers S1, S2, S3, and S4. But if there aren't any purple parts, then every supplier supplies all of them (see the discussion of empty ranges in the previous chapter)-even supplier S5, who supplies no parts at all. And the foregoing division can't possibly return supplier number S5, because it extracts supplier numbers from SP instead of S, and supplier S5 isn't currently represented in SP. So the informal characterization of that division as "Get supplier numbers for suppliers who supply all purple parts" is incorrect; it should be, rather, "Get supplier numbers for suppliers who supply at least one part and also supply all purple parts." As this example demonstrates, therefore (and to repeat something I said in Chapter 7), the divide operator doesn't really solve the problem it was originally, and explicitly, intended to solve.

## EXAMPLE 6: MORE ON NAMING SUBEXPRESSIONS

I'll give another example to illustrate the usefulness of introducing symbolic names for subexpressions. The query is "Get suppliers such that every part they supply is in the same city as that supplier." Here's a logical formulation:

```
{ SX } WHERE FORALL PX
    ( IF EXISTS SPX ( SPX.SNO = SX.SNO AND SPX.PNO = PX.PNO )
        THEN PX.CITY = SX.CITY )
```

("suppliers SX such that, for all parts PX, if there's a shipment of PX by SX, then PX.CITY = SX.CITY").
This time I'll just show the transformations without naming the transformation laws involved at each step (I'll leave that as an exercise for you):

```
{ SX } WHERE FORALL PX ( IF exp1 THEN exp2 )
{ SX } WHERE NOT EXISTS PX ( NOT ( IF exp1 THEN exp2 ) )
{ SX } WHERE NOT EXISTS PX ( NOT ( NOT ( exp1 ) OR exp2 ) )
{ SX } WHERE NOT EXISTS PX ( NOT ( NOT ( exp1 AND NOT ( exp2 ) ) ) )
{ SX } WHERE NOT EXISTS PX ( exp1 AND NOT ( exp2 ) )
```

Now expand expl and exp2 and map to SQL:

```
SELECT *
FROM S AS SX
WHERE NOT EXISTS
    ( SELECT *
        FROM P AS PX
        WHERE PX.CITY <> SX.CITY
        AND EXISTS
            ( SELECT *
                FROM SP AS SPX
                WHERE SPX.SNO = SX.SNO
                AND SPX.PNO = PX.PNO ) )
```

Result:

| SNO | SNAME | STATUS | CITY |
| :---: | :--- | ---: | :--- |
| S3 | Blake | 30 | Paris |
| S5 | Adams | 30 | Athens |

By the way, if you find this result a little surprising, note that supplier S3 supplies just one part, part P2, and supplier S5 supplies no parts at all; logically speaking, therefore, both of these suppliers do indeed satisfy the condition that "every part they supply" is in the same city.

Here for interest is a Tutorial $\mathbf{D}$ version of the same example:

```
S WHERE RELATION { TUPLE { CITY CITY } } = ( ( !!SP ) JOIN P ) { CITY }
```


## EXAMPLE 7: DEALING WITH AMBIGUITY

As we saw in Chapter 10, natural language is often ambiguous. For example, consider the following query: "Get suppliers such that every part they supply is in the same city." First of all, notice the subtle (?) difference between this example and the previous one. Second, and more important, note that this natural language formulation is indeed ambiguous! For the sake of definiteness, I'm going to assume it means the following:

Get suppliers SX such that for all parts PX and PY, if SX supplies both of them, then PX.CITY= PY.CITY.
Observe that a supplier who supplies just one part will qualify under this interpretation. (So will a supplier who supplies no parts at all, incidentally.) Alternatively, the query might mean:

Get suppliers SX such that (a) SX supplies at least two distinct parts and (b) for all pairs of distinct parts PX and PY, if SX supplies both of them, then PX.CITY = PY.CITY.

Now a supplier who supplies just one part or no parts at all won't qualify.
As I've said, I'm going to assume the first interpretation, just to be definite. But note that ambiguities of this kind are quite common with complex queries and complex business rules, and another advantage of logic in the context at hand is precisely that it can pinpoint and help resolve such ambiguities.

Here then is a logical formulation for the first interpretation:

```
{ SX } WHERE FORALL PX ( FORALL PY
    ( IF EXISTS SPX ( SPX.SNO = SX.SNO AND SPX.PNO = PX.PNO )
        AND EXISTS SPY ( SPY.SNO = SX.SNO AND SPY.PNO = PY.PNO )
        THEN PX.CITY = PY.CITY ) )
```

And here are the transformations (again I'll leave it to you to decide just which law is being applied at each stage):

```
{ SX } WHERE FORALL PX ( FORALL PY
    ( IF exp1 AND exp2 THEN exp3 ) )
{ SX } WHERE NOT EXISTS PX ( NOT FORALL PY
    ( IF exp1 AND exp2 THEN exp3 ) )
{ SX } WHERE NOT EXISTS PX ( NOT ( NOT EXISTS PY ( NOT
    ( IF exp1 AND exp2 THEN exp3 ) ) ) )
{ SX } WHERE NOT EXISTS PX ( EXISTS PY ( NOT
    ( IF exp1 AND exp2 THEN exp3 ) ) )
{ SX } WHERE NOT EXISTS PX ( EXISTS PY ( NOT
    ( NOT ( exp1 AND exp2 ) OR exp3 ) ) )
{ SX } WHERE NOT EXISTS PX ( EXISTS PY ( NOT
    ( NOT ( exp1 ) OR NOT ( exp2 ) OR exp3 ) ) )
{ SX } WHERE NOT EXISTS PX ( EXISTS PY (
    ( exp1 AND exp2 AND NOT ( exp3 ) ) ) )
```

SQL equivalent:

```
SELECT *
FROM S AS SX
WHERE NOT EXISTS
    ( SELECT *
        FROM P AS PX
        WHERE EXISTS
            ( SELECT *
                FROM P AS PY
                WHERE EXISTS
                            ( SELECT *
                        FROM SP AS SPX
                        WHERE SPX.SNO = SX.SNO
                        AND SPX.PNO = PX.PNO )
                AND EXISTS
            ( SELECT *
                        FROM SP AS SPY
                        WHERE SPY.SNO = SX.SNO
                            AND SPY.PNO = PY.PNO )
                AND PX.CITY <> PY.CITY ) )
```

By the way, I used two distinct range variables SPX and SPY, both ranging over SP, in this example purely for reasons of clarity; I could perfectly well have used the same one (say SPX) twice over-it would have made no logical difference at all. Anyway, here's the result:

| SNO | SNAME | STATUS | CITY |
| :---: | :---: | ---: | :--- |
| S3 | Blake | 30 | Paris |
| S5 | Adams | 30 | Athens |

At this point, I'd like to remind you of another transformation law that's sometimes useful: the contrapositive law (I mentioned this one in the previous chapter). Consider the implication IF NOT $q$ THEN NOT $p$. By definition, this expression is equivalent to NOT (NOT $q$ ) OR NOT $p$-which is the same as $q$ OR NOT $p$-which is the same as NOT $p$ OR $q$-which is the same as IF $p$ THEN $q$. So we have:

```
IF p THEN q \equiv IF NOT q THEN NOT p
```

Note that this law does make intuitive sense: If the truth of $p$ implies the truth of $q$, then the falsity of $q$ must imply the falsity of $p$. For example, if "It's raining" implies "The streets are wet," then "The streets aren't wet" must imply "It isn't raining."

In the example at hand, then, another possible way of stating the interpretation previously assumed ("Get suppliers SX such that for all parts PX and PY, if SX supplies both of them, then PX.CITY = PY.CITY") is:

Get suppliers SX such that for all parts PX and PY, if PX.CITY $\neq$ PY.CITY, then SX doesn't supply both of them. ${ }^{6}$

This perception of the query will very likely lead to a different (though logically equivalent) SQL formulation. I'll leave the details as an exercise.

## EXAMPLE 8: USING COUNT

Now, there's still a little more to be said about the previous example. Let me state the query again: "Get suppliers such that every part they supply is in the same city." Here's yet another possible natural language interpretation of this query:

Get suppliers SX such that the number of cities for parts supplied by SX is less than or equal to one.
Note that "less than or equal to," by the way-"equal to" alone would correspond to a different interpretation of the query (right?). Logical formulation:

```
{ SX } WHERE COUNT ( PX.CITY WHERE EXISTS SPX
    ( SPX.SNO = SX.SNO AND SPX.PNO = PX.PNO ) ) \leq 1
```

This is the first example in this chapter to make use of an aggregate operator. As I think you can see, however, the mapping is quite straightforward. An equivalent SQL formulation is:

[^126]```
SELECT *
FROM S AS SX
WHERE ( SELECT COUNT ( DISTINCT PX.CITY )
        FROM P AS PX
        WHERE EXISTS ( SELECT *
                                FROM SP AS SPX
                                WHERE SPX.SNO = SX.SNO
    AND SPX.PNO = PX.PNO ) ) <= 1
```

The result is as shown under Example 7. However, I remind you from the previous chapter that as a general rule it's wise, for performance reasons, to be careful over the use of COUNT; in particular, don't use it where EXISTS would be more logically correct.

Here are some questions for you: First, given the foregoing SQL formulation of the query, is that DISTINCT in the COUNT invocation really necessary? Second, try to formulate the query in terms of GROUP BY and HAVING. If you succeed, what were the logical steps you went through to construct that formulation? (See Example 12 for further discussion of GROUP BY and HAVING.)

## EXAMPLE 9: JOIN QUERIES

This time, for practice, I'll just present the query and the SQL formulation and leave you to give the logical formulation and the derivation process. The query is "Get suppliers such that every part they supply is in the same city (as in Examples 7 and 8), together with the city in question." Here's the SQL formulation:

```
SELECT DISTINCT SX.* , PX.CITY
FROM S AS SX , P AS PX
WHERE EXISTS
    ( SELECT *
        FROM SP AS SPX
        WHERE SPX.SNO = SX.SNO
        AND NOT EXISTS
            ( SELECT *
                FROM SP AS SPY
                WHERE SPY.SNO = SPX.SNO
            AND EXISTS
                ( SELECT *
                            FROM P AS PY
                                    WHERE PY.PNO = SPY.PNO
                                    AND PY.CITY <> PX.CITY ) ) )
```

Result:

| SNO | SNAME | STATUS | CITY |
| :---: | :--- | ---: | :--- |
| S3 | Blake | 30 | Paris |

Exercise: Is the DISTINCT necessary in this example? And why is this section called "Join Queries"?

## EXAMPLE 10: UNIQUE QUANTIFICATION

Recall this example from Chapter 10 (a logical formulation of the constraint that there's exactly one supplier for each shipment):

```
CONSTRAINT CX6 FORALL SPX ( UNIQUE SX ( SX.SNO = SPX.SNO ) ) ;
```

Recall too that the logic expression

```
EXISTS SX ( bx )
```

maps to the SQL expression

```
EXISTS ( SELECT * FROM S AS SX WHERE ( sbx ) )
```

where $s b x$ is the SQL analog of the boolean expression $b x$. However, the logic expression

```
UNIQUE SX ( bx )
```

does not map to the SQL expression

```
UNIQUE ( SELECT * FROM S AS SX WHERE ( sbx ) )
```

(There's an obvious trap for the unwary here.) Instead, it maps to:

```
UNIQUE ( SELECT k FROM S AS SX WHERE ( sbx ) )
AND
EXISTS ( SELECT * FROM S AS SX WHERE ( sbx ) )
```

where $k$ denotes an arbitrary constant value. ${ }^{7}$ (The UNIQUE invocation says there's at most one, the EXISTS invocation says there's at least one-where by "one" I mean one row in table S for which the boolean expression $s b x$ evaluates to TRUE.) So constraint CX6 might map to:

```
CREATE ASSERTION CX6 CHECK
    ( NOT EXISTS
            ( SELECT *
                FROM SP AS SPX
                WHERE NOT UNIQUE
                        ( SELECT SX.SNO
                        FROM S AS SX
                        WHERE SX.SNO = SPX.SNO )
        OR NOT EXISTS
            ( SELECT SX.SNO
                FROM S AS SX
                        WHERE SX.SNO = SPX.SNO ) ) ) ;
```

[^127]Note: As in one of the examples in Chapter 10, the UNIQUE invocation here-even though it might not look like it-is in fact of the form UNIQUE (SELECT constant FROM ...), thanks to the boolean expression in the inner WHERE clause. ${ }^{8}$

Incidentally, I think this example illustrates very well my claim that the SQL formulations produced by the techniques I'm describing in this chapter can be hard to understand. The foregoing constraint might be transcribed into stilted natural language like this:

There exists no shipment such that either there's not at most one corresponding supplier or there's not at least one corresponding supplier.

Well, I don't know about you, but I think it's far from immediately obvious that this extremely tortuous sentence is logically equivalent to the following one:

Every shipment has exactly one corresponding supplier.
By the way, there's another equivalence we might appeal to here-the logic expression UNIQUE SX $(b x)$ is clearly equivalent (as we saw in Chapter 10) to:

```
COUNT ( SX WHERE ( bx ) ) = 1
```

As a result we can simplify the foregoing SQL CREATE ASSERTION to:

```
CREATE ASSERTION CX6 CHECK
    ( NOT EXISTS
        ( SELECT *
            FROM SP AS SPX
            WHERE ( SELECT COUNT ( * )
                FROM S AS SX
                        WHERE SX.SNO = SPX.SNO ) <> 1 ) ) ;
```

Here for interest is yet another SQL formulation, one that uses neither UNIQUE nor COUNT. Try to convince yourself it's correct.

```
CREATE ASSERTION CX6 CHECK
    ( NOT EXISTS
        ( SELECT *
        FROM SP AS SPX
        WHERE NOT EXISTS
            ( SELECT *
                FROM S AS SX
                    WHERE SX.SNO = SPX.SNO
                    AND NOT EXISTS
                        ( SELECT *
                FROM S AS SY
                        WHERE SY.SNO = SX.SNO
```

[^128]```
AND ( SY.SNAME <> SX.SNAME OR
    SY.STATUS <> SX.STATUS OR
    SY.CITY <> SX.CITY ) ) ) ) ) ;
```

Note carefully, however, that this formulation relies on the fact that duplicate rows are prohibited (in table $S$ in particular); it doesn't work otherwise. Avoid duplicate rows!

## EXAMPLE 11: ALL OR ANY COMPARISONS

You probably know that SQL supports what are called generically $A L L$ or $A N Y$ comparisons (or, more formally, quantified comparisons, but I prefer to avoid this term because of possible confusion with SQL's EXISTS and UNIQUE operators). An ALL or ANY comparison is an expression of the form $r x \theta t s q$, where:

- $r x$ is a row expression.
- $t s q$ is a table subquery. (Subqueries of all kinds are discussed further in Chapter 12.)

■ $\theta$ is any of the usual scalar comparison operators supported in SQL ("=", " $<>$ ", " $<"$ ", " $<=", ~ ">", ~ ">=")$ followed by one of the keywords ALL, ANY, or SOME. (As mentioned in Chapter 7, in a footnote, SOME is just an alternative spelling for ANY in this context.)

The semantics are as follows:

- An ALL comparison returns TRUE if and only if the corresponding comparison without the ALL returns TRUE for all of the rows in the table represented by $t s q$. If that table is empty, the ALL comparison returns TRUE. ${ }^{9}$
- An ANY comparison returns TRUE if and only if the corresponding comparison without the ANY returns TRUE for at least one of the rows in the table represented by $t s q$. If that table is empty, the ANY comparison returns FALSE.

Here's an example ("Get part names for parts whose weight is greater than that of every blue part"):

```
SELECT DISTINCT PX.PNAME
FROM P AS PX
WHERE PX.WEIGHT >ALL ( SELECT PY.WEIGHT
                                    FROM P AS PY
                                    WHERE PY.COLOR = 'Blue' )
```

Result:

[^129]| PNAME |
| :--- |
| Bolt <br> Screw <br> Cog |

As this example suggests, the "row expression" $r x$ in the ALL or ANY comparison $r x \theta$ tsq is often-almost always, in fact-just a simple scalar expression, in which case the scalar value denoted by that expression is effectively coerced to a row that contains just that scalar value. (Incidentally, note that even if $r x$ doesn't consist of a simple scalar expression but actually does denote a row of degree greater than one, $\theta$ can still be something other than " $=$ " or " $<>$ ", though the practice isn't recommended. See Chapter 3 for further discussion of this point.)

Recommendation: Don't use ALL or ANY comparisons-they're error prone, and in any case their effect can always be achieved by other methods. As an illustration of the first point, consider the fact that a natural language formulation of the foregoing query might very well use any in place of every-"Get part names for parts whose weight is greater than that of any blue part"-which could lead to the incorrect use of >ANY in place of $>$ ALL. As another example, illustrating both points, consider the following SQL expression:

```
SELECT DISTINCT SNAME
FROM S
WHERE CITY <>ANY ( SELECT CITY FROM P )
```

This expression could easily be read as "Get names of suppliers whose city isn't equal to any part city"-but that's not what it means. Instead, it's logically equivalent ${ }^{10}$ to the following ("Get names of suppliers where there's at least one part in a different city"):

```
SELECT DISTINCT SNAME
FROM S
WHERE EXISTS ( SELECT *
    FROM P
    WHERE P.CITY <> S.CITY )
```

Result:

```
SNAME
    Smith
    Jones
    Jones
    Clark
    Adams
```

In fact, ALL or ANY comparisons can always be transformed into equivalent expressions involving EXISTS, as the foregoing example suggests. They can also usually be transformed into expressions involving MAX or MIN-because certainly (e.g.) a value is greater than all of the values in some set if and only if it's greater than the maximum value in that set-and expressions involving MAX and MIN are often easier to understand, intuitively speaking, than ALL or ANY comparisons. The table overleaf summarizes the possibilities in this regard. Note in

[^130]particular from the table that =ANY and <>ALL are equivalent to IN and NOT IN, respectively, and so these two are important exceptions to the overall recommendation to avoid ALL and ANY comparisons in general-i.e., you can use $=$ ANY and IN interchangeably, and you can use $<>$ ALL and NOT IN interchangeably too. (Personally, I think IN and NOT IN are much clearer than their alternatives, but it's your choice.) By contrast, =ALL and <>ANY have no analogous equivalents; however, expressions involving those operators can always be replaced by expressions involving EXISTS instead, as already noted.

|  | ANY | ALL |
| :---: | :---: | :---: |
| $=$ | IN |  |
| $<>$ |  | NOT IN |
| $<$ | $<$ MAX | $<$ MIN |
| $<=$ | $<=$ MAX | $<=$ MIN |
| $>$ | $>$ MIN | $>$ MAX |
| $>=$ | $>=$ MIN | $>=$ MAX |

Caveat: Unfortunately, the transformations involving MAX and MIN aren't guaranteed to work if the MAX or MIN argument happens to be an empty set. The reason is that SQL defines the MAX and MIN of an empty set to be null. For example, here again is the formulation shown earlier for the query "Get part names for parts whose weight is greater than that of every blue part":

```
SELECT DISTINCT PX.PNAME
FROM P AS PX
WHERE PX.WEIGHT >ALL ( SELECT PY.WEIGHT
                                    FROM P AS PY
                                    WHERE PY.COLOR = 'Blue' )
```

And here's a transformed "equivalent":

```
SELECT DISTINCT PX.PNAME
FROM P AS PX
WHERE PX.WEIGHT > ( SELECT MAX ( PY.WEIGHT )
    FROM P AS PY
    WHERE PY.COLOR = 'Blue' )
```

Now suppose there are no blue parts. Then the first of the foregoing expressions will return all part names in table P , but the second will return an empty result. ${ }^{11}$

Anyway, to make the transformation in the example valid after all, use COALESCE-e.g., as follows:

[^131]```
SELECT DISTINCT PX.PNAME
FROM P AS PX
WHERE PX.WEIGHT > ( SELECT COALESCE ( MAX ( PY.WEIGHT ) , 0.0 )
    FROM P AS PY
    WHERE PY.COLOR = 'Blue' )
```

By way of another example, consider the query "Get part names for parts whose weight is less than that of some part in Paris." Here's a logical formulation:

```
{ PX.PNAME } WHERE EXISTS PY ( PY.CITY = 'Paris' AND
    PX.WEIGHT < PY.WEIGHT )
```

Here's a corresponding SQL formulation:

```
SELECT DISTINCT PX.PNAME
FROM P AS PX
WHERE EXISTS ( SELECT *
    FROM P AS PY
    WHERE PY.CITY = 'Paris'
    AND PX.WEIGHT < PY.WEIGHT )
```

But this query too could have been expressed in terms of an ALL or ANY comparison, thus:

```
SELECT DISTINCT PX.PNAME
FROM P AS PX
WHERE PX.WEIGHT <ANY ( SELECT PY.WEIGHT
                                    FROM P AS PY
                                    WHERE PY.CITY = 'Paris' )
```

Result:

| PNAME |
| :--- |
| Nut <br> Screw <br> Cam |

As this example suggests (and indeed as already stated), expressions involving ALL and ANY comparisons can always be transformed into equivalent expressions involving EXISTS instead. Some questions for you:

- Are you sure "<ANY" is the correct comparison operator in this example? (Was "less than any" the phrase used in the natural language version? Should it have been? Recall too that "less than any" maps to "<ALL"-right?)
- Which of the various formulations do you think is the most "natural"?
- Are the various formulations equivalent if the database permits nulls? Or duplicates?


## EXAMPLE 12: GROUP BY AND HAVING

As promised earlier, there's a little more I want to say about the GROUP BY and HAVING clauses. Consider this query: "For each part supplied by no more than two suppliers, get the part number and city and the total quantity supplied of that part." Here's a possible logical (relational calculus) formulation:

```
{ PX.PNO , PX.CITY ,
    TPQ := SUM ( SPX.QTY WHERE SPX.PNO = PX.PNO , QTY ) }
    WHERE COUNT ( SPY WHERE SPY.PNO = PX.PNO ) \leq 2
```

SQL formulation:

```
SELECT PX.PNO , PX.CITY ,
    ( SELECT COALESCE ( SUM ( SPX.QTY ) , 0 )
                        FROM SP AS SPX
                                WHERE SPX.PNO = PX.PNO ) AS TPQ
FROM P AS PX
WHERE ( SELECT COUNT ( * )
    FROM SP AS SPY
    WHERE SPY.PNO = PX.PNO ) <= 2
```

Result:

| PNO | CITY | TPQ |
| :--- | :--- | :--- |
| P1 | London | 600 |
| P3 | Oslo | 400 |
| P4 | London | 500 |
| P5 | Paris | 500 |
| P6 | London | 100 |

As the opening to this section suggests, however, the interesting thing about this example is that it's one that might appear to be more easily-certainly more succinctly-expressed using GROUP BY and HAVING, thus:

```
SELECT PX.PNO , PX.CITY , COALESCE ( SUM ( SPX.QTY ) , 0 ) AS TPQ
FROM P AS PX , SP AS SPX
WHERE PX.PNO = SPX.PNO
GROUP BY PX.PNO
HAVING COUNT ( * ) <= 2
```

But:

- In that GROUP BY / HAVING formulation, is the appearance of PX.CITY in the SELECT item commalist legal? Answer: Yes, it is-at least according to the standard-though it used not to be. (I did mention this point in Chapter 7, but I'll repeat it here for convenience.) Let $S$ be a SELECT expression with a GROUP BY clause, and let column $C$ be referenced in the SELECT clause of $S$. In earlier versions of SQL, then, $C$ had to be one of the grouping columns (or be referenced inside a "set function" invocation, but let's agree to ignore that possibility for simplicity). In the current version, by contrast, it's required only that $C-$ or $\{C\}$, rather-be functionally dependent on the grouping columns.

■ Do you think the GROUP BY / HAVING formulation is easier to understand? (Debatable.)

- Does the GROUP BY / HAVING formulation work correctly for parts that aren't supplied by any suppliers at all? (No, it doesn't.)
- Are the formulations equivalent if the database permits nulls? Or duplicates?

As a further exercise, give SQL formulations (a) using GROUP BY and HAVING, (b) not using GROUP BY and HAVING, for the following queries:

■ Get supplier numbers for suppliers who supply $N$ different parts for some $N>3$.
■ Get supplier numbers for suppliers who supply $N$ different parts for some $N<4$.
What do you conclude from this exercise?

## EXERCISES

11.1 If you haven't already done so, complete the exercises included inline in the body of the chapter.
11.2 Take another look at the various SQL expressions in the body of the chapter. From those SQL formulations alone (i.e., without looking at the problem statements), see if you can come up with a natural language interpretation of what the SQL expressions mean. Then compare your interpretations with the problem statements as given in the chapter.
11.3 Try applying the techniques described in this chapter to some genuine SQL problems from your own work environment. Note: This exercise is important. The techniques described in this chapter can seem a little daunting or hard to follow at first; in order to become familiar and comfortable with them, therefore, there's really no substitute for "getting your hands dirty" and applying them for yourself.
11.4 Let relvar EMP have attributes ENO and HEIGHT and predicate Employee ENO has height HEIGHT. Here's a relational calculus formulation of the quota query (see Exercise 7.14) "Get the employee number for the three shortest employees":

```
{ EX.ENO } WHERE COUNT ( EY WHERE EY.HEIGHT < EX.HEIGHT ) < 3
```

And here's a fairly direct transliteration of this expression into SQL:

```
SELECT EX.ENO
FROM EMP AS EX
WHERE ( SELECT COUNT ( * )
    FROM EMP AS EY
    WHERE EY.HEIGHT < EX.HEIGHT ) < 3
```

Here by contrast are three GROUP BY / HAVING expressions:

```
SELECT EX.ENO
FROM EMP AS EX , EMP AS EY
WHERE EX.HEIGHT >= EY.HEIGHT
GROUP BY EX.ENO
HAVING 3 <= COUNT ( * )
SELECT EX.ENO
FROM EMP AS EX , EMP AS EY
WHERE EX.HEIGHT > EY.HEIGHT
GROUP BY EX.ENO
HAVING 3 > COUNT ( * )
SELECT EX.ENO
FROM EMP AS EX , EMP AS EY
WHERE EX.HEIGHT > EY.HEIGHT
OR EX.ENO = EY.ENO
GROUP BY EX.ENO
HAVING 3 >= COUNT ( * )
```

Do you think these expressions are easier to understand than the relational calculus expression? More to the point, do they accurately represent the desired query? Also, what happens in each case if there aren't exactly three shortest employees?
11.5 Some of the examples discussed in the present chapter-or others very much like them-were also discussed in earlier chapters, but the SQL formulations I gave in those chapters were often more "algebra like" than "calculus like." Can you come up with any transformation laws that would allow the calculus formulations to be mapped into algebraic ones or vice versa?
11.6 In this chapter, I've discussed techniques for mapping relational calculus expressions into SQL equivalents. However, the mapping process was always carried out "by hand," as it were. Do you think it could be mechanized?

## Chapter 12

# Miscellaneous SQL Topics 

I explained that we are calling the White Paper "Open Government" because you always dispose of the difficult bit in the title. It does less harm there than on the statute books.<br>-Sir Humphrey Appleby, in Open Government<br>(first episode of the BBC TV series Yes Minister, by Antony Jay and Jonathan Lynn, 1981)

This last chapter is something of a potpourri; it discusses a few SQL features that, for one reason or another, don't fit very neatly into any of the previous chapters. It also gives a simplified BNF grammar for SQL table expressions and SQL boolean expressions, for purposes of reference.

Also, this is as good a place as any to define two terms that you need to watch out for. The terms in question are implementation defined and implementation dependent, and they're both used heavily in the SQL standard. Here are the definitions:

Definition: An implementation defined feature is one whose semantics can vary from one implementation to another, but do at least have to be specified for any individual implementation. In other words, the implementation is free to decide how it will implement the feature in question, but the result of that decision must be documented. An example is the maximum length of a character string.

Definition: An implementation dependent feature, by contrast, is one whose semantics can vary from one implementation to another and don't even have to be specified for any individual implementation. In other words, the term effectively means undefined; the implementation is free to decide how it will implement the feature in question, and the result of that decision doesn't need to be documented (it might vary from release to release, or even more frequently). An example is the full effect of an ORDER BY clause if the specifications in that clause fail to specify a total ordering, as in, e.g, SELECT SNO FROM S ORDER BY CITY.

## SELECT *

Use of the "SELECT *" form of the SQL SELECT clause is acceptable in situations where the specific columns involved, and their left to right ordering, are both irrelevant-for example, in an EXISTS invocation. It can be dangerous in other situations, however, because the meaning of that "*" can change if (e.g.) new columns are added to an existing table. Recommendation: Be on the lookout for such situations and try to avoid them. In particular, don't use "SELECT *" at the outermost level in a cursor definition-instead, always name the pertinent columns explicitly. A similar remark applies to view definitions also. (However, if you adopt the strategy suggested under the discussion of column naming in Chapter 3 of always accessing the database via views-the "operate via views" strategy-then it might be safe to use "SELECT *" anywhere you like other than in the definitions of those views themselves.)

## EXPLICIT TABLES

An explicit table in SQL is an expression of the form TABLE $T$, where $T$ is the name of a base table or view or an "introduced name" (see the discussion of WITH in Chapter 6). It's logically equivalent to the following:

```
( SELECT * FROM T )
```

Here's a fairly complicated example that makes use of explicit tables ("Get all parts-but if the city is London, show it as Oslo and show the weight as double"):

```
WITH T1 AS ( SELECT PNO , PNAME , COLOR , WEIGHT , CITY
        FROM P
        WHERE CITY = 'London' ) ,
        T2 AS ( SELECT PNO , PNAME , COLOR , WEIGHT , CITY ,
                2 * WEIGHT AS NEW_WEIGHT , 'Oslo' AS NEW_CITY
            FROM T1 )
        T3 AS ( SELECT PNO', PNAME , COLOR ,
                NEW WEIGHT AS WEIGHT , NEW_CITY AS CITY
        FROM T2 ) ,
        T4 AS ( TABLE P EXCEPT CORRESPONDING TABLE T1 )
TABLE T4 UNION CORRESPONDING TABLE T3
```


## NAME QUALIFICATION

Column names in SQL can usually be dot qualified by the name of the applicable range variable (see the next section). However, SQL allows that qualifier to be omitted in many situations, in which case an implicit qualifier is assumed by default. But:

- The SQL rules regarding implicit qualification aren't always easy to understand. As a result, it isn't always obvious what a particular unqualified name refers to.
- What's unambiguous today might be ambiguous tomorrow (e.g., if new columns are added to an existing table).
- In Chapter 3 I recommended, strongly, that columns that represent the same kind of information be given the same name whenever possible. If that recommendation is followed, then unqualified names will often be ambiguous anyway, and dot qualification will therefore be required.

So a good general rule is: When in doubt, qualify. Unfortunately, however, there are certain contexts in which qualification isn't allowed. The contexts in question are, loosely, ones in which the name serves as a reference to the column per se, rather than to the data contained in that column. Here's a partial list of such contexts (note the last two in particular):

- A column definition within a base table definition
- A key or foreign key specification

The column name commalist, if specified (but it shouldn't be-see Chapter 8), in CREATE VIEW

- The column name commalist, if specified (but it usually shouldn't be-see the next section), following the definition of a range variable
- The column name commalist in JOIN ... USING
- The column name commalist, if specified (and it should be-see Chapter 5), on INSERT
- The left side of a SET assignment on UPDATE

It might help to note that most of the contexts listed above are ones in which no range variable, as such, is available for dot qualification use anyway. The point is, however, that an unsuspecting user might expect to be able to use table names as qualifiers in these contexts, on the grounds-I suppose-that SQL often uses table names as if they were range variable names anyway, as explained in the next section.

## RANGE VARIABLES

As we saw in Chapter 10, a range variable in the relational model is a variable-a variable in the sense of logic, that is, not the usual programming language sense-that ranges over the set of tuples in some relation (or the set of rows in some table, in SQL terms). In SQL, such variables are defined by means of AS specifications in the context of either FROM or explicit JOIN. Here's a simple example of the FROM case:

```
SELECT SX.SNO
FROM S AS SX
WHERE SX.STATUS > 15
```

SX here is a range variable that ranges over table S ; in other words, its permitted values are rows of table S . You can think of the SELECT expression overall as being evaluated as follows. First, the range variable takes on one of its permitted values, say the row for supplier S1. Is the status value in that row greater than 15 ? If it is, then supplier number S1 appears in the result. Next, the range variable moves on to another row of table $S$, say the row for supplier S2; again, if the status value in that row is greater than 15 , then the relevant supplier number appears in the result. And so on, exhaustively, until variable SX has taken on all of its permitted values.

Note: SQL calls a name such as SX in the example a correlation name. However, it doesn't seem to have a term for the thing that such a name names; certainly there's no such thing in SQL as a "correlation." (Note in particular that the term doesn't necessarily have anything to do with correlated subqueries, which are discussed in the next section.) I prefer the term range variable.

Incidentally, it's worth noting that SQL requires SELECT expressions always to be formulated in terms of range variables; if no such variables are specified explicitly, it assumes the existence of implicit ones with the same names as the corresponding tables. For example, the SELECT expression

```
SELECT SNO
FROM S
WHERE STATUS > 15
```

-arguably a more "natural" SQL formulation of the example discussed above-is treated as shorthand for this expression (note the text in bold):

```
SELECT S.SNO
FROM S AS S
WHERE S.STATUS > 15
```

In this latter formulation, the "S" dot qualifiers and the "S" in the specification "AS S" do not denote table S; rather, they denote a range variable called $S$ that ranges over the table with the same name. ${ }^{1}$

Now, the BNF grammar defined later in this chapter refers to the items in the commalist in a FROM clause-i.e., the items following the keyword FROM itself-as table specifications. ${ }^{2}$ The operand expressions in an explicit JOIN are also table specifications. Let $t s$ be such a table specification. Then, if the table expression portion of $t s$ is a table subquery (see the next section), then $t s$ must also include an AS clause-even if the range variable introduced by that AS clause is never explicitly mentioned anywhere else in the overall expression. Here's a JOIN example:

```
( SELECT SNO , CITY FROM S ) AS TEMP1
    NATURAL JOIN
( SELECT PNO , CITY FROM P ) AS TEMP2
```

Here's another example (repeated from Chapter 7):

```
SELECT PNO , GMWT
FROM ( SELECT PNO, WEIGHT * 454 AS GMWT
WHERE GMWT > 7000.0
```

For interest, here's the same example with all implicit qualifiers made explicit:

```
SELECT TEMP.PNO , TEMP.GMWT
FROM ( SELECT P.PNO , P.WEIGHT * 454 AS GMWT
        FROM P ) AS TEMP
WHERE TEMP.GMWT > 7000.0
```

Note: A range variable definition in SQL can always optionally include a column name commalist that defines column names for the table the range variable ranges over, as in this example (see the last two lines):

```
SELECT TEMP.SNO , TEMP.SNAME , TEMP.STATUS, TEMP.SCITY ,
    TEMP.PNO , TEMP.PNAME , TEMP.COLOR , TEMP.WEIGHT , TEMP.PCITY
FROM ( SELECT * FROM S JOIN P ON S.CITY > P.CITY ) AS TEMP
    ( SNO , SNAME , STATUS , SCITY ,
        PNO , PNAME , COLOR , WEIGHT , PCITY )
```

The introduced column names here (SNO, SNAME, STATUS, SCITY, PNO, PNAME, COLOR, WEIGHT, and PCITY) effectively rename columns SNO, SNAME, STATUS, S.CITY, PNO, PNAME, COLOR, WEIGHT,

[^132]and P.CITY, respectively (see the explanation of JOIN ... ON in Chapter 6). ${ }^{3}$ However, it shouldn't be necessary to introduce column names in this way very often if other recommendations in this book are followed.

Recommendation: Favor the use of explicit range variables, especially in "complex" expressions-they can aid clarity, and sometimes they can save keystrokes. ${ }^{4}$ Be aware, however, that SQL's name scoping rules for such variables can be quite hard to understand (but this is true regardless of whether the variables in question are explicit or implicit).

Caveat: Many SQL texts refer to range variable names (or correlation names) as aliases, and describe them as if they were just alternative names for the tables they range over. But such a characterization seriously misrepresents the true state of affairs-indeed, it betrays a serious lack of understanding of what's really going on-and is strongly deprecated on that account. Be on your guard against this sloppy manner of speaking.

## SUBQUERIES

A subquery in SQL is a table expression, $t x$ say, enclosed in parentheses; if the table denoted by $t x$ is $t$, the table denoted by the subquery is $t$ also. Note, however, that (as mentioned in Chapters 1 and 6 ) the expression $t x$ can't be an explicit JOIN expression. Thus, for example,
( A NATURAL JOIN B )
isn't a legal subquery. ${ }^{5}$ By contrast, the following expression is a legal subquery:
( SELECT * FROM A NATURAL JOIN B )
Subqueries fall into three categories (though the syntax is the same in every case). The details, partly repeated from earlier chapters, are as follows:

- A table subquery is a subquery that's neither a row subquery nor a scalar subquery.
- A row subquery is a subquery appearing in a position where a row expression is expected. Let $r s q$ be such a subquery; then $r s q$ must denote a table with just one row. Let the table in question be $t$, and let the single row in $t$ be $r$; then $r s q$ behaves as if it denoted that row $r$ (in other words, $t$ is coerced to $r$ ). Note: If $r s q$ doesn't denote a table with just one row, then (a) if it denotes a table with $n$ rows ( $n>1$ ), an error is raised; (b) if it denotes a table with no rows at all, then that table is treated as if it contained just one row, where the row in question contains a null in every column position.
- A scalar subquery is a subquery appearing in a position where a scalar expression is expected. Let ssq be such a subquery; then ssq must denote a table with just one row and just one column. Let the table in question be $t$, let the single row in $t$ be $r$, and let the single value in $r$ be $v$; then $s s q$ behaves as if it denoted that value $v$ (in other words, $t$ is coerced to $r$, and then $r$ is coerced to $v$ ). Note: If $s s q$ doesn't denote a table with just one row and just one column, then (a) if it denotes a table with $m$ columns ( $m>1$ ), an error is raised

[^133](probably at compile time); (b) if it denotes a table with one column and $n$ rows ( $n>1$ ), an error is raised (probably at run time); (c) if it denotes a table with one column and no rows at all, then that table is treated as if it contained just one row, where the row in question contains a single null.

The following examples involve, in order, a table subquery, a row subquery, and a scalar subquery:

```
SELECT SNO
FROM S
WHERE CITY IN
    ( SELECT CITY /* table subquery */
        FROM P
        WHERE COLOR = 'Red' )
UPDATE S
SET ( STATUS , CITY ) =
    ( SELECT STATUS , CITY /* row subquery */
        FROM S
        WHERE SNO = 'S1' )
WHERE CITY = 'Paris' ;
SELECT SNO
FROM S
WHERE CITY =
    ( SELECT CITY /* scalar subquery */
        FROM P
        WHERE PNO = 'P1' )
```

Next, a correlated subquery is a special kind of (table, row, or scalar) subquery; to be specific, it's a subquery that includes a reference to some "outer" table. In the following example, the parenthesized expression following the keyword IN is a correlated subquery, because it includes a reference to the outer table S (the query is "Get names of suppliers who supply part P1"):

```
SELECT DISTINCT S.SNAME
FROM S
WHERE 'P1' IN
    ( SELECT PNO /* correlated subquery */
        FROM SP
        WHERE SP.SNO = S.SNO )
```

As noted in Chapter 11, correlated subqueries are often contraindicated from a performance point of view, because-conceptually, at any rate-they have to be evaluated once for each row in the outer table instead of just once and for all. (In the example, if the overall expression is evaluated as stated, the subquery will be evaluated $n$ times, where $n$ is the number of rows in table S.) For that reason, it's a good idea to avoid correlated subqueries if possible. In the case at hand, it's very easy to reformulate the query to achieve this goal:

```
SELECT DISTINCT S.SNAME
FROM S
WHERE SNO IN
    ( SELECT SNO /* noncorrelated subquery */
        FROM SP
        WHERE PNO = 'P1' )
```

Finally, a "lateral" subquery is a special kind of correlated subquery. To be specific, it's a correlated subquery that (a) appears in a FROM clause and (b) includes a reference to an "outer" table that's defined by a table
specification appearing earlier in that same FROM clause. For example, consider the query "For each supplier, get the supplier number and the number of parts supplied by that supplier." Here's one possible formulation of that query in SQL :

```
SELECT S.SNO , TEMP.PCT
FROM S , LATERAL ( SELECT COUNT ( PNO ) AS PCT
    FROM SP
    WHERE SP.SNO = S.SNO ) AS TEMP
```

The purpose of the keyword LATERAL is to tell the system that the subquery to which it's prefixed is correlated with something previously mentioned in the very same FROM clause (in the example, that "lateral" subquery yields exactly one value-namely, the applicable count-for each SNO value in table S). Given the sample values in Fig. 1.1 in Chapter 1, the result looks like this:

| SNO | PCT |
| ---: | ---: |
| S1 | 6 |
| S2 | 2 |
| S3 | 1 |
| S4 | 3 |
| S5 | 0 |

Now, there's something going on here that you might be forgiven for finding a bit confusing. The items in a FROM clause are table specifications, and so they denote tables. In the example, though, the particular table specification that begins with the keyword LATERAL-more precisely, what remains of that table specification if the keyword LATERAL is removed-looks more like what might be called a scalar specification, or more precisely a scalar subquery; certainly it could used as such, should the context demand such an interpretation (e.g., in a SELECT clause). In fact, however, it's a table subquery. The table it denotes, for a given value of S.SNO, is called TEMP; that table has just one column, called PCT, and just one row, and hence in fact contains a single scalar value. Then the expression TEMP.PCT in the SELECT clause causes that scalar value to become the contribution of table TEMP to the applicable result row (just as the expression S.SNO in that same SELECT clause causes the applicable SNO value to become the contribution of table S to that result row).

Following on from the foregoing rather complicated explanation, I feel bound to add that it's not exactly clear why "lateral" subqueries are needed anyway. Certainly the foregoing example can easily be reformulated in such a way as to avoid the "need" (?) for any such thing:

```
SELECT S.SNO , ( SELECT COUNT ( PNO )
    FROM SP
    WHERE SP.SNO = S.SNO ) AS PCT
FROM S
```

The subquery has moved from the FROM clause to the SELECT clause; it still refers to something else in the same clause (S.SNO, to be specific), but now the keyword LATERAL is no longer needed (?). However, do note what's happened to the specification AS PCT, which appeared inside the subquery in the LATERAL formulation but has now moved outside (this point is discussed in more detail in an aside in the section "Summarization" in Chapter 7).

Finally: I've defined the term subquery; perhaps it's time to define the term query, too!-even though I've used that term ubiquitously throughout previous chapters. So here goes: A query is a retrieval request; in the SQL context, in other words, it's either a table expression-though such expressions can also be used in contexts other
than queries per se-or a statement, such as a SELECT statement in "direct" (i.e., interactive) SQL, that asks for such an expression to be evaluated. Note: The term is sometimes used (though not in this book!) to refer to an update request also. It's also used to refer to the natural language version of some retrieval or update request.

## "POSSIBLY NONDETERMINISTIC" EXPRESSIONS

As we saw in Chapter 2, an SQL table expression is "possibly nondeterministic" if it might give different results on different evaluations, even if the database hasn't changed in the interim. Here's the standard's own definition:

A <query expression> or <query specification> is possibly nondeterministic if an implementation might, at two different times where the state of the SQL-data is the same, produce results that differ by more than the order of the rows due to General Rules that specify implementation dependent behavior.

Actually this definition is a trifle odd, inasmuch as tables-which is what <query expressions>s and <query specifications>s are supposed to produce-aren't supposed to have a row ordering anyway. But let's overlook this detail; the important point is that "possibly nondeterministic" expressions aren't allowed in integrity constraints, ${ }^{6}$ a state of affairs that could have serious practical implications if true.

The standard's rules for labeling a given table expression "possibly nondeterministic" are quite complex, and full details are beyond the scope of the present discussion. However, a table expression $t x$ is certainly considered to be "possibly nondeterministic" if any of the following is true:"

- $t x$ is a union, intersection, or difference, and the operand tables include a column of type character string.
$t x$ is a SELECT expression, the SELECT item commalist in that SELECT expression includes an item ( $C$ say) of type character string, and at least one of the following is true:
a. The SELECT item commalist is preceded by the keyword DISTINCT.
b. $\quad C$ involves a MAX or MIN invocation.
c. $t x$ directly includes a GROUP BY clause and $C$ is one of the grouping columns.
- $t x$ is a SELECT expression that directly includes a HAVING clause and the boolean expression in that HAVING clause includes either (a) a reference to a grouping column of type character string or (b) a MAX or MIN invocation in which the argument is of type character string.
- $t x$ is a JOIN expression and either or both of the operand expressions is possibly nondeterministic.

Note, however, that these rules are certainly stronger than they need be. For example, suppose NO PAD applies to the collation in effect and that collation is one in which there are no characters that are "equal but

[^134]distinguishable"; then, e.g., SELECT MAX $(C)$ FROM $T$, where column $C$ of table $T$ is of the character string type in question, is surely well defined.

## EMPTY SETS

The empty set is the set containing no elements. This concept is both ubiquitous and extremely important in the relational world, but SQL commits a number of errors in connection with it. Unfortunately there isn't much you can do about most of those errors, but you should at least be aware of them. Here they are (this is probably not a complete list):

- A VALUES expression isn't allowed to contain an empty row expression commalist.

■ The SQL "set functions" all return null if their argument is empty (except for COUNT(*) and COUNT, which correctly return zero in such a situation).

- If a scalar subquery evaluates to an empty table, that empty table is coerced to a null.
- If a row subquery evaluates to an empty table, that empty table is coerced to a row of all nulls.
- If the set of grouping columns and the table being grouped are both empty, GROUP BY produces a result containing just one (necessarily empty) group, whereas it should produce a result containing no groups at all.
- A key can't be an empty set of columns (nor can a foreign key, a fortiori).
- A table can't have an empty heading.
- A SELECT item commalist can't be empty.
- A FROM item commalist can't be empty.
- The set of common columns in UNION CORRESPONDING, INTERSECT CORRESPONDING, and EXCEPT CORRESPONDING can't be empty (though it can be for JOIN).

A row can't have an empty set of components.

## A SIMPLIFIED BNF GRAMMAR

For purposes of reference, it seems appropriate to close this chapter, and the main part of this book, with a simplified BNF grammar for SQL table expressions and SQL boolean expressions. ${ }^{8}$ The grammar is deliberately somewhat conservative, in that it fails to define as valid certain expressions that are so, according to the SQL standard. (However, I don't believe it defines as valid any expressions that aren't so according to that standard.) To be more specific, constructs that I've previously advised you not to use-including in particular everything to do with nulls and 3VL-are deliberately omitted; so too are certain somewhat esoteric features (e.g., recursive queries). Also, for

[^135]reasons explained in Chapter 1, almost all of the syntactic categories in what follows have names that differ from their counterparts in the standard. The following simplifying abbreviations are used:

```
exp for expression
spec for specification
```

All syntactic categories of the form <... name $>$ are assumed to be $<$ identifier $>$ s and are defined no further here. The category <scalar exp> is also left undefined-though it might help to recall in particular that:

- A scalar subquery is a legal scalar expression.
- Most "row expressions" that occur in practice are actually scalar expressions.
- Boolean expressions are scalar expressions too.


## Table Expressions

As you can see, the grammar in this subsection begins with a production for < with exp>, a construct not mentioned (as such) in the body of the book. I introduce this syntactic category partly in order to capture the fact that join expressions can't appear without being nested inside some other table expression-but it does mean that the construct referred to throughout earlier parts of the book as a table expression doesn't directly correspond to anything defined in the grammar! (I mean, there's no production for a syntactic category called <table exp>.) I apologize if you find this state of affairs confusing, but it's the kind of thing that always happens when you try to define a grammar for a language that violates orthogonality.

```
<with exp>
    ::= [ <with spec> ] <nonjoin exp>
<with spec>
    ::= WITH <name intro commalist>
<name intro>
    ::= <table name> AS <nonjoin exp>
<nonjoin exp>
    ::= <nonjoin term>
    | <nonjoin exp> UNION [ DISTINCT ]
                                    [ CORRESPONDING ] <nonjoin term>
            | <nonjoin exp> EXCEPT [ DISTINCT ]
                        [ CORRESPONDING ] <nonjoin term>
<nonjoin term>
    ::= <nonjoin primary>
    | <nonjoin term> INTERSECT [ DISTINCT ]
                                    [ CORRESPONDING ] <nonjoin primary>
<nonjoin primary>
    ::= TABLE <table name>
            | <table selector>
            | <select exp>
            | ( <nonjoin exp> )
```

```
<table selector>
    ::= VALUES <row exp commalist>
<row exp>
    ::= <scalar exp>
        | <row selector>
        | <row subquery>
<row selector>
    ::= ( <scalar exp commalist> )
<row subquery>
    ::= <subquery>
<subquery>
    ::= ( <nonjoin exp> )
<select exp>
    ::= SELECT [ DISTINCT ] [ * | <select item commalist> ]
                FROM <table spec commalist>
                [ WHERE <boolean exp> ]
                        [ GROUP BY <column name commalist> ]
                            [ HAVING <boolean exp> ]
<select item>
    ::= <scalar exp> [ AS <column name> ]
        | <range variable name>.*
<table spec>
    ::= <table name> [ AS <range variable name> ]
        | [ LATERAL ] <table subquery> AS <range variable name>
        <join exp>
        | ( <join exp> )
<table subquery>
    ::= <subquery>
<join exp>
    ::= <table spec> CROSS JOIN <table spec>
        | <table spec> NATURAL JOIN <table spec>
        | <table spec> JOIN <table spec> ON <boolean exp>
        | <table spec> JOIN <table spec>
                        USING ( <column name commalist> )
```


## Boolean Expressions

Note that the grammar in this subsection agrees with the SQL standard in giving AND higher precedence than OR; thus, the boolean expression (e.g.) $p$ AND $q$ OR $r$ is understood as ( $p$ AND $q$ ) OR $r$, not as $p$ AND ( $q$ OR $r$ ). But it's probably better always to specify parentheses explicitly in such expressions.

```
<boolean exp>
    ::= <boolean term>
            | <boolean exp> OR <boolean term>
```

284 Chapter 12 / Miscellaneous Topics

```
<boolean term>
    ::= <boolean factor>
        | <boolean term> AND <boolean factor>
<boolean factor>
    ::= [ NOT ] <boolean primary>
<boolean primary>
    ::= <boolean literal>
        | <boolean variable name>
        | <boolean column name>
        | <condition>
        | ( <boolean exp> )
<boolean literal>
    ::= TRUE | FALSE
<condition>
    ::= <simple comparison exp>
        | <between exp>
        | <like exp>
        | <in exp>
        | <match exp>
        | <all or any exp>
        | <exists exp>
        | <unique exp>
<simple comparison exp>
    ::= <row exp> <simple comp op> <row exp>
<simple comp op>
    ::= = | < | <= | > | >= | <>
<between exp>
    ::= <row exp> [ NOT ] BETWEEN <row exp> AND <row exp>
<like exp>
    ::= <scalar exp> [ NOT ] LIKE <scalar exp> [ ESCAPE <scalar exp> ]
```

The $<$ scalar $\exp >$ s must denote character strings. For ESCAPE, that string must be of length one.

```
<in exp>
    ::= <row exp> [ NOT ] IN <table subquery>
        | <scalar exp> [ NOT ] IN ( <scalar exp commalist> )
<match exp>
    ::= <row exp> MATCH [ UNIQUE ] <table subquery>
<all or any exp>
    ::= <row exp> <scalar comp op> <all or any> <table subquery>
<all or any>
    ::= ALL | ANY | SOME
<exists exp>
    ::= EXISTS <table subquery>
```

```
<unique exp>
    ::= UNIQUE <table subquery>
```


## EXERCISES

12.1 According to the BNF grammar given in the body of the chapter, which of the following are legal as "stand alone" expressions (i.e., expressions not nested inside other expressions) and which not, syntactically speaking? ( $A$ and $B$ are table names, and you can assume the tables they denote satisfy the requirements for the operator in question in each case.)

```
A NATURAL JOIN B
A INTERSECT B
SELECT * FROM A NATURAL JOIN B
SELECT * FROM A INTERSECT B
SELECT * FROM ( A NATURAL JOIN B )
SELECT * FROM ( A INTERSECT B )
SELECT * FROM ( SELECT * FROM A INTERSECT SELECT * FROM B )
SELECT * FROM ( A NATURAL JOIN B ) AS C
SELECT * FROM ( A INTERSECT B ) AS C
TABLE A NATURAL JOIN TABLE B
TABLE A INTERSECT TABLE B
SELECT * FROM A INTERSECT SELECT * FROM B
( SELECT * FROM A ) INTERSECT ( SELECT * FROM B )
( SELECT * FROM A ) AS AA INTERSECT ( SELECT * FROM B ) AS BB
```

What do you conclude from this exercise? Perhaps I should remind you that, relationally speaking, intersection is a special case of natural join.
12.2 Take another look at the expressions in Exercise 12.1. In which of those expressions would it be syntactically legal to replace $A$ or $B$ or both by "table literals" (i.e., appropriate VALUES invocations)?
12.3 Let X and Y both be of the same character string type and be subject to the same collation; let PAD SPACE apply to that collation (not recommended, of course); and let X and Y have the values ' 42 ' and ' 42 ', respectively
(note the trailing space in the second of these). Then we know from Chapter 2 that although X and Y are clearly distinct, the expression $\mathrm{X}=\mathrm{Y}$ gives TRUE. But what about the expression X LIKE Y ?
12.4 Given our usual sample values, what do the following expressions return?

```
SELECT DISTINCT STATUS
FROM S
WHERE STATUS BETWEEN 10 AND 30
SELECT DISTINCT CITY
FROM S
WHERE CITY LIKE 'L%'
SELECT DISTINCT CITY
FROM S
WHERE CITY BETWEEN 'Paris' AND 'Athens'
```

12.5 The following is intended to be an SQL expression of type BOOLEAN. Is it legal?

```
( SELECT CITY FROM S WHERE STATUS < 20 )
    =
( SELECT CITY FROM P WHERE WEIGHT = 14.0 )
```

12.6 In the body of the chapter I recommended circumspection in the use of asterisk notation in the SELECT clause. For brevity, however, I didn't always follow my own advice in this respect in earlier chapters. Take a look through those chapters and see if you think any of my uses of the asterisk notation were unsafe.
12.7 Consider any SQL product available to you. Does that product support (a) the UNIQUE operator, (b) explicit tables, (c) lateral subqueries, (d) "possibly nondeterministic" expressions?
12.8 With regard to possibly nondeterministic expressions, recall that SQL prohibits the use of such expressions in integrity constraints. Take another look at the examples in Chapter 8 and/or the answers to the exercises from that chapter in Appendix F. Do any of those examples or answers involve any possibly nondeterministic expressions? If so, what can be done about it?
12.9 Throughout this book I've taken the term $S Q L$ to refer to the official standard version of that language specifically (though my treatment of the standard has deliberately been a very long way from exhaustive). But every product on the market departs from the standard in various ways, either by omitting some standard features or by introducing proprietary features of its own or (almost certainly in practice) both. Again, consider any SQL product available to you. Identify as many departures from the standard in that product as you can.

## Appendix A

## The Relational Model

I believe quite strongly that if you think about the issue at the appropriate level of abstraction, you're inexorably led to the position that databases must be relational. Let me immediately try to justify this very strong claim! ${ }^{1}$ My argument goes like this:

■ First of all, we saw in Chapter 5 that a database, despite the name, isn't really just a collection of data; rather, it's a collection of "true facts," or (rather more respectably, since "facts" are supposed to be true by definition) true propositions-for example, the proposition "Joe's salary is 50K."

■ Propositions like "Joe's salary is 50 K " are easily encoded as ordered pairs-e.g., the ordered pair (Joe, 50 K ), in the case at hand (where "Joe" is a value of type NAME, say, and " 50 K " is a value of type MONEY, say).

- But we don't want to record just any old propositions; rather, we want to record all propositions that happen to be true instantiations of certain predicates. In the case of "Joe's salary is 50 K ," for example, the pertinent predicate is " $x$ 's salary is $y$," where $x$ is a value of type NAME and $y$ is a value of type MONEY.

In other words, we want to record the extension of the predicate " $x$ 's salary is $y$," which we can do in the form of a set of ordered pairs.

But a set of ordered pairs is, precisely, a binary relation, in the mathematical sense of that term. Here's the definition:

Definition: A (mathematical) binary relation over two sets $A$ and $B$ is a subset of the cartesian product of $A$ and $B$; in other words, it's a set of ordered pairs $(a, b)$, such that the first element $a$ is a value from $A$ and the second element $b$ is a value from $B$.

- A binary relation in the foregoing sense can be depicted as a table. Here's an example:


[^136](As an aside, I remark that this particular example is not just a relation but a function, because each person has just one salary. A function is a special case of a binary relation.) So we can regard this picture as depicting a subset of the cartesian product of the set of all names ("type NAME") and the set of all money values ("type MONEY"), in that order.

Given the argument so far, then, we can see we're talking about some fairly humble (but very solid) beginnings. However, in 1969-1970, Codd realized that:

- We need to deal with $n$-adic, not just dyadic, predicates and propositions (e.g., "Joe has salary 50 K , works in department D4, and was hired in 1993 "). So we need to deal with n-ary relations, not just binary ones, and $n$-tuples (tuples for short), not just ordered pairs.
- Left to right ordering might be acceptable for pairs but soon gets unwieldy for $n>2$; so let's replace that ordering concept by the concept of attributes (identified by name), and let's redefine the relation concept accordingly. The example now looks like this:


From this point forward, then, you can take the term relation to mean a relation in this revised and extended sense, barring explicit statements to the contrary.

Data representation alone isn't the end of the story-we need operators for deriving further relations from the given ("base") ones, so that we can do queries and the like (e.g., "Get all persons with salary 60 K "). But since a relation is both a logical construct (the extension of a predicate) and a mathematical one (a special kind of set), we can apply both logical and mathematical operators to it. Thus, Codd was able to define both a relational calculus (based on logic) and a relational algebra (based on set theory). And the relational model was born.

## THE RELATIONAL MODEL vs. OTHERS

Perhaps you can begin to see now why it's my opinion that (to repeat something I said in Chapter 5) the relational model is rock solid, and "right," and will endure. A hundred years from now, I fully expect database systems still to be based on Codd's relational model. Why? Because the foundations of that model-namely, set theory and predicate logic-are themselves rock solid in turn. Elements of predicate logic in particular go back well over 2,000 years, at least as far as Aristotle (384-322 BCE).

So what about other data models?-the "object oriented model," for example, or the "hierarchic model," or the CODASYL "network model," or the "semistructured model"? In my view, these other models are just not in the
same ballpark. Indeed, I seriously question whether they deserve to be called models at all. ${ }^{2}$ The hierarchic and network models in particular never really existed in the first place!-as abstract models, I mean, preceding any implementations. Instead, they were invented after the fact; that is, hierarchic and network products were built first, and the corresponding models were defined afterward, by a process of induction-here just a polite term for guesswork-from those products. As for the object oriented and semistructured models, it's entirely possible that the same criticism applies; I suspect it does, but it's hard to be sure. One problem is that there doesn't seem to be any consensus on what those models might consist of. ${ }^{3}$ It certainly can't be claimed, for example, that there's a unique, clearly defined, and universally accepted object oriented model, and similar remarks apply to the semistructured model also. (Actually, some people have claimed there isn't a unique relational model, either. I'll deal with that argument in a few moments.)

Aside: The following quote from "The Object Oriented Database Sysem Manifesto," by Malcolm Atkinson, François Bancilhon, David DeWitt, Klaus Dittrich, David Maier, and Stanley Zdonik (Proc. 1st International Conference on Deductive and Object Oriented Databases, Kyoto, Japan, 1989) lends weight to my suggestion that in the case of the object oriented model, at least, implementations came first and the model itself was-or indeed, the quote rather strongly suggests, should be (?)-defined afterward:

With respect to the specification of the system, we are taking a Darwinian approach: We hope that, out of the set of experimental prototypes being built, a fit model will emerge. We also hope that viable implementation technology for that model will evolve simultaneously.

In other words, the authors are suggesting that the code should be written first, and that a model might possibly be developed later by abstracting from that code. End of aside.

Another important reason why I don't believe those other models really deserve to be called models at all is the following. First, I hope you agree it's undeniable that the relational model is indeed a model and thus not, by definition, concerned with implementation issues. By contrast, those other models all fail, much of the time, to make a clear distinction between issues that truly are model issues and issues that have to do with matters of implementation; at the very best, they muddy that distinction considerably (they're all much "closer to the metal," as it were). ${ }^{4}$ As a consequence, they're harder to use and understand, and they give implementers far less freedom-far less than the relational model does, I mean-to adopt inventive or creative approaches to questions of implementation.

So what of those claims to the effect that there are several relational models, too? One example of such a claim can be found in the book Joe Celko's Data and Databases: Concepts in Practice (Morgan Kaufmann, 1999), where the author, Joe Celko, says this:

There is no such thing as the relational model for databases anymore [sic] than there is just one geometry.
And to bolster his argument, he goes on to identify what he says are six "different relational models."

[^137]Now, I wrote an immediate response to these claims when I first encountered them. Here's a lightly edited version of what I said at the time:

It's true there are several different geometries (euclidean, elliptic, hyperbolic, and so forth). But is the analogy a valid one? That is, do those "different relational models" differ in the same way those different geometries differ? It seems to me the answer to this question is no. Elliptic and hyperbolic geometries are often referred to, quite explicitly, as noneuclidean geometries; ${ }^{5}$ for the analogy to be valid, therefore, it would seem that at least five of those "six different relational models" would have to be nonrelational models, and hence, by definition, not "relational models" at all. (Actually, I would agree that several of those "six different relational models" are indeed not relational. But then it can hardly be claimed-at least, it can't be claimed consistently-that they're different relational models.)

And I went on to say this (again somewhat edited here):
But I have to admit that Codd did revise his own definitions of what the relational model was, somewhat, throughout the 1970s and 1980s. One consequence of this fact is that critics have been able to accuse Codd in particular, and relational advocates in general, of "moving the goalposts" far too much. For example, Mike Stonebraker has written (in his introduction to Readings in Database Systems, 2nd edition, Morgan Kaufmann, 1994) that "one can think of four different versions" of the model:

- Version 1: Defined by the 1970 CACM paper
- Version 2: Defined by the 1981 Turing Award paper
- Version 3: Defined by Codd's 12 rules and scoring system
- Version 4: Defined by Codd's book

Let me interrupt myself briefly to explain the references here. They're all by Codd. The 1970 CACM paper is "A Relational Model of Data for Large Shared Data Banks," CACM 13, No. 6 (June 1970), and it's discussed in a little more detail in Appendix G of the present book. The 1981 Turing Award paper is "Relational Database: A Practical Foundation for Productivity," CACM 25, No. 2 (February 1982). The 12 rules and the accompanying scoring system are described in Codd's Computerworld articles "Is Your DBMS Really Relational?" and "Does Your DBMS Run By The Rules?" (October 14th and October 21st, 1985). Finally, Codd's book is The Relational Model for Database Management Version 2 (Addison-Wesley, 1990). Now back to my response:

Perhaps because we're a trifle sensitive to such criticisms, Hugh Darwen and I have tried to provide, in our book Databases, Types, and the Relational Model: The Third Manifesto, our own careful statement of what we believe the relational model is (or ought to be!). Indeed, we'd like our Manifesto to be seen in part as a definitive statement in this regard. I refer you to the book itself for the details; here just let me say that we see our contribution in this area as primarily one of dotting a few $i$ 's and crossing a few $t$ 's that Codd himself left undotted or uncrossed in his own work. We most certainly don't want to be thought of as departing in any major respect from Codd's original vision; indeed, the whole of the Manifesto is very much in the spirit of Codd's ideas and continues along the path that he originally laid down.

To all of the above I'd now like to add another point, which I think clearly refutes Celko's original argument. I agree there are several different geometries. But the reason why those geometries are all different is: They start from different axioms. By contrast, we've never changed the axioms for the relational model. We have made a number of changes over the years to the model itself-for example, we've added relational comparisons-but the

[^138]axioms (which are basically those of classical set theory and classical predicate logic) have remained unchanged ever since Codd's first papers. Moreover, what changes have occurred have all been, in my view, evolutionary, not revolutionary, in nature. Thus, I really do claim there's only one relational model, even though it has evolved over time and will presumably continue to do so. As I said in Chapter 1, it can be seen as a small branch of mathematics; as such, it grows over time as new theorems are proved and new results discovered. What's more-as with mathematics in general-those new theorems and results can be proved and discovered by anyone who's competent to do so. The relational model began as the brainchild of one man, but now belongs to the world. ${ }^{6}$

So what are those evolutionary changes? Here are some of them:

- As already mentioned, we've added relational comparisons.
- We've clarified the logical difference between relations and relvars.
- We've clarified the concept of first normal form; as a consequence, we've embraced the concept of relation valued attributes in particular.
- We have a better understanding of the nature of relational algebra, including the relative significance of various operators and an appreciation of the importance of relations of degree zero, and we've identified certain useful new operators (for example, extend and semijoin).
- We've added the concept of image relations.
- We have a better understanding of updating, including view updating in particular.
- We have a better understanding of the fundamental significance of integrity constraints in general, and we have many good theoretical results regarding certain important special cases.
- We've clarified the nature of the relationship between the model and predicate logic.
- Finally, we have a clearer understanding of the relationship between the relational model and type theory (more specifically, we've clarified the nature of domains).


## THE SIGNIFICANCE OF THEORY

Note: The bulk of this section consists of an abbreviated and slightly revised version of material from an interview I did in 2005 (published in my book Date on Database: Writings 2000-2006, Apress, 2006).

The relational model, whatever else it might be, is certainly a theory-so I'd like to say a few words about the significance of theory in general before getting into details of the relational model in particular. As I said in the preface to the present book, it's an article of faith with me that theory is practical. The purpose of relational theory in particular is not just theory for its own sake; the purpose of that theory is to allow us to build systems that are 100 percent practical. Thus, I believe that, in the relational context specifically, departures from the underlying theory are A Big Mistake.

Unfortunately, however, the term "theory" has two quite different meanings. In common parlance, it's almost pejorative-"oh, that's just your theory." Indeed, in such contexts it's effectively just a synonym for opinion

[^139](and the adverb merely-it's merely your opinion-is often implied, too). But to a scientist, the term has a very different meaning. To a scientist, a theory is a set of ideas or principles that explain some set of observable phenomena, such as the motion of the planets. Of course, when I say I it explains something, I mean it does so coherently: It fits the facts, as it were. Moreover (and very importantly), it doesn't just explain something, it also makes predictions-predictions that can be tested and (at least in principle) can be shown to be false. And if any of those predictions do indeed turn out to be false, then we move on: Either we modify the existing theory, or we adopt a new one. That's the scientific method:

- First, we observe certain phenomena, empirically.
- We construct a theory or hypothesis to explain those phenomena.
- We use that theory to make predictions.
- We test the accuracy of those predictions.
- Based on the results of those tests, we refine our theory (or reject it, in extreme cases).
- And we iterate.

That's how the Copernican system replaced epicycles; how Einstein's cosmology replaced Newton's; how general relativity replaced special relativity; and so on. Incidentally, Carl Sagan has a nice observation in this regard:

> In science it often happens that scientists say, "You know, that's a really good argument, my position is mistaken," and then they actually change their minds, and you never hear that old view from them again. They really do it. It doesn't happen as often as it should, because scientists are human and change is sometimes painful. But it happens every day. I cannot recall the last time something like that happened in politics or religion.

Anyway, I claim the relational model is indeed a theory in the scientific sense; more specifically, I claim it's a mathematical theory. Now, mathematical theories are a little special, in a way. First of all, the observed phenomena they're supposed to explain tend to be rather abstract-not nearly as concrete as something like the motion of the planets, for example. Second, the predictions they make are essentially the theorems that can be proved within the theory; thus, those "predictions" can be falsified only if there's something wrong with the premises, or axioms, on which the theorems are based. But even this does happen from time to time! For example, in euclidean geometry, you can prove that every triangle has angles that sum to 180 degrees. So if we ever found a triangle that didn't have this property, we would have to conclude that the premises-the axioms of euclidean geometry-must be wrong. And in a sense exactly that happened: Triangles on the surface of a sphere (for example, on the surface of the Earth) turned out to have angles that sum to more than 180 degrees. And the problem turned out to be the euclidean axiom regarding parallel lines. Riemann replaced that axiom by a different one and thereby defined a different (but equally valid) kind of geometry.

In the same kind of way, the theory that's the relational model might be falsified in some way-but I think it's pretty unlikely, because (as I said in the previous section) the premises on which the relational model is based are essentially those of set theory and predicate logic, and those premises have stood up pretty well for a very long time.

So, to get to the real point of this section: Given that the relational model is a scientific theory, the question is whether that theory is really important. Of course, my own answer to this question is yes. In fact, I'd like to turn the question on its head ... First of all, database management is a field in which some solid theory does exist. Furthermore, we know the value of that theory; we know the benefits that accrue if we follow that theory. We also
know there are costs associated with not following that theory (we might not know exactly what those costs are-I mean, it might be hard to quantify them-but we do know there are going to be costs).

If you're traveling on an airplane, you'd like to be sure it's been constructed in accordance with the principles of physics and aerodynamics. If you live or work in a high rise building, you'd like to be sure it's been constructed in accordance with sound engineering and architectural principles. In the same kind of way, if you're using a DBMS, wouldn't you like to be sure it's been constructed in accordance with solid database principles? If it hasn't, you know things will go wrong. And while it might be hard to say exactly what will go wrong, and it might be hard to say whether things will go wrong in a major or minor way, you know-it's guaranteed-that things will go wrong.

So I don't think people should be asking "What's the business value of implementing the relational model?" Rather, I think they should be asking, or perhaps trying to explain, what the business value is of not implementing it. In other words, those who ask "What's the value of the relational model?" are basically saying "What's the value of theory?"-and I hereby challenge them to tell me what the value is of not abiding by the theory.

## THE RELATIONAL MODEL DEFINED

Now I'd like to give a precise definition of just what it is that constitutes the relational model. The trouble is, the definition I'll give is indeed reasonably precise: so much so, in fact, that I think it would have been pretty hard to understand if I'd given it in Chapter 1. (As Bertrand Russell once memorably said: Writing can be either readable or precise, but not at the same time.) Now, I did give a definition in Chapter 1-a definition, that is, of what I there called "the original model"-but I frankly don't think that definition is even close to being good enough, for the following reasons among others:

For starters, it was much too long and rambling. (Well, that was fair enough, given the intent of that preliminary chapter; but now I want a definition that's reasonably succinct, as well as being precise.)

I don't really much care for the idea that the model should be thought of as consisting of "structure plus integrity plus manipulation"; in some ways, in fact, I think it's actively misleading to think of it in such terms. The truth is, those three aspects of the model are inextricably intertwined. For example, the relvars in any given database (structural piece) will be subject to a variety of integrity constraints (integrity piece), and those constraints will be expressed using a variety of relational operators (manipulative piece). Moreover, those relvars will (or should) have been designed in accordance with relational design theory, which likewise involves all three pieces. And of course they'll be subject to update, which again involves all three pieces.

- "The original model" included a few things I'm not too comfortable with: for instance, divide, nulls, the entity integrity rule, the idea of being forced to choose one key and make it primary, and the idea (still argued on occasion) that domains and types might somehow be different things. Regarding nulls, incidentally, I note that Codd invented the relational model in 1969 and didn't introduce nulls until 1979; in other words, the model managed perfectly well-in my opinion, better-for some ten years without any notion of nulls at all. What's more, early languages and implementations managed perfectly well without them, too.
- The original model also omitted a few things I now consider vital. For example, it excluded any mention-at least, any explicit mention-of all of the following: predicates, constraints (other than key and foreign key constraints), relation variables, relational comparisons, relation type inference and associated features, image relations, certain algebraic operators (especially rename, extend, summarize (?), semijoin, and semidifference), and the important relations TABLE_DUM and TABLE_DEE. (On the other hand, I think it could fairly be argued that these features at least weren't precluded by the original model; it might even be
argued in some cases that they were in fact included, in a kind of embryonic form. For example, it was certainly always intended that implementations should include support for constraints other than just key and foreign key constraints. Relational comparisons too were at least implicitly required, even in Codd's very first paper.)

Without further ado, then, let me give my own preferred definition.
Definition: The relational model consists of five components:

1. An open ended collection of scalar types, including type BOOLEAN in particular ${ }^{7}$
2. A relation type generator and an intended interpretation for relations of types generated thereby
3. Facilities for defining relation variables of such generated relation types
4. A relational assignment operator for assigning relation values to such relation variables
5. A relationally complete (but otherwise open ended) collection of generic relational operators for deriving relation values from other relation values

The following subsections elaborate on each of these components in turn. First, however, a word of caution. As John Muir once said, when we try to pick out anything by itself, we find it hitched to everything else in the universe (often quoted in the form "Everything is connected to everything else"). John Muir was referring to the natural world, of course, but he might just as well have been talking about the relational model. ${ }^{8}$ The fact is, the various features of the relational model are highly interconnected-remove just one of them, and the whole edifice crumbles. Translated into concrete terms, this metaphor means that if we build a "relational" DBMS that fails to support some aspect of the model, the resulting system (which shouldn't really be called relational, anyway) will be bound to display behavior on occasion that's certainly undesirable and possibly unforeseeable. I can't stress the point too strongly: Every feature of the model is there for solid practical reasons; if we choose to ignore some detail, then we do so at our own peril.

## Scalar Types

Scalar types can be either system defined or user defined, in general; thus, a means must be available for users to define their own scalar types (this requirement is implicit in the fact that the set of scalar types is open ended). A means must therefore also be available for users to define their own operators, since types without operators are useless. The set of system defined scalar types is required to include type BOOLEAN-the most fundamental type of all, containing precisely two values (viz., the truth values TRUE and FALSE)-but a real system will surely support others as well (INTEGER, CHAR, and so on). Support for type BOOLEAN implies support for the usual logical operators (NOT, AND, OR, and so on) as well as other operators, system or user defined, that return boolean values. In particular, the equality comparison operator " $=$ " (which is a boolean operator by definition) must be available in connection with every type, nonscalar as well as scalar, for without it we couldn't even say what the

[^140]values are that constitute the type in question. What's more, the model prescribes the semantics of that operator, too. To be specific, if $v 1$ and $v 2$ are values of the same type, then $v l=v 2$ returns TRUE if $v l$ and $v 2$ are the very same value and FALSE otherwise.

Aside: The following is a logical consequence of the foregoing definition of equality that can be very helpful in practice. Let $O p$ be an operator with a parameter $P$; let $P$ be of type $T$, so that the argument corresponding to $P$ in any given invocation of $O p$ is also of type $T$; and let $v 1$ and $v 2$ be values of type $T$. If two successful invocations of $O p$ that are identical in all respects except that the argument corresponding to $P$ is $v l$ in one invocation and $v 2$ in the other are somehow distinguishable in their effect, then $v 1=v 2$ will (in fact, must) evaluate to FALSE. End of aside.

Let $T$ be some scalar type. Associated with type $T$, then, there's at least one selector operator, with the properties that (a) every invocation of that operator returns a value of type $T$ and (b) every value of type $T$ is returned by some invocation of that operator (more specifically, by some corresponding literal-recall that a literal is a special case of a selector invocation).

## Relation Types

The relation type generator allows users to specify individual relation types as desired: for example, as the type for some relation variable or some relation valued attribute. The intended interpretation for a given relation of a given type, in a given context, is as a set of propositions; each such proposition (a) constitutes an instantiation of some predicate that corresponds to the relation heading, (b) is represented by a tuple in the relation body, and (c) is assumed to be true. If the context in question is some relvar-that is, if we're talking about the relation that happens to be the current value of some relvar-then the predicate in question is the relvar predicate for that relvar. Relvars in particular are interpreted in accordance with The Closed World Assumption (see later in this appendix, also Chapter 5).

Let $R T$ be some relation type. Associated with $R T$, then, there's a relation selector operator, with the properties that (a) every invocation of that operator returns a relation of type $R T$ and (b) every relation of type $R T$ is returned by some invocation of that operator (more specifically, by some relation literal). Also, since the equality comparison operator " $=$ " is available in connection with every type, it's available in connection with type $R T$ in particular. So too is the relational inclusion operator " $\subseteq$ "; if relations $r 1$ and $r 2$ are of the same type, then $r l$ is included in $r 2$ if and only if the body of $r 1$ is a subset of that of $r 2$.

## Relation Variables

As noted above, one use-a particularly important one-for the relation type generator is in specifying the type of a relation variable, or relvar, when that relvar is defined. Such a variable is the only kind permitted in a relational database; all other kinds of variables, scalar variables or tuple variables or any other kind, are prohibited. (In programs that access such a database, by contrast, they're not prohibited-in fact, they're probably required.)

The statement that the database contains nothing but relvars is one possible formulation of what Codd originally called The Information Principle, though I don't think it's a formulation he ever used himself. Instead, he usually stated the principle like this:

The entire information content of the database at any given time is represented in one and only one way: namely, as explicit values in attribute positions in tuples in relations.

I heard Codd refer to this principle on more than one occasion as the fundamental principle underlying the relational model. Any violation of it thus has to be seen as serious. ${ }^{9}$ Database tables that involve top to bottom row ordering or left to right column ordering, or contain duplicate rows, or pointers, or nulls, or have anonymous columns or duplicate column names, all constitute such violations. But why is the principle so important? The answer is bound up with the observations I made in Chapter 5 to the effect that (along with types) relations are both necessary and sufficient to represent any data whatsoever at the logical level. In other words, the relational model gives us everything we need in this respect, and it doesn't give us anything we don't need.

I'd like to pursue this point a moment longer. In general, it's axiomatic that if we have $n$ different ways of representing data, then we need $n$ different sets of operators. For example, if we had arrays as well as relations, we'd need a full complement of array operators as well as a full complement of relational ones. ${ }^{10}$ If $n$ is greater than one, therefore, we have more operators to implement, document, teach, learn, remember, and use (and choose among). But those extra operators add complexity, not power! There's nothing useful that can be done if $n$ is greater than one that can't be done if $n$ equals one (and in the relational model, of course, $n$ does equal one).

What's more, not only does the relational model give us just one construct, the relation itself, for representing data, but that construct is-to quote Codd himself (see the section "Objectives of the Relational Model," later in this appendix)-of spartan simplicity: It has no ordering to its tuples, it has no ordering to its attributes, it has no duplicate tuples, it has no pointers, and (at least as far as I'm concerned) it has no nulls. Any contravention of these properties is tantamount to introducing another way of representing data, and therefore to introducing more operators as well. In fact, SQL is living proof of this observation. For example, SQL has nine different union operators (and ought by rights to have 18, if not 27), while the relational model has just one.

Aside: Perhaps I should explain these last remarks. First of all, SQL supports six different unions for tables as such-UNION DISTINCT, UNION DISTINCT CORRESPONDING, UNION DISTINCT
CORRESPONDING BY, and three variants on these in which DISTINCT is replaced by ALL. The funny thing is, the one kind of union it doesn't support for tables as such is true bag union! For the record, here's the definition: Let $b 1$ and $b 2$ be bags; let $x$ appear exactly $n 1$ times in $b 1$ and exactly $n 2$ times in $b 2$; and let $b$ be the bag union of $b 1$ and $b 2$. Then $x$ appears exactly $n$ times in $b$, where-to adopt a self-explanatory notation $-n=\operatorname{MAX}(n 1, n 2)$. So SQL ought by rights to support BAG (or some such keyword) as an alternative to DISTINCT and ALL, giving us three more unions. Nine so far. Next, SQL supports two different unions for what it calls "multiset values" (as opposed to tables), viz., MULTISET UNION DISTINCT and MULTISET UNION ALL; but it ought really to support seven more possibilities here too (involving BAG, CORRESPONDING, and so on), at least if those "multiset values" are multisets of rows specifically. Now we're up to 18. Finally, SQL also supports a union "set function" (for use in summarization), though it calls it not UNION but FUSION. FUSION has no variants, but by rights the same possibilities that apply to tables should apply here too (again, if the "multiset values" in question are multisets of rows specifically). Total: 27. ${ }^{11}$ End of aside.

[^141]As you can see, then, The Information Principle is certainly important-but it has to be said that its name hardly does it justice. Other names that have been proposed, mainly by Hugh Darwen or myself or both, include The Principle of Uniform Representation and The Principle of Uniformity of Representation. (This latter is clumsy, I admit, but at least it's accurate.)

There's one more point I should mention under the heading of "Relation Variables." As Darwen and I demonstrate in our book on The Third Manifesto, the database isn't really just "a container for relvars," even though we usually talk about it as if it were. Rather, it's a variable. After all, it can certainly be updated-and that means it's a variable by definition! Logically speaking, in other words, the database in its entirety is one (typically rather large) variable in itself, which we might call a dbvar. I'll elaborate on this concept in the section "Database Variables" later in this appendix.

## Relational Assignment

Like the equality comparison operator " $=$ ", the assignment operator " $:=$ " must be available in connection with every type (for without it we would have no way of assigning values to a variable of the type in question), and again relation types are no exception to this rule. The operators INSERT, DELETE, and UPDATE (likewise D_INSERT and I_DELETE) are permitted and indeed useful, but strictly speaking they're only shorthands. What's more, support for relational assignment (a) must include support for multiple relational assignment in particular and (b) must abide by both The Assignment Principle and The Golden Rule.

## Relational Operators

The "generic relational operators" are the operators that make up the relational algebra (or something logically equivalent to the algebra), and they're therefore built in-though there's no inherent reason why users shouldn't be able to define additional operators of their own, if desired. Precisely which operators are included isn't specified, but they're required to provide, in their totality, at least the expressive power of the relational calculus. (In other words, they're required to be relationally complete-see further discussion below.)

Now, there seems to a widespread misconception concerning the purpose of the algebra. To be specific, many people seem to think it's meant just for writing queries-but it's not; rather, it's for writing relational expressions. Those expressions in turn serve many purposes, including query but certainly not limited to query alone. Here are some other important ones (this isn't an exhaustive list):

- Defining views and snapshots
- Defining the set of tuples to be inserted into, deleted from, or updated in, some relvar (or, more generally, defining the set of tuples to be assigned to some relvar)
- Defining constraints (though here the relational expression in question will be just a subexpression of some boolean expression, typically but not invariably an IS_EMPTY invocation)
- Serving as a basis for investigations into other areas, such as optimization and database design

The algebra also serves as a kind of yardstick against which the expressive power of database languages can be measured. Essentially, a language is said to be relationally complete if and only if it's at least as powerful as the algebra (or the calculus-it comes to the same thing), meaning its expressions permit the definition of every relation that can be defined by means of expressions of the algebra (or the calculus). As noted in Chapter 10, relational completeness is a basic measure of the expressive capability of a language; if a language is relationally complete, it
means (among other things, and speaking a trifle loosely) that queries of arbitrary complexity can be formulated without having to resort to branching or iterative loops. In other words, as I also said in that earlier chapter, it's relational completeness that allows end users-at least in principle, though possibly not in practice-to access the database directly, without having to go through the potential bottleneck of the IT department.

## DATABASE VARIABLES

Note: This section consists of a revised version of material from Appendix D ("What Is a Database?") from the book Databases, Types, and the Relational Model: The Third Manifesto, by Hugh Darwen and myself (see Appendix G).

I mentioned in the previous section that databases are really variables (as I said in that section, if a database can be updated, then it's a variable by definition). In other words, we can draw a distinction between database values and database variables, precisely analogous to the one we already draw between relation values and relation variables. As a matter of fact, we-i.e., Darwen and myself-did draw exactly such a distinction in the first version of The Third Manifesto. Here's an edited quote:

> The first version of this Manifesto distinguished databases per se (i.e., database values) from database variables ... It suggested that the unqualified term database be used to mean a database value specifically, and it introduced the term dbvar as shorthand for "database variable." While we still believe this distinction to be a valid one, we found it had little direct relevance to other aspects of the Manifesto. We therefore decided, in the interests of familiarity, to revert to more traditional terminology. [In other words, we went on to use the term "database" to mean a database variable rather than a database value, and we didn't use the terms "database variable" or "dbvar" at all.]

And of course I've done the same thing-I mean, I've used the term database in the traditional way, and I haven't used the terms database variable or dbvar at all-throughout the present book, prior to this point. However, the most recent edition of the Manifesto book, after quoting the foregoing text, goes on to say:

Now this bad decision has come home to roost! With hindsight, it would have been much better to "bite the bullet" and adopt the more logically correct terms database value and database variable (or dbvar), despite their lack of familiarity.

That same book gives arguments in support of this position, of course, but I don't need to get into those arguments here; the simple fact is, a database simply is a variable (its value changes over time), regardless of whether we call it a "dbvar" or just a database.

Now, it follows from the foregoing that when we "update some relvar" (within some database), what we're really doing is updating the pertinent dbvar. (For clarity, I'll adopt the term $d b v a r$ for the remainder of the present section.) For example, the Tutorial D statement

```
DELETE SP WHERE QTY < 150 ;
```

"updates the shipments relvar SP" and thus really updates the entire suppliers-and-parts dbvar (the "new" database value for that dbvar being the same as the "old" one except that certain shipment tuples have been removed). In other words, while we might say a database "contains variables" (viz., the applicable relvars), such a manner of speaking is only approximate, and in fact quite informal. A more formal and more accurate way of characterizing the situation is this:

## A dbvar is a tuple variable.

The tuple variable in question has one attribute for each relvar in the dbvar (and no other attributes), and each of those attributes is relation valued. In the case of suppliers and parts, for example, we can think of the entire dbvar as a tuple variable of the following tuple type:

```
TUPLE { S RELATION { SNO CHAR , SNAME CHAR ,
    STATUS INTEGER, CITY CHAR } ,
    P RELATION { PNO CHAR , PNAME CHAR ,
    COLOR CHAR , WEIGHT RATIONAL , CITY CHAR } ,
    SP RELATION { SNO CHAR , PNO CHAR , QTY INTEGER } }
```

Suppose we call the suppliers-and-parts dbvar (or tuple variable) SPDB. Then the DELETE statement shown above might be regarded as shorthand for the following tuple assignment:

```
SPDB := TUPLE { S ( S FROM SPDB ) ,
    P ( P FROM SPDB ) ,
    SP ( ( SP FROM SPDB ) WHERE NOT (QTY < 150 ) ) } ;
```

Explanation: The expression on the right side of this assignment is a tuple selector invocation, and it denotes a tuple with three attributes called $\mathrm{S}, \mathrm{P}$, and SP , each of which is relation valued. Within that tuple, the value of attribute $S$ is the current value of relvar $S$; the value of attribute $P$ is the current value of relvar $P$; and the value of attribute SP is the current value of relvar SP , minus tuples for which the quantity is less than 150 .

In sum: A dbvar is a tuple variable, and a database (i.e., the value of some given dbvar at some given time) is a tuple. What's more, given a relational assignment of the form

$$
R:=r x
$$

(where $R$ is a relvar reference-i.e., a relvar name-and $r x$ is a relational expression), that relvar reference $R$ is really a pseudovariable reference (see the paragraph immediately following). In other words, the relational assignment is shorthand for an assignment that "zaps" one component of the corresponding dbvar (which is, to repeat, really a tuple variable). It follows that "relation variables" (at least, relation variables in the database) aren't really variables at all; rather, they're a convenient fiction that gives the illusion that the database-or the dbvar, rather-can be updated in a piecemeal fashion, individual relvar by individual relvar.

A note on pseudovariables: Essentially, a pseudovariable reference consists of an operational expression appearing in an assignment in the target position. For example, let X be a variable of type CHAR, and let 'Middle' be the current value of $X$. Then the assignment $\operatorname{SUBSTR}(X, 2,1):=$ ' $u$ ' has the effect of "zapping" the second character position within $X$, replacing the ' $i$ ' by a ' $u$ '. The expression on the left side of that assignment is a pseudovariable reference. The paper "On the Logical Differences Between Types, Values, and Variables" (see Appendix G) discusses the concept in detail.

## OBJECTIVES OF THE RELATIONAL MODEL

For purposes of reference if nothing else, it seems appropriate in this appendix to document Codd's own stated objectives in introducing his relational model. The following list is based on one he gave in his paper "Recent Investigations into Relational Data Base Systems" (an invited paper to the 1974 IFIP Congress), but I've edited it just slightly here:

1. To provide a high degree of data independence
2. To provide a community view of the data of spartan simplicity, so that a wide variety of users in an enterprise, ranging from the most computer naïve to the most computer sophisticated, can interact with a common model (while not prohibiting superimposed user views for specialized purposes)
3. To simplify the potentially formidable job of the DBA
4. To introduce a theoretical foundation, albeit modest, into database management (a field sadly lacking in solid principles and guidelines)
5. To merge the fact retrieval and file management fields in preparation for the addition at a later time of inferential services in the commercial world
6. To lift database application programming to a new level-a level in which sets (and more specifically relations) are treated as operands instead of being processed element by element

I'll leave it to you to judge to what extent you think the relational model meets these objectives. Myself, I think it does pretty well.

## SOME DATABASE PRINCIPLES

In Chapter 1, I said I was interested in principles, not products, and we've encountered several principles at various points in the book. Here I collect them together for ease of reference.

- The Information Principle (also known as The Principle of Uniform Representation or The Principle of Uniformity of Representation): The database contains nothing but relvars; equivalently, the entire information content of the database at any given time is represented in one and only one way-namely, as explicit values in attribute positions in tuples in relations. ${ }^{12}$
- The Closed World Assumption: Let relation $r$ correspond to predicate $P$. If tuple $t$ appears in $r$, then the proposition $p$ corresponding to $t$ is assumed to be true. Conversely, if tuple $t$ plausibly could appear in $r$ but doesn't in fact appear, then the proposition $p$ corresponding to $t$ is assumed to be false. Note: In Chapter 5 I explained The Closed World Assumption in terms of relvars, not relations, but the definition just given is slightly more general. Note that it applies to relations that are the current values of relvars in particular, but it isn't limited to such relations.

The Principle of Interchangeability: There must be no arbitrary and unnecessary distinctions between base and virtual relvars.

The Assignment Principle: After assignment of the value $v$ to the variable $V$, the comparison $V=v$ must evaluate to TRUE.

[^142]The Golden Rule: No update operation must ever cause the database constraint for any database to evaluate to FALSE.

The Principle of Identity of Indiscernibles: Let $a$ and $b$ be any two things (any two "entities," if you prefer); then, if there's no way whatsoever of distinguishing between $a$ and $b$, there aren't two things but only one. ${ }^{13}$ Note: I didn't mention this principle earlier in the book, but I appealed to it tacitly on many occasions. It can alternatively be stated thus: Every entity has its own unique identity. In the relational model, such identities are represented in the same way as everything else-namely, by means of attribute values (see The Information Principle above)-and numerous benefits accrue from this simple fact.

## WHAT REMAINS TO BE DONE?

All of the above is not to say we won't continue to make progress or there isn't still work to be done in this important field. In fact, I see at least four areas, somewhat interrelated, where developments are either under way or are needed: implementation, foundations, higher level abstractions, and higher level interfaces.

## Implementation

In some ways the message of this book can be summed up very simply:

## Let's implement the relational model!

To elaborate: First of all, I think it's clear from the body of the book that it's being extremely charitable to SQL to describe it as a relational language. It follows that SQL products can be considered relational only to a first approximation. The truth is, the relational model has never been properly implemented in commercial form (at least, not in any mainstream product), and users have never really enjoyed the benefits that a truly relational product would bring. Indeed, that's one of the reasons why I wrote this book, and it's also one of the reasons why Hugh Darwen and I have been working for so long on The Third Manifesto. The Third Manifesto-the Manifesto for short-is a formal proposal for a solid foundation for future DBMSs. And it goes without saying that what it really does, in as careful and precise a manner as the authors are capable of, is define the relational model and spell out some of the implications of that definition. (It also goes into a great deal of detail on the impact of type theory on that model; in particular, it proposes a comprehensive model of type inheritance as a logical consequence of that type theory.)

So we'd really like to see the ideas of the Manifesto implemented properly in commercial form ("we" here meaning, primarily, Hugh Darwen and myself). ${ }^{14}$ We believe such an implementation would serve as a solid basis on which to build so many other things-for example, "object/relational" DBMSs; spatiotemporal DBMSs; DBMSs used in connection with the World Wide Web; and "rule engines" (also known as "business logic servers"), which some see as the next generation of general purpose DBMS products. We further believe we would then have the right framework for supporting the other items that are suggested below as also being desirable. Personally, in fact, I would go further; I would suggest that trying to implement those items in any other kind of framework is likely to

[^143]prove more difficult than doing it right. To quote the well known mathematician Gregory Chudnovsky: "If you do it the stupid way, you will have to do it again" (from an article in The New York Times, December 24th, 1997).

## Foundations

There's still much interesting work to be done on theoretical foundations (in other words, it's certainly not the case that all of the foundation problems have been solved). Here are three examples:

- Let $r x$ be some relational expression. By definition, the relation $r$ denoted by $r x$ satisfies a constraint $r c$ that's derived from the constraints satisfied by the relations in terms of which $r x$ is expressed. To what extent can the process of determining that constraint $r c$ be mechanized?
- Can we inject more science into the database design process? In particular, can we come up with a precise and operationally useful characterization of the notion of redundancy? Note: The book Normal Forms and All That Jazz: A Database Professional's Guide to Database Design Theory (see Appendix G) offers some proposals in this connection.
- Can we come up with a good way-that is, a way that's robust, logically sound, and ergonomically satisfactory-of dealing with the "missing information" problem? Note: Appendix C of the present book offers some suggestions in this regard.


## Higher Level Abstractions

One way we make progress in computer languages and applications is by raising the level of abstraction. For example, I pointed out in Chapter 5 that the familiar KEY and FOREIGN KEY specifications are really just shorthand for constraints that can be expressed more longwindedly using the general integrity features of any relationally complete language like Tutorial D. But those shorthands are useful: Quite apart from the fact that they save us some writing, they also serve to raise the level of abstraction, by allowing us to talk in terms of certain bundles of concepts that belong naturally together. In a sense, they make it easier to see the forest as well as the trees.

By way of another illustration, consider the relational algebra. I showed in Chapters 6 and 7 that many of the operators of the algebra-including ones we use all the time, even if we don't realize it, like semijoin-are really shorthand for certain combinations of other operators. ${ }^{15}$ In other words, what's really going on here is again a raising of the level of abstraction (rather as macros raise the level of abstraction in a conventional programming language).

Raising the level of abstraction in the relational world can be regarded as building on top of the relational model; it doesn't change the model, but it does make it more directly useful for certain tasks. And one area where this approach looks as if it's going to prove really fruitful is temporal databases. In our book Temporal Data and the Relational Model (see Appendix G), Hugh Darwen, Nikos Lorentzos, and I-building on original work by Lorentzos-introduce interval types as a basis for supporting temporal data in a relational framework. For example, consider the "temporal relation" in Fig. A. 1 opposite, which shows that certain suppliers supplied certain parts during certain intervals of time (you can read $d 04$ as "day 4," d06 as "day 6," and so on; likewise, you can read [d04:d06] as "the interval from day 4 to day 6 inclusive," and so on). Attribute DURING in that relation is interval valued.

[^144]| SNO | PNO | DURING |
| :---: | :---: | :---: |
| S1 | P1 | [d04: d06] |
| S1 | P1 | [d09:d10] |
| S1 | P3 | [d05:d10] |
| S2 | P1 | [d02:d04] |
| S2 | P1 | [d08:d10] |
| S2 | P2 | [d03: d03] |
| S2 | P2 | [d09:d10] |

Fig. A.1: A relation with an interval valued attribute
Support for interval attributes, and hence for temporal databases, involves among other things support for generalized versions of the regular algebraic operators. For reasons that aren't important here, we call those generalized operators "U_ operators"; thus, there's a $U_{-}$restrict operator, a $U_{\_}$join operator, a $U_{-}$union operator, and so on. But-and here comes the point-those U_ operators are all, in the last analysis, nothing but shorthand for certain combinations of regular (i.e., conventional) algebraic operators, as described in this book. Once again, then, what's fundamentally going on is a raising of the level of abstraction.

Two further points on this topic: First, our approach to temporal data involves not just "U_" versions of the algebraic operators but also (a) "U_" keys and foreign keys, (b) "U_" comparison operators, and (c) "U_" versions of INSERT, DELETE, and UPDATE-but, again, all of these constructs turn out to be essentially just shorthand. Second, it also turns out that the Manifesto's type inheritance model has a crucial role to play in that temporal support-and so once again we see an example of the interconnectedness of all of these issues.

## Higher Level Interfaces

There's another way in which we can build on the relational model, and that's by means of various kinds of applications that run on top of the relational interface and provide various specialized services. One example might be decision support; another might be data mining; another might be a natural language front end. For the users of such applications, the relational model will disappear under the covers, at least to some degree. (Though even if it does, and even if most users interact with the database only through some such front end, it seems to me that database design and the like will still necessarily be based on solid relational principles. At least, I certainly hope so.)

By the way: Suppose it's your job to implement one of those front end applications. Which would you prefer as a target?-a relational DBMS, or some other kind (an object oriented DBMS, say)? And if you opt for the former, as I obviously think you should, which would you prefer?-a DBMS that supports the relational model as such, or one that supports SQL?

In case it's not clear, my point is this: We've come a long way from the early days when SQL was being touted as a language that end users could use for themselves, ${ }^{16}$ and I know many people will dismiss my numerous criticisms of SQL as mere carping for that very reason. Real users don't use it anyway, right? Only programmers use it. And in any case, much of the SQL code that's actually executed is never written by a human programmer at all but is generated by some kind of front end application. However, it seems to me that SQL is bad as a target

[^145]language for all of the same reasons that it's bad as a source language. And it further seems to me, therefore, that my criticisms are still germane.

## So What about SQL?

SQL is incapable of providing the kind of firm foundation we need for future growth and development. Instead, it's the relational model that has to provide that foundation. In The Third Manifesto, therefore, Darwen and I reject SQL as such; in its place, we argue that some truly relational language like Tutorial $\mathbf{D}$ should be implemented as soon as possible. Of course, we aren't so naïve as to think that SQL will ever disappear. Rather, we hope that Tutorial D, or some other true relational language, will be sufficiently superior that it will become the database language of choice (by a process of natural selection), and SQL will become "the database language of last resort." In fact, we see a parallel with the world of programming languages, where COBOL has never disappeared (and never will); but COBOL has become "the programming language of last resort" for developing applications, because better alternatives exist. We see SQL as a kind of database COBOL, and we would like to see some other language become available as a better alternative to it.

To say it again, we do realize that SQL databases and applications are going to be with us for quite a long time-to think otherwise would be quite unrealistic-and so we do have to pay some attention to the question of what to do about today's SQL legacy. The Manifesto therefore does include some specific proposals in this regard. In particular, it offers some suggestions for implementing SQL on top of a true relational language, so that existing SQL applications can continue to work. Detailed discussion of those proposals would be out of place here; suffice it to say, however, that we believe we can simulate various nonrelational features of SQL-even things like duplicates and nulls-without having to support such concepts directly in the underlying relational language.

## Appendix B

## SQLDepartures from the Relational Model

In this appendix I summarize, mainly for purposes of reference and with little by way of additional commentary, some of the ways in which SQL-by which I mean, as always in this book, the standard version of that language except where otherwise noted-departs from the relational model. Now, I know there are those who will quibble over specific items in this list; it's not easy to compile such a list, especially if it's meant to be orthogonal (i.e., if an attempt is made to keep the various items all independent of one another). But I don't think such quibbling is important. What's important is the cumulative effect, which quite frankly I think is overwhelming. ${ }^{1}$

- SQL fails to distinguish adequately between table values and table variables.
- SQL tables aren't the same as relations (or relvars), because they either permit or require, as the case may be, (a) duplicate rows; (b) nulls; (c) left to right column ordering; (d) anonymous columns; (e) duplicate column names; (f) pointers; and-at least in some products, though not in the standard as such-(g) hidden columns. Note that most if not all of these differences constitute violations of The Assignment Principle.
- SQL has no proper table literals.
- SQL often seems to think views aren't tables.
- SQL tables (views included!) must have at least one column.
- SQL has no explicit table assignment operator.
- SQL has no explicit multiple table assignment a fortiori (nor does it have an INSERT/DELETE analog).
- SQL violates The Assignment Principle in numerous different ways (some but not all of them having to do with nulls).
- SQL violates The Golden Rule in numerous different ways (some but not all of them having to do with nulls).
- SQL has no proper "table type" notion. As a consequence, its support for table type inference (i.e., determining the type of the result of some table expression) is very incomplete.

[^146]■ SQL has no "=" operator for tables; in fact, it has no table comparison operators at all.
■ SQL supports "reducible keys" (i.e., it allows proper superkeys to be declared as keys).
■ Numerous SQL operators are "possibly nondeterministic."

- SQL supports various row level operators (cursor updates, row level triggers).
- Although the SQL standard doesn't, the dialects of SQL supported in various commercial products do sometimes refer to certain storage level constructs (e.g., indexes).
- SQL's view definitions include mapping information as well as structural information.
- SQL's support for view updating is weak, ad hoc, and incomplete.
- SQL fails to distinguish properly between types and representations.
- SQL's "structured types" are sometimes encapsulated and sometimes not. (This issue wasn't discussed in the body of this book.)
- SQL fails to distinguish properly between types and type generators.
- Although the SQL standard does support type BOOLEAN, commercial SQL products typically don't.
- SQL's support for " $=$ " is seriously deficient. To be more specific, SQL's " $=$ " operator (a) can give TRUE even when the comparands are clearly distinct; (b) can fail to give TRUE even when the comparands aren't clearly distinct; (c) can have user defined, and hence arbitrary, semantics (for user defined types); (d) isn't supported at all for the system defined type XML; and (e) in some products, isn't supported for certain other types as well.
- SQL is based on three-valued logic (sort of), whereas the relational model is based on two-valued logic.
- SQL isn't relationally complete.

The foregoing list is not exhaustive.

## Appendix C

## A Relational Approach

## to Missing Information

The book Database Explorations: Essays on The Third Manifesto and Related Matters, by Hugh Darwen and myself (see Appendix G), describes a variety of approaches to the problem of missing information, all of which avoid the use of, or apparent need for, SQL-style nulls. The present appendix is based on a chapter from that book, and it describes one of those approaches in detail. The approach in question is known as the decomposition approach, because it involves decomposing, in a variety of ways, relvars that might appear to require nulls (or something like them) into ones that don't. In other words, the emphasis is on designing the database in such a way as to avoid a perceived need for nulls. As a consequence, the approach:

- Has no notion of null or any other construct that's allowed to appear anywhere a value is expected and yet isn't itself a value
- Relies exclusively on classical two-valued logic (2VL), instead of three-valued logic (3VL) or, more generally, $n$-valued logic ( $n \mathrm{VL}$ ) for some $n>2$
- Abides by The Information Principle-see Appendix A of this book and elsewhere-in that, at all times, the database contains relations and nothing but relations

Is capable of dealing with missing information of any number of different kinds
Note: Before going any further, I should mention that the approach I'm going to be describing is similar but not identical to one proposed by David McGoveran in 1994 in a series of papers with the overall title "Nothing from Nothing" (again, see Appendix G).

Consider Fig. C. 1 overleaf, which shows a version of our usual suppliers table in which certain information is missing (indicated in the figure, as in Chapters 1 and 4, by shading the pertinent entries). Note that I can't say the figure shows a relation, precisely because of those shaded entries; hence my use of the term table, and the related terms column and row, here and throughout this appendix. Now, I said in Chapter 5 that the predicate for suppliers was as follows:

Supplier SNO is under contract, is named SNAME, has status STATUS, and is located in city CITY.
For present purposes, however, I'll simplify this predicate slightly by dropping the bit about the supplier being under contract. The predicate becomes:

Supplier SNO is named SNAME, has status STATUS, and is located in city CITY.

| SNO | SNAME | STATUS | CITY |
| :---: | :---: | :---: | :---: |
| S1 | Smith | 20 | London |
| S2 | Jones | 10 |  |
| S3 | Blake | §/4alald | Paris |
| S4 | Clark |  | . $\sim$ |

Fig. C.1: Table S-sample value

This predicate is at best approximate, however. It would be appropriate if it weren't for those shaded entries. After all, the following-obtained from the predicate by substituting values from the row in Fig. C. 1 for supplier S1-is a meaningful instantiation of it (i.e., it's a meaningful proposition):

Supplier S1 is named Smith, has status 20, and is located in city London.
But if we substitute values from the row for, say, supplier S2, we obtain:
Supplier $S 2$ is named Jones, has status 10, and is located in city $\qquad$
And this certainly isn't a meaningful instantiation or proposition; in fact, it doesn't make sense at all.
Another interesting question is: What are the data types for columns STATUS and CITY? (I'm assuming here for the sake of the example, and I'll continue to assume throughout the rest of this appendix, that shaded entries don't appear, and won't ever appear, in the other two columns, SNO and SNAME.) In SQL in particular, the shaded entries in columns STATUS and CITY can be interpreted as meaning the pertinent entries are null; elsewhere in this book, however, I've said the (SQL) data types of those columns are INTEGER and VARCHAR(20), respectively, and null certainly isn't a value of either type INTEGER or type VARCHAR(20). In fact, of course, null isn't a value at all, and so it can't be said to be of any type at all. ${ }^{1}$

From this preliminary discussion, it should be clear that what we need to do is get rid of those shaded entries. Two kinds of decomposition, vertical and horizontal, can be used to achieve this goal.

## VERTICAL DECOMPOSITION

The first step in the process of getting rid of those shaded entries is to apply vertical decomposition to produce a set of tables with the property that no table ever has more than one column with any such entries. (Note: Vertical decomposition-vertical because the dividing lines in the decomposition are between columns, so to speak-is essentially what we do when we do classical normalization.) For the table in Fig. C.1, the result of this step is tables SN, ST, and SC as shown in Fig. C.2. (For obvious reasons I use T, not S, as an abbreviation for STATUS throughout this appendix.)

[^147]SN

| SNO | SNAME |
| :---: | :---: |
| $=\overline{\text { S1 }}$ | Smith |
| S2 | Jones |
| S3 | Blake |
| S4 | Clark |


| SNO | STATUS |
| :---: | :---: |
| S1 | 20 |
| S2 | 10 |
| S3 |  |
| S 4 | \MAM, |


| SNO | CITY |
| :---: | :---: |
| S1 | London |
| S 2 | «. |
| S 3 | Paris |
| S 4 | \aldals |

Fig. C.2: Vertically decomposing table S

The "obvious" (?) predicates for the tables in Fig. C. 2 are as follows:

- SN: Supplier SNO is named SNAME.
- ST: Supplier SNO has status STATUS.
- SC: Supplier SNO is located in city CITY.

However, the predicates for ST and SC here are still only approximate, because of those shaded entries-and that's why we need horizontal decomposition, which I'll get to in the next section. Note first, however, that each of tables SN, ST, and SC has just two columns. But this state of affairs is a fluke, in a way; it's a direct result of my choice of example. If the example were different-e.g., if we knew that column STATUS, as well as columns SNO and SNAME, will never contain any shaded entries-then the appropriate vertical decomposition would be as shown in Fig. C. 3 below. Note: I've assumed in Fig. C.3, just for the sake of the revised example (but in accordance with our usual sample values), that suppliers S3 and S4 have status 30 and 20, respectively.

SNT

| SNO | SNAME | STATUS |
| :---: | :---: | ---: |
| $=$ S1 | Smith | 20 |
| S2 | Jones | 10 |
| S3 | Blake | 30 |
| S4 | Clark | 20 |

SC

| SNO | CITY |
| :---: | :---: |
| S1 | London |
| S2 |  |
| S3 | Paris |
| S4 |  |

Fig. C.3: Vertically decomposing table S, if every supplier has a known status

## HORIZONTAL DECOMPOSITION

In horizontal decomposition, the dividing lines in the decomposition are between rows (so to speak) instead of between columns. The basic motivation for such decomposition is this: We shouldn't try to use the same table to represent two or more different predicates. For example, consider table SC again as shown in either Fig. C. 2 or Fig. C.3. In that table, the row for supplier S1 means: Supplier S1 is located in London. By contrast, the row for supplier S2 means: We don't know where supplier S2 is located (at any rate, let's agree that's what it means for the
time being). So different rows correspond to different predicates, and the predicate I gave for SC earlier-Supplier SNO is located in city CITY-doesn't in fact apply to every row.

Now, we might try a different predicate, perhaps like this (note the OR, which I've shown in uppercase bold for emphasis):

Supplier SNO is located in city CITY OR we don't know where supplier SNO is located.
But this predicate doesn't work either. If we try to instantiate it with values (or "values," rather) from the row for supplier S2, we get:

Supplier $S 2$ is located in city $\approx \not \approx \otimes \not \approx \otimes \otimes \boldsymbol{O R}$ we don't know where supplier $S 2$ is located.
 $\prod_{\pi / \pi}^{\infty} \sim \pi$ isn't a legitimate city name and can't legitimately be substituted as an argument for the CITY parameter in the putative predicate. So what we need to do is break the predicate into two separate pieces, as it were (more precisely, we need to break the two disjuncts apart); that is, we need to apply horizontal decomposition to table SC, to obtain one table for each of those disjuncts. The result of this step is the tables shown in Fig. C.4:


Fig. C.4: Horizontally decomposing table SC
As you can see, we now have two tables: (a) an abbreviated version of table SC (for which I've retained the name SC, for convenience), containing just the original SC rows that had no shaded entries in column CITY; and (b) another table SUC (suppliers with an unknown city), containing just the original SC rows that did have shaded entries in column CITY-except that the CITY column in that table, if we kept it, would contain nothing but shaded entries, and so we can discard it without losing any information. The predicates for these tables are as follows:

- SC: Supplier SNO is located in city CITY.

■ SUC: We don't know where supplier SNO is located.
Observe in particular that the predicate for (this revised version of) table SC has two parameters, SNO and CITY, and that table has two columns accordingly; by contrast, the predicate for table SUC has just one parameter, SNO, and that table has just one column accordingly.

Of course, we can and should perform an analogous horizontal decomposition on table ST from Fig. C.2. The result is shown in Fig. C.5:


Fig. C.5: Horizontally decomposing table ST
The predicates for the tables in Fig. C. 5 are as follows:
■ ST: Supplier SNO has status STATUS.
■ SUT: We don't know supplier SNO's status.

## WHAT DO THE SHADED ENTRIES MEAN?

Let's ignore status values for the moment and concentrate on cities. So far, then, I've said that shaded entries in the CITY column (as shown in, e.g., Fig. C.2) mean we don't know the applicable supplier city-i.e., the supplier does have a city, but we don't know what it is. But our not knowing is only one of many possible reasons why we might not be able to use a genuine city name as some entry in that column. For example, it might be that the notion of having a city simply doesn't apply to some suppliers (perhaps they conduct their business entirely online). If so, we might say, very loosely, that table SC, with those shaded entries in the CITY column (i.e., table SC as shown in Fig. C.2), has a predicate looking something like this:

Supplier SNO is located in city CITY OR we don't know where supplier SNO is located OR supplier SNO isn't located anywhere.

Note, therefore, that those shaded entries now potentially have two distinct interpretations: Some of them mean we don't know the applicable city, others mean the property of having a city doesn't apply. So, again, we apply horizontal decomposition, this time to obtain three tables: SC (suppliers with a known city), SUC (suppliers with an unknown city), and SNC (suppliers with no city). The predicates are:

- SC: Supplier SNO is located in city CITY.
- SUC: We don't know where supplier SNO is located.
- SNC: Supplier SNO doesn't have a location.

If we assume for the sake of the example that supplier S 2 has an unknown city and supplier S 4 doesn't have a city at all, the result of this decomposition is as shown in Fig. C.6:


Fig. C.6: Horizontally decomposing table SC, allowing for suppliers with no city
In other words, the decomposition approach allows us to represent as many different kinds of missing information as we like. To be specific, if there are $n$ distinct reasons for supplier cities to be missing, there'll be $n+1$ tables having to do with suppliers and cities. Two possible objections to the approach thus immediately spring to mind:

1. Aren't some queries going to get awfully complex? For example, suppose we just want to retrieve everything in the database having to do with suppliers (the analog of SELECT * FROM S in SQL); aren't we going to have to do a lot of joins, or (worse) outer joins?
2. Aren't we going to wind up with an awful lot of tables?

I'll come back to the first of these issues in the section "Queries," later. As for the second, well, there are several points I want to make. Let $C$ be an SQL column for which nulls are allowed. Then:

- If the nulls in column $C$ all represent the same kind of missing information, and if the same is true for all such columns $C$, then the number of tables resulting from the decomposition approach is exactly the same as the number resulting from a good relational design. (To paraphrase something I said earlier, the presence of such a column $C$ in a table $T$ means table $T$ is certainly not a relational table. Proper relational design requires elimination of such columns.)
- The situation is worse if the nulls in some such column $C$ represent two or more distinct kinds of missing information but proper decomposition isn't done. If it isn't, there'll certainly be fewer tables-but the apparent simplicity of such a design is spurious: Those tables aren't relational, they don't faithfully reflect the real world, they no longer have a clear predicate, and queries are more susceptible to errors of formulation or errors of interpretation or both.
- There's a tactic we might consider, if we want to reduce the number of tables, which I'll illustrate with reference to Fig. C.6. In terms of that example, the tactic would involve combining tables SUC and SNC into a single table with two columns, SNO and REASON, where REASON indicates the reason why the applicable supplier has no recorded city:

| SNO | REASON |
| :--- | :--- |
| $=$S2 <br> S4 | $d / k$ |
| $n / a$ |  |

But now we have to define appropriate values, and spell out their interpretations, for column REASON (in the example, I've used $d / k$ for "don't know" and $n / a$ for "not applicable"). In fact, if the decomposition approach requires $n$ missing information tables, the combination approach requires $n$ missing information reasons. So the combination approach is in some respects no less complex than the decomposition approach.

## CONSTRAINTS

So far, then, our suggested overall design for the running example looks like Fig. C. 7 below.

SN

| SNO | SNAME |
| :---: | :---: |
| $=$S1 <br> S2 <br> S3 <br> S4 | Smith |
|  | Blake |
|  | Clark |

ST

| SNO | STATUS |
| :---: | ---: |
| S1 | 20 |
| S2 | 10 |

SC

| SNO | CITY |
| :--- | :--- |
| $\overline{\text { S1 }}$ | London <br> S3 <br> Paris |

SUT

| SNO |
| :---: |
| S3 |
| S4 |

SUC

| SNO |
| :---: |
| $=$ |

Fig. C.7: Fully decomposing table S
I'm assuming here, and will continue to assume for the rest of this appendix, that there's just one reason why STATUS values might be missing (viz., we don't know the value) and just two reasons why CITY values might be missing (viz., either we don't know the value or no such value exists). Note, however, that the design of Fig. C. 7 requires certain constraints to be satisfied in order to hold it together, so to speak. To be specific, the following constraints need to be stated and enforced:

1. Each table has $\{\mathrm{SNO}\}$ as a key.
2. Each row in SN has a matching row in exactly one of ST and SUT, and conversely.
3. Each row in SN has a matching row in exactly one of SC, SUC, and SNC, and conversely.

Of course, the first of these is just a conventional key constraint on each of the six tables; it can thus be expressed by means of conventional KEY specifications. As for the other two, they can easily be expressed in Tutorial D using D_UNION, as follows: ${ }^{2}$

[^148]```
CONSTRAINT EQD2
    SN { SNO } = D_UNION { ST { SNO } , SUT { SNO } } ;
CONSTRAINT EQD3
    SN { SNO } = D_UNION { SC { SNO } , SUC { SNO } , SNC { SNO } } ;
```

Aside: Actually it might not be a good idea to use D_UNION in a constraint as I've just done. After all, if some update violates the constraint in question, we don't want a run-time error, we just want the constraint to evaluate to FALSE and the update to be rejected. So constraint EQD2, for example, might better be formulated as follows:

```
CONSTRAINT EQD2 ARE_DISJOINT { ST { SNO } , SUT { SNO } } AND
    SN { SNO } = UN\overline{ION { ST { SNO } , SUT { SNO } } ;}
```

(The operator ARE_DISJOINT is defined to give FALSE if and only if two or more of its argument relations have any tuples in common.) End of aside.

## QUERIES

Now I return to the question I raised earlier: Given a design like that of Fig. C.7, aren't some queries going to get awfully complex? In particular, what's involved with that design in doing a query analogous to the "simple" SQL query SELECT * FROM S?

Before I address that issue, let me first point out that some queries-queries, I venture to suggest, that are more likely to be needed in practice than ones like SELECT * FROM S—are actually easier to formulate with the design of Fig. C.7. As a trivial example, the query "For suppliers for whom CITY is both applicable and known, get supplier numbers and cities" becomes just-

```
SELECT SNO , CITY
FROM SC
```

-instead of:

```
SELECT SNO , CITY
FROM S
WHERE CITY IS NOT NULL
```

What's more, the query "Get suppliers for whom CITY is applicable but unknown" is not only simpler with the design of Fig. C.7, it can't be done at all with the original design of Fig. C.1. (In other words, not only does the design of Fig. C. 1 not deal very well with the missing information problem in general, it actually manages to lose information!)

Be that as it may, let's now consider the "SELECT * FROM S" question. More precisely, let's see how a respectable version of the table in Fig. C. 1 can be obtained from those in Fig. C.7-where by respectable, I mean the table will contain proper and informative data values everywhere (no shaded entries! no nulls!), as indicated in Fig. C. 8 below.
S

| SNO | SNAME | XSTATUS | XCITY |
| :--- | :--- | :--- | :--- |
| $=\mathbf{S 1}$ | Smith | 20 | London |
| S2 | Jones | 10 | $d / k$ |
| S3 | Blake | $d / k$ | Paris |
| S4 | Clark | $d / k$ | $n / a$ |

Fig. C.8: Revised (respectable) version of table S
Now, however, I'll switch to Tutorial D (doing the example in SQL would make it too hard to see the forest for the trees). I'll show the solution a step at a time, using the values from Fig. C. 7 as a basis for illustrating the result of each step in turn; then I'll bring all the steps together at the end.

1. WITH ( T1 := EXTEND ST : \{ XSTATUS := CAST_AS_CHAR ( STATUS ) \} ) :

T1

| SNO | STATUS | XSTATUS |
| :---: | ---: | :--- |
| S1 | 20 | 20 |
| S2 | 10 | 10 |

```
/* STATUS values are integers; */
/* XSTATUS values are character strings */
```

2. WITH ( T2 := T1 \{ ALL BUT STATUS \} ) :

## T2

| SNO | XSTATUS |
| :--- | :--- |
| $=$S1 20 <br> S2 10${ }^{2}$ |  |

3. WITH ( T3 := EXTEND SUT : \{ XSTATUS := $\left.\mathfrak{d} / k^{\prime}\right\}$ ) :

T3

| SNO | XSTATUS |
| :--- | :--- |
| S3 | $d / k$ |
| S4 | $d / k$ |

4. WITH ( T4 : = UNION \{T2 , T3 \} ) :

T4

| SNO | XSTATUS |
| :--- | :--- |
| $=$S1 20 <br> S2 10 <br> S3 $d / k$ <br> S4 $d / k$${ }^{2}$ |  |

5. WITH ( T5 := SC RENAME \{ CITY AS XCITY \} ) :

T5

| SNO | XCITY |
| :---: | :--- |
| $=\overline{\text { S1 }}=$ | London |
| S3 | Paris |

6. WITH ( T6 := EXTEND SUC : \{ XCITY := 'd/k' \} ) :

T6

| $=\frac{\text { SNO }}{}$ | XCITY |
| :--- | :--- |
| S2 | $d / k$ |

7. WITH ( T7 := EXTEND SNC : \{XCITY $:=$ ' $\left.n / a^{\prime}\right\}$ ) : T7

| SNO | XCITY |
| :---: | :--- |
| $=\frac{\text { S4 }}{}$ | $n / a$ |

8. WITH ( T8 := UNION \{ T5 , T6 , T7 \} ) :

T8

| SNO | XCITY |
| :---: | :--- |
| S1 | London |
| S2 | d/k |
| S3 | Paris |
| S4 | $n / a$ |

9. WITH ( $\mathrm{S}:=$ JOIN $\{\mathrm{SN}, \mathrm{T} 4, \mathrm{~T} 8\}$ ) $: \mathrm{S}$

S

| SNO | SNAME | XSTATUS | XCITY |
| :--- | :--- | :--- | :--- |
| S1 | Smith | 20 | London |
| S2 | Jones | 10 | $d / k$ |
| S3 | Blake | $d / k$ | Paris |
| S4 | Clark | $d / k$ | $n / a$ |

Putting all of these steps together and simplifying slightly:

```
WITH ( T1 := EXTEND ST : { XSTATUS := CAST_AS_CHAR ( STATUS ) } ,
    T2 := T1 { ALL BUT STATUS } ,
    T3 := EXTEND SUT : { XSTATUS := `d/ k' } ,
    T4 := UNION {T2 , T3 } ,
    T5 := SC RENAME { CITY AS XCITY } ,
    T6 := EXTEND SUC : { XCITY := ' d/k' } ,
    T7 := EXTEND SNC : { XCITY := 'n/a' } ,
    T8 := UNION {T5 ,T6, T7 } ,
    S := JOIN { SN , T4 , T8 } ) :
S
```

Now, it's certainly true that this expression looks a little complicated (or tedious, at any rate), and it would look even more so if I hadn't formulated it a step at a time, using WITH. However:

- Various shorthands could be defined, if desired, that could be used to simplify it.
- I frankly doubt whether tables such as that in Fig. C. 8 would ever be wanted much in practice anyway, except perhaps as the basis for some kind of periodic report.
- In any case, the complexity, such as it is, can always be concealed by making the table a view.


## MORE ON PREDICATES

In this section, ${ }^{3}$ I show how it's possible to get "don't know" answers out of a database without nulls, even if there aren't any tables like table SUC (suppliers with an unknown city) that explicitly represent the fact that something is unknown. For simplicity, suppose our database consists in its entirety of just table SC (suppliers with a known city), as shown in Fig. C. 9 below.

[^149]

Fig. C.9: Table SC (suppliers with a known city)
Now consider the following query on table SC:

## Is supplier S1 in London?

## In Tutorial D: ${ }^{4}$

```
( SC WHERE SNO = 'S1' AND CITY = 'London' ) { }
```

Clearly, this expression evaluates to either TABLE_DEE or TABLE_DUM: TABLE_DEE if supplier S1 is in London, TABLE_DUM otherwise. Note, therefore, that-as I mentioned in Chapter 3-TABLE_DEE and TABLE_DUM can be interpreted as yes and no, respectively. Note too the implicit appeal to The Closed World Assumption!-in effect, we're saying that if the row (S1,London) fails to appear in table SC, we're allowed to conclude that it's not the case that supplier S1 is in London.

Now, I said previously that the predicate for table SC was Supplier SNO is located in city CITY. But it isn't-not really. To see why not, consider what happens if some user tries to introduce a new row into the table, perhaps as follows:

INSERT SC RELATION \{ TUPLE \{ SNO 'S6' , CITY 'Madrid' \} \} ;
In effect, the user here is telling the system there's a new supplier, S6, with city Madrid. Now, the system obviously has no way of knowing whether the user's assertion is true; as explained in Chapter 8, all it can do (and does do) is check that the requested insertion, if performed, doesn't cause any integrity constraints to be violated. If it doesn't, then the system accepts the row, and interprets it as representing a true fact from this point forward.

We see, therefore, that rows in table SC don't necessarily represent actual states of affairs in the real world; rather, they represent what the user tells the system about the real world, or in other words the user's knowledge of the real world. Thus, the predicate for relvar S isn't really just Supplier SNO is located in city CITY; rather, it's We know that supplier SNO is located in city CITY. And the effect of a successful INSERT is to make the system aware of something the user already knows. Thus, the database doesn't contain the real world (of course not); what it contains is, rather, the system's knowledge of the real world. And the system's knowledge is derived in turn from the user's knowledge (of course!-there's no magic here). ${ }^{5}$

So when we pose a query to the system, by definition that query can't be a query about the real world; instead, it is-it has to be-a query about the system's knowledge of the real world. For example, consider again the

[^150]query discussed above: Is supplier S1 in London? This rather imprecise natural language formulation has to be understood as shorthand for the following more accurate one:

Do we know that supplier S1 is in London?
In practice, of course, we almost never talk in such precise terms; we usually elide qualifiers like "Do we know that" (or "According to the system's knowledge, is it true that," or "Does the database say that," and so on). But even if we do elide them, we certainly need to understand that, conceptually, they're there-for otherwise we'll be really confused. (Though perhaps I should add that such confusions aren't exactly unknown in practice.)

It follows from the foregoing discussion that the Tutorial D expression I showed earlier-

```
( S WHERE SNO = 'S1' AND CITY = 'London' ) { }
```

-doesn't really represent the query Is supplier S1 in London? after all. Rather, it represents the query Do we know that supplier S1 is in London? And, appealing again to The Closed World Assumption, it follows further that:

- If the result is TABLE_DEE (yes), it means we do know supplier S1 is in London.
- If the result is TABLE_DUM (no), it means we don't know whether supplier S 1 is in London. And that's a "don't know" answer if ever you saw one.

Of course, if a row for supplier S1 does appear in the table but the CITY value in that row isn't London, we know supplier S1 isn't in London (I'm appealing here to the fact that $\{\mathrm{SNO}\}$ is a key for table SC). Putting it all together, then, we have the following:

- If a row for supplier S1 appears in table SC and the CITY value in that row is London, it means yes, we know supplier S1 is in London.
- If a row for supplier S 1 appears in table SC but the CITY value in that row is something other than London, it means no, we know supplier S1 isn't in London.
- And if no row for supplier S1 appears in table SC at all, it means we don't know whether supplier S1 is in London.

Given The Closed World Assumption, then, we can formulate queries that return a true / false / don't know answer, and we don't need nulls or 3VL to do so. Here's a Tutorial D formulation for the example under discussion:

```
( EXTEND ( S WHERE SNO = 'S1' AND CITY = 'London' ) { } :
    { RESULT := 'true , } ) { RESULT }
    UNION
( EXTEND ( S WHERE SNO = 'S1' AND CITY #= 'London' ) { } :
    { RESULT := 'false , } ) { RESULT }
    UNION
( EXTEND ( RELATION { TUPLE { SNO 'S1' } } MINUS S { SNO } ) { } :
    { RESULT := 'unknown' } ) { RESULT }
```

As you can see, this expression takes the form $a$ UNION $b$ UNION $c$, where each of $a$, $b$, and $c$ is a table of just one column. Moreover, it should be clear that exactly one of $a, b$, and $c$ contains just one row and the other two
contain no rows at all. The overall result is thus a one-column, one-row table; the single column, RESULT, is of type character string, and the single row contains the appropriate RESULT value. And the trick-though it isn't really a trick at all-is that the RESULT value is a character string, not a truth value. As a consequence, there's no need to get into the 3 VL quagmire in order to formulate queries that can yield true, false, or unknown answers, if that's what we really want.

For completeness, here's an SQL analog of the foregoing Tutorial D expression:

```
SELECT 'true , AS RESULT
FROM ( SELECT S.*
        FROM S
        WHERE SNO = 'S1'
        AND CITY = 'London' ) AS POINTLESS1
UNION CORRESPONDING
SELECT 'false , AS RESULT
FROM ( SELECT S.*
        FROM S
        WHERE SNO = 'S1'
        AND CITY <> 'London' ) AS POINTLESS2
UNION CORRESPONDING
SELECT 'unknown' AS RESULT
FROM ( VALUES ( 'S1' )
    EXCEPT
    SELECT S.SNO
    FROM S ) AS POINTLESS3
```

Incidentally, if you're wondering about those AS POINTLESS specifications in this SQL expression, I remind you that SQL has a syntax rule to the effect that a subquery in the FROM clause must be accompanied by an explicit AS clause that defines an associated range variable, even if that range variable is never explicitly referenced anywhere in the overall expression. Note also that specifying CORRESPONDING on the EXCEPT in the final portion of this expression would actually be incorrect! It could be made correct by replacing the specification VALUES ('S1') by an expression of the form SELECT DISTINCT 'S1' AS SNO FROM $T$ where $T$ is some arbitrary-but necessarily nonempty-named table.

## EXERCISES

C. 1 Give SQL versions of constraints EQD2 and EQD3 from the section "Constraints" in the body of the appendix.
C. 2 Give an SQL version of the Tutorial D expression near the end of the section Queries in the body of the appendix.
C. 3 Why would it be incorrect to specify CORRESPONDING on the EXCEPT in the final portion of the SQL expression at the end of the section immediately preceding these exercises?

## Appendix D

## A Tutorial D Grammar

For purposes of reference, this appendix gives a BNF grammar for Tutorial D relational expressions and assignments (nonrelational operations are mostly omitted, as are definitional operations such as those used to define types, base relvars, views, and constraints). The following are also omitted:

- TUPLE FROM, because it doesn't return a relation
- THE _ operators and <attribute name> FROM, because these operators too don't return relations (except in the unusual special case where the specified possrep component or attribute, as applicable, happens to be relation valued)
- DIVIDEBY and SUMMARIZE, because (as explained in Chapter 7) these operators are both somewhat deprecated

Also, the grammar is simplified in certain respects. In particular, it makes no attempt to say where image relations can and can't be used, nor does it pay any attention to operator precedence rules. (As a result of this latter point, certain constructs permitted by the grammar-for example, the expression $r l$ MINUS $r 2$ MINUS $r 3$-are potentially ambiguous. Additional syntax rules are needed to resolve such issues, but such rules are omitted here. Of course, parentheses can always be used to guarantee a desired order of evaluation anyway.) A few points of detail:

- The shorthand $\exp$ is used as an abbreviation for expression.
- All syntactic categories of the form <... name $>$ are assumed to be $<$ identifier $>$ s and are defined no further here.
- The categories <tuple exp> and <bool exp> are also left undefined-though it might help to recall in particular that a relational comparison is a special case of a boolean expression.
- As usual, all of the various commalists mentioned in what follows are allowed to be empty.


## Relational Expressions

```
<relation exp>
    ::= <with exp> | <nonwith exp>
<with exp>
    ::= WITH ( <name intro commalist> ) : <relation exp>
<name intro>
    ::= <relvar name> := <relation exp>
<nonwith exp>
    ::= <image exp> | <relation op> | ( <relation op> )
```

```
<image exp>
    ::= !!<nonwith exp> | ( <image exp> )
<relation op>
    ::= <relation selector> | <monadic op> | <dyadic op> | <n-adic op>
<relation selector>
    ::= RELATION [ <heading> ] { <tuple exp commalist> }
        | TABLE_DUM | TABLE_DEE
<heading>
    ::= { <attribute commalist> }
<attribute>
    ::= <attribute name> <type name>
<monadic op>
    ::= <relvar name> | <rename> | <where> | <project>
        | <extend> | <group> | <ungroup> | <tclose>
<rename>
    ::= <relation exp> RENAME { <renaming commalist> }
<renaming>
    ::= <attribute name> AS <attribute name>
<where>
    ::= <relation exp> WHERE <bool exp>
<project>
    ::= <relation exp> { [ ALL BUT ] <attribute name commalist> }
<extend>
    ::= EXTEND <relation exp> : { <attribute assign commalist> }
<attribute assign>
    ::= <attribute name> := <exp>
```

Note: An alternative form of <attribute assign>, syntactically identical to a <relation assign> except that the pertinent <attribute name $>$ appears in place of the target $<$ relvar name $>$ in that $<$ relation assign $>$, is also supported if the attribute in question is relation valued.

```
<group>
    ::= <relation exp> GROUP
        ( { [ ALL BUT ] <attribute name commalist> }
                        AS <attribute name> )
<ungroup>
    ::= <relation exp> UNGROUP ( <attribute name> )
<tclose>
    ::= TCLOSE ( <relation exp> )
<dyadic op>
    ::= <relation exp> <dyadic op name> <relation exp>
```

```
<dyadic op name>
    ::= UNION | D_UNION | XUNION | INTERSECT | MINUS | I_MINUS
        | JOIN | TI\overline{MES | MATCHING | NOT MATCHING}
<n-adic op>
    ::= <n-adic op name> [ <heading> ] { <relation exp commalist> }
<n-adic op name>
    ::= UNION | D_UNION | XUNION | INTERSECT | JOIN | TIMES
<relation comp>
    ::= <relation exp> <relation comp op> <relation exp>
<relation comp op>
    ::= = | | | \subseteq | \subset | \ | つ
```


## Assignments

```
<relation assignment>
    ::= [ WITH ( <name intro commalist> ) : ]
                                <relation assign commalist> ;
<relation assign>
    ::= <relvar name> := <relation exp>
        | <insert> | <d_insert>| <delete> | <i_delete>| <update>
<insert>
    ::= INSERT <relvar name> <relation exp>
<d_insert>
    ::= D_INSERT <relvar name> <relation exp>
<delete>
    ::= DELETE <relvar name> <relation exp>
        | DELETE <relvar name> [ WHERE <bool exp> ]
<i_delete>
    ::= I_DELETE <relvar name> <relation exp>
<update>
    ::= UPDATE <relvar name> [ WHERE <bool exp> ] :
                                    { <attribute assign commalist> }
```


## Appendix E

## Summaryof Recommendations

In this appendix I present for purposes of quick reference a brief summary of the recommendations from Chapters 1-12. The page numbers against the various items show where the individual recommendations are discussed in the body of the text.

■ Don't use SQL like a simple access method. (Page 13)

- Avoid the use of any SQL construct that references physical access paths such as indexes. (Pages 13-14)

■ Don't use table to mean a base table specifically unless your intended meaning is clear from the context. Don't think of views as if they were somehow different from tables. (Page 19)

- Avoid coercions wherever possible. (Page 41)
- Ensure that columns with the same name are of the same type. (Page 41)
- Avoid type conversions where possible. When they can't be avoided, do them explicitly if you can. (Pages 41-42)
- Don't use PAD SPACE. (Page 43)
- Avoid possibly nondeterministic expressions. (Page 43)

■ Don't use "typed tables," reference values, REF types, or any SQL construct related to these features. (Page 45)

■ If you must talk about nulls, call them nulls, not "null values." (Page 51)
■ Don't use the comparison operators " $<$ ", " $<=$ ", " $>$ ", and " $>=$ " on rows of degree greater than one. (Page 55)

- Use AS specifications whenever necessary (and possible) to give proper column names to columns that otherwise (a) wouldn't have a name at all or (b) would have a name that wasn't unique. (Pages 62, 110)
- If two columns represent the same kind of information, give them the same name wherever possible. (Page 63)

Never write code that relies on left to right column positioning. (Pages 63-64)

- Avoid duplicates. Make sure you know when SQL eliminates duplicates without you asking it to; when you do have to ask, make sure you know whether it matters if you don't; when it does matter, specify DISTINCT; and never specify ALL. (Pages 73-74,117)
- Avoid nulls: (Pages 77-78)
a. Specify NOT NULL, explicitly or implicitly, for every column in every base table.
b. Don't use the keyword NULL anywhere other than in the context of such a NOT NULL specification.
c. Don't use the keyword UNKNOWN in any context whatsoever.
d. Don't omit the ELSE clause from a CASE expression unless omitting it makes no logical difference.
e. Don't use NULLIF.
f. Don't use the keywords OUTER, FULL, LEFT, and RIGHT on JOIN (except, just possibly, in connection with COALESCE).
g. Don't use union join.
h. Don't specify PARTIAL or FULL on MATCH; don't use MATCH on foreign key constraints; and don't use IS DISTINCT FROM.
i. Don't use IS TRUE, IS NOT TRUE, IS FALSE, or IS NOT FALSE.
j. Use COALESCE on every scalar expression that might "evaluate to null" without it.
- Don't use DELETE or UPDATE through a cursor unless you can be certain that integrity constraint problems will never arise in connection with such use. (Page 87)
- Avoid operations that are inherently row level (e.g., row level triggers). (Pages 87,110)
- Specify target columns explicitly on INSERT. (Page 90)
- Don't define as a key some column combination that you know not to be irreducible. (Page 92)

■ Use UNIQUE and/or PRIMARY KEY specifications to ensure that every base table has at least one key. (Page 93)

- Ensure that foreign key columns have the same name as the corresponding key columns wherever possible. (Page 95)
- Don't use triggers if they violate The Assignment Principle. (Page 96)

■ Don't use any operation that violates the relational closure property if you want the result to be amenable to further relational processing. (Page 108)

- Use NATURAL JOIN in preference to other methods of formulating a join (but make sure columns with the same name are of the same type). (Page 115)
- If you use JOIN ON, make sure columns with the same name are of the same type, and make sure you rename columns appropriately. (Pages 115-116)
- If you use JOIN USING, make sure columns with the same name are of the same type. (Page 116)
- If you use CROSS JOIN, make sure there aren't any common column names. (Page 116)
- For UNION, INTERSECT, and EXCEPT, make sure corresponding columns have the same name and type. (Page 117)
- For UNION, INTERSECT, and EXCEPT, always specify CORRESPONDING if possible. If it isn't possible, then make sure columns line up properly. Preferably avoid use of the BY option, unless it makes no difference anyway. (Page 117)
- If you use GROUP BY or HAVING, make sure the table you're summarizing is the one you really want to summarize; also, be on the lookout for the possibility that some summarization is being done on an empty set, and use COALESCE wherever necessary. (Page 150)

■ Where possible, use database constraints to make up for SQL's lack of support for type constraints. (Page 173)

- Specify constraints declaratively whenever you can. (Page 179)
- Use immediate checking whenever you can. If checking has to be deferred on some constraint, make sure the check is done before doing any operation that might rely on the constraint being satisfied. (Page 184)
- In CREATE VIEW, don't use the option that allows you to specify the view column names immediately following the view name itself. (Page 194)

■ Specify WITH CASCADED CHECK OPTION on view definitions whenever possible. (Pages 194, 205-206)

- Specify constraints that apply to views (e.g., key constraints) in the form of comments-typically on the view definition. (Page 200)

■ Never use view, unqualified, to mean a snapshot; never use materialized view; and watch out for violations of these recommendations on the part of others. (Page 213)

- Be careful over the use of COUNT; in particular, don't use it where EXISTS would be more logically correct. (Page 242)
- Use the techniques described in Chapter 11, at least for formulating "complex" SQL expressions. (Pages 247ff)
- Don't use ALL or ANY comparisons. (Page 260)

■ Don't use "SELECT *" at the outermost level in a cursor definition or view definition. (Page 273)

- Favor the use of explicit range variables, especially in "complex" expressions. (Page 277)

Well ... after this rather lengthy list of admonitions, it seems only right to close this appendix by reminding you of what in Chapter 1 I called the overriding rule:

## You can do what you like, so long as you know what you're doing.

## Appendix F

## Answers to Exercises

Note: All mistakes in this appendix are deliberate <joke>.

## CHAPTER 1

1.1 Here are a few examples of statements from the early part of the chapter in which every occurrence of the term relation (highlighted here in bold) should be replaced by the term relvar:

- "[Every] relation has at least one candidate key."

■ "[A] foreign key is a combination, or set, of attributes $F K$ in some relation $r 2$ such that each $F K$ value is required to be equal to some value of some key $K$ in some relation $r 1$ ( $r 1$ and $r 2$ not necessarily distinct)."

- "[The] relational assignment operator ... allows the value of some relational expression ... to be assigned to some relation."
- "A view (also known as a virtual relation) is a named relation whose value at any given time $t$ is the result of evaluating a certain relational expression at that time $t$."

And so on.
1.2 E. F. Codd (1923-2003) was the inventor of the relational model, among many other things. In December 2003 I published a brief tribute to him and his achievements, which you can find on the ACM SIGMOD website www.acm.org/sigmod and elsewhere. Note: An expanded version of that tribute appears in my book Date on Database: Writings 2000-2006 (Apress, 2006).
1.3 A domain can be thought of as a conceptual pool of values from which actual attributes in actual relations take their actual values. In other words, a domain is a type, and the terms domain and type are effectively interchangeable-but personally I much prefer type, as having a longer pedigree (in the computing world, at least), as well as being slightly more succinct. Domain is the term used in most of the older database literature, however. Note: Don't confuse domains as understood in the relational world with the construct of the same name in SQL, which can be regarded at best as a very weak kind of type (see Chapter 2-in particular, the answer to Exercise 2.1 later in this appendix). Also, be aware that some older writings (including certain very early ones by myself) unfortunately and mistakenly use the term domain when what they really mean is attribute. Be on your lookout for confusion in this area.
1.4 A database satisfies the referential integrity rule if and only if for every tuple containing a reference (i.e., a foreign key value) there exists a referent (i.e., a tuple in the pertinent "target" relvar with that same value as a value for the pertinent target key). Loosely: If $B$ references $A$, then $A$ must exist. See Chapters 5 and 8 for further discussion.
1.5 Let $R$ be a relvar. Then every relation $r$ that can legally be assigned to $R$ must have the same heading, and hence a fortiori the same attributes and the same degree (see Chapters 2 and 3 for further discussion); and the heading, attributes, and degree of $R$ are, respectively, the heading, attributes, and degree of every such relation $r$. They can therefore (and in practice always are) specified as part of the definition of $R$.

Now let the relation that's assigned to $R$ at some particular time $t$ be $r$. Then the body, tuples, and cardinality of $R$ at that time $t$ are, respectively, the body, tuples, and cardinality of $r$. Note that the body, tuples, and cardinality of a relvar vary over time, while the heading, attributes, and degree don't.

By the way, it follows from the foregoing that if we use SQL's ALTER TABLE to add a column to or drop a column from some base table $T$, then the effect, logically speaking, is to replace that table $T$ by some distinct table $T^{\prime}$ (the term table being, in such contexts, SQL's counterpart to the relational term relvar). $T^{\prime}$ ' is not "the same table as before"-speaking purely from a logical point of view, that is. But it's convenient to overlook this nicety in informal contexts.

### 1.6 See the section "Model vs. Implementation" in the body of the chapter.

1.7 (a) Physical data independence is the independence of users and application programs from the way the data is physically stored and accessed. It's a logical consequence of keeping a rigid separation between the model and its implementation. To the extent that such separation is observed, and hence to the extent that physical data independence is achieved, we have the freedom to make changes to the way the data is physically stored and accessed-typically for performance reasons-without at the same time having to make corresponding changes in queries and application programs. Such independence is desirable because it translates into protecting investment in training, applications, and logical database designs.
(b) The model is the abstract machine with which users interact; the implementation is the realization of that abstract machine on some physical computer system. Users have to understand the model, since it defines the interface they have to deal with; they don't have to understand the implementation, because that's under the covers (at least, it should be). The following analogy might help: In order to drive a car, you don't have to know what goes on under the hood-all you have to know is how to steer, how to shift gear, and so on. So the rules for steering, shifting, and the rest are the model, and what's under the hood is the implementation. (It's true that you might drive better if you do have some understanding of what goes on under the hood, but you don't have to know. Analogously, you might use a data model better if you have some knowledge of how it's implemented-but ideally, at least, you shouldn't have to know.) Note: The term architecture is sometimes used with a meaning very similar to that of model as defined here.
1.8 Rows in tables are ordered top to bottom but tuples in relations aren't; columns in tables are ordered left to right but attributes in relations aren't; tables might have duplicate rows but relations never have duplicate tuples. Also, relations contain values, but tabular pictures don't (they don't even contain "occurrences" or "appearances" of such values); rather, they contain symbols that denote such appearances-for example, the symbol (or numeral) 5 , which denotes an appearance of the value five. See the answer to Exercise 3.5 later in this appendix for several further differences.

### 1.9 No answer provided.

1.10 Throughout this book I use the term relational model to mean the abstract machine originally defined by Codd (though that abstract machine has been refined, clarified, and extended somewhat since Codd's original vision). I don't use the term to mean just a relational design for some particular database. There are lots of relational models in the latter sense but only one in the former sense. (As noted in the body of the chapter, you can find quite a lot more on this issue in Appendix A.)

### 1.11 Here are some:

- The relational model has nothing to say about "stored relations" at all; in particular, it categorically doesn't say which relations are stored and which not. In fact, it doesn't even say that relations as such have to be stored-there might be a better way to do it (and indeed there is, though the specifics are beyond the scope of this book).
- Even if we agree that the term "stored relation" might make some kind of sense-meaning a user visible relation that's represented in storage in some direct and efficient manner, without getting too specific on just what direct and efficient might mean-which relations are "stored" should be of no significance whatsoever at the relational (i.e., user) level of the system. In particular, the relational model categorically does not say that "tables" (meaning, more specifically, base tables, or rather base relations) are stored and views aren't.
- The extract quoted doesn't mention the crucial logical difference between relations and relvars.
- The extract also seems to assume that table and base table are interchangeable terms (and concepts)-a very serious error, in my opinion.
- The extract also seems to distinguish between tables and relations (and/or relvars). If "table" means, specifically, an SQL table, then I certainly agree there are some important distinctions to be observed, but they're not the ones the extract seems to be interested in.
- "[It's] important to make a distinction between stored relations ... and virtual relations": Actually, it's extremely important from the user's perspective (and from the perspective of the relational model, come to that) not to make any such distinction at all!
1.12 Here are a few things that are wrong with it:
- The relational model as such doesn't "define tables" at all, in the sense meant by the extract quoted. It doesn't even "define" relations (or relvars, rather). Instead, such definitions are supplied by some user. And anyway: What's a "simple" table? Are there any complex ones?

What on earth does the phrase "each relation and many to many relationships" mean? What does it mean to "define tables" for such things?

- The following concepts aren't part of the model, so far as I know: entities, relationships between entities, linking tables, "cross-reference keys." (It's true that Codd's original model had a rule called "entity integrity," but that name was only a name, and I reject that rule in any case.) It's also true that it's possible to put some charitable interpretations on all of these terms, but the statements that result from such interpretations are usually wrong. For example, relations don't always represent "entities" (what "entity" is represented by the relation that's the projection of suppliers on \{STATUS,CITY\}?).
- Primary and secondary indexes and rapid access to data are all implementation notions-they're nothing to do with the model. In particular, primary (or, more generally, candidate) keys shouldn't be equated with "primary indexes."

■ "Based upon qualifications"? Would it be possible to be a little more precise? It's truly distressing, in the relational context above all others (where precision of thought and articulation was always a key objective),
to find such dreadfully sloppy phrasing. Well, yeah, you know, a relation is kind of like a table, or a kind of a table, or something ... if you see what I mean.

- Finally, what about the operators? It's an all too common error to think the relational model has to do with structure only and to forget about the operators. But the operators are crucial! As Codd himself once observed: "Structure without operators is ... like anatomy without physiology."

As a kind of postscript to the foregoing, I remark that the relational model certainly seems to have received more than its fair share of misunderstanding or misrepresentation in the literature over the years. Here are a few more quotes to illustrate the point:

- "Relational model: A scheme for defining databases in which data elements are organized into relations, typically viewed as rows in tables." As I wrote when I first had occasion to comment on this "definition" (which appears in a book on object technology): Never mind the inaccuracies-you mean that's $i t$ ? What about the operators? What about integrity? What about declarative query? What about views? What about the model's set level nature? What about optimization? And so on and so forth.
- "A newer form of database manager, the relational model, ... [removes] information about complex relationships from the database ... Although the relational model is much more flexible than its predecessors, it pays a price for this flexibility. The information about complex relationships that was removed from the database must be expressed as procedures in every program that accesses the database, a clear violation of the independence required for modularity." I'll leave comments on this one to you (it's taken from that same book on object technology).
- "Consider a data relationship in which a part can have multiple suppliers and vice versa ... There are two base tables: a part table and a supplier table. Then there is a cross-reference table from part to supplier and another cross-reference table from supplier to part" (emphasis added). This quote is from what has to be one of the worst textbooks I've ever read.
1.13 Here are some possible CREATE TABLE statements. Regarding the column data types, see Chapter 2. Note: These CREATE TABLE statements, along with their Tutorial D counterparts, are repeated in Chapter 5, where further pertinent discussion can be found. See also the answer to Exercise 2.15 later in this appendix.

```
CREATE TABLE S
    ( SNO VARCHAR(5) NOT NULL,
        SNAME VARCHAR(25) NOT NULL ,
        STATUS INTEGER NOT NULL ,
        CITY VARCHAR(20) NOT NULL,
        UNIQUE ( SNO ) ) ;
CREATE TABLE P
    ( PNO VARCHAR(6) NOT NULL ,
        PNAME VARCHAR(25) NOT NULL,
        COLOR CHAR(10) NOT NULL,
        WEIGHT NUMERIC(5,1) NOT NULL,
        CITY VARCHAR(20) NOT NULL,
        UNIQUE ( PNO ) ) ;
```

```
CREATE TABLE SP
    ( SNO VARCHAR(5) NOT NULL ,
        PNO VARCHAR(6) NOT NULL ,
        QTY INTEGER NOT NULL ,
        UNIQUE ( SNO , PNO ) ,
        FOREIGN KEY ( SNO ) REFERENCES S ( SNO ) ,
        FOREIGN KEY ( PNO ) REFERENCES P ( PNO ) ) ;
```

Note that SQL encloses the column definitions and the key and foreign key specifications all inside the same set of parentheses (contrast this with what Tutorial D does-again, see Chapters 2 and 5). Note too that by default SQL columns permit nulls; if we want to prohibit them, therefore (and I do), we have to specify an explicit constraint to that effect. There are various ways of defining such a constraint; specifying NOT NULL as part of the column definition is probably the easiest.
1.14 Tutorial D (I can't show this in SQL, because SQL doesn't support relational assignment):

```
SP := SP UNION RELATION { TUPLE { SNO 'S5' , PNO 'P6', QTY 250 } } ;
```

The text between the keyword UNION and the closing semicolon is a relation selector invocation (see Chapter 3), and it denotes the relation that contains just the tuple to be inserted. See Chapter 5 for further discussion.
1.15 I'll give an answer here for completeness (Tutorial Dagain), but I'll defer detailed explanations to Chapter 7:

```
WITH ( R1 := S WHERE CITY = 'Paris' ,
    R2 := EXTEND R1 : { STATUS := 25 } ) :
S := ( S MINUS R1 ) UNION R2 ;
```

1.16 First consider the generic assignment:

```
R := rx ;
```

Here $R$ is a relvar name and $r x$ is a relational expression, denoting the relation to be assigned to relvar $R$. An SQL analog might look like this-

```
DELETE FROM T ;
INSERT INTO T (...) tx ;
```

-where $T$ is an SQL table corresponding to relvar $R$ and $t x$ is an SQL table expression corresponding to the relational expression $r x$. Note the need for the preliminary DELETE; note too that anything could happen, loosely speaking, between that DELETE and the subsequent INSERT, whereas there's no notion in the relational case of there being anything "between the DELETE and the INSERT" (the assignment is a semantically atomic operation). In other words, the foregoing DELETE / INSERT combination, unlike the assignment it's trying to mimic, is a sequence of two distinct statements. One implication of this fact is that a failure could occur between those two statements, something that couldn't happen with the assignment as such.

As for the question "Can all relational assignments be expressed in terms of INSERT and/or DELETE and/or UPDATE?", the answer is yes (though in fact we don't need UPDATE as such at all). To be specific, the generic assignment-

```
R := rX ;
```

-is logically equivalent to:

```
R}:=(R\mathrm{ MINUS ( R MINUS rx ) ) UNION (rX MINUS R );
```

To elaborate slightly, let $d$ and $i$ be the relations denoted by the expressions $R$ MINUS $r x$ and $r x$ MINUS $R$, respectively. Then the original assignment is logically equivalent to the following one:

```
R := ( R MINUS d ) UNION i ;
```

This latter assignment is effectively equivalent to deleting $d$ from $R$ and then inserting $i$ into $R$. Do note, however, that the DELETE and INSERT in question are both done as part of the same statement, not as two separate statements. See the discussion of multiple assignment in Chapter 8.

There's another point I need to clear up here, too. In the body of the chapter, I said that SQL doesn't support relational assignment directly, and that's true. However, one reviewer of that chapter objected that, for example, the following SQL expression "could be thought of as relational assignment" (I've simplified the reviewer's example somewhat):

```
SELECT LS.*
FROM ( SELECT SNO , SNAME , STATUS
        FROM S
        WHERE CITY = 'London' ) AS LS
```

In effect, the reviewer was suggesting that this expression is assigning some table value to a table variable called LS. But it isn't. In particular, it isn't possible to go on and do further queries or updates on LS; LS isn't an independent table in its own right, it's just a temporary table that's conceptually materialized as part of the process of evaluating the overall SELECT expression. That expression is not a relational assignment. (In any case, assignment of any kind is a statement, not an expression. Statement vs. expression is another of those important logical differences. See Exercise 2.24 in Chapter 2.)

And one further point: The SQL standard supports a variant of CREATE TABLE, "CREATE TABLE AS," that allows the base table being created to be initialized to the result of some query, thereby not only creating the table in question but also assigning an initial value to it. Once initialized, however, the table in question behaves just like any other base table; thus, CREATE TABLE AS doesn't really constitute support for relational assignment, as such, either.
1.17 The discussions that follow are based on more extensive ones to be found in my book An Introduction to Database Systems (see Appendix G).

Duplicate tuples: Essentially, the concept makes no sense. Suppose for simplicity that the suppliers relvar had just two attributes, SNO and CITY, and suppose it contained a tuple showing that "it's a true fact" that supplier S1 is located in London. Then if it also contained a duplicate of that tuple (if that were possible), it would simply be informing us of that same "rrue fact" a second time. But as Chapter 4 observes, if something is true, saying it twice doesn't make it more true! For further discussion, see Chapter 4 or the paper "Double Trouble, Double Trouble" mentioned in Appendix G.

Tuple ordering: The lack of tuple ordering means there's no such thing as "the first tuple" or "the fifth tuple" or "the 97th tuple" of a relation, and there's no such thing as "the next tuple"; in other words, there's no concept of
positional addressing, and no concept of "nextness." If we did have such concepts, we would need certain additional operators as well-for example, "retrieve the $n$th tuple," "insert this tuple here," "move this tuple from here to there," and so on. As a matter of fact (to lift some text from Appendix A), it's axiomatic that if we have $n$ different ways to represent information, then we need $n$ different sets of operators. ${ }^{1}$ And if $n>1$, then we have more operators to implement, document, teach, learn, remember, and use (and choose among). But those extra operators add complexity, not power! There's nothing useful that can be done if $n>1$ that can't be done if $n=1$.

By the way, another good argument against ordering (of any kind) is that positional addressing is fragile-the addresses change as insertions and deletions are performed.

Attribute ordering: The lack of attribute ordering means there's no such thing as "the first attribute" or "the second attribute" (and so on), and there's no "next attribute" (i.e., there's no concept of "nextness")-attributes are always referenced by name, never by position. As a result, the scope for errors and obscure programming is reduced. For example, there's no way to subvert the system by somehow "flopping over" from one attribute to another. This situation contrasts with that found in certain programming systems, where it might be possible to exploit the physical adjacency of logically discrete items, deliberately or otherwise, in a variety of subversive ways. Note: Many other negative consequences of attribute ordering (or column ordering, rather, in SQL) are discussed in subsequent chapters. See also the paper "A Sweet Disorder," mentioned in Appendix G.

In the interest of accuracy, I should add that for reasons that don't concern us here, relations in mathematics, unlike their counterparts in the relational model, do have a left to right ordering to their attributes. A similar remark applies to tuples also. See Appendix A for further discussion.

## CHAPTER 2

2.1 A type is a named, finite set of values-all possible values of some specific kind: for example, all possible integers, or all possible character strings, or all possible supplier numbers, or all possible XML documents, or all possible relations with a certain heading (etc., etc.). There's no difference between a domain and a type. Note: SQL does draw a distinction between domains and types, however. In particular, it supports both a CREATE TYPE statement and a CREATE DOMAIN statement. To a first approximation, CREATE TYPE is SQL's counterpart to the TYPE statement of Tutorial D, which I'll be discussing in Chapter 8 (though there are many, many differences, not all of them trivial in nature, between the two). CREATE DOMAIN might be regarded, very charitably, as SQL's attempt to provide a tiny part of the total functionality of CREATE TYPE (it was introduced in SQL:1992, while CREATE TYPE wasn't introduced until SQL:1999); now that CREATE TYPE exists, there seems little reason to use, or even support, CREATE DOMAIN at all.
2.2 Every type has at least one associated selector; a selector is an operator that allows us to select, or specify, an arbitrary value of the type in question. Let $T$ be a type and let $S$ be a selector for $T$; then every value of type $T$ must be returned by some successful invocation of $S$, and every successful invocation of $S$ must return some value of type T. See Chapter 8 for further discussion. Note: Selectors are provided "automatically" in Tutorial D-since they're required by the relational model, at least implicitly-but not, in general, in SQL. In fact, although the selector concept necessarily exists, SQL doesn't really have a term for it; certainly selector as such isn't an SQL term. Further details are beyond the scope of this book.

A literal is a "self-defining symbol"; it denotes a value that can be determined at compile time. More precisely, a literal is a symbol that denotes a value that's fixed and determined by the symbol in question (and the

[^151]type of that value is therefore also fixed and determined by the symbol in question). Here are some Tutorial D examples:

```
4
'XYZ'
FALSE
5.0
POINT ( 5.0 , 2.5 )
```

```
/* a literal of type INTEGER */
/* a literal of type CHAR */
/* a literal of type BOOLEAN */
/* a literal of type RATIONAL */
/* a literal of type POINT */
```

(The last of these involves the user defined type POINT from the body of the chapter.)
Every value of every type, tuple and relation types included, must be denotable by means of some literal. A literal is a special case of a selector invocation; to be specific, it's a selector invocation all of whose arguments are themselves specified as literals in turn (implying in particular that a selector invocation with no arguments at all, like the INTEGER selector invocation 4, is a literal by definition). Note finally that there's a logical difference between a literal as such and a constant-a constant is a value, while a literal is a symbol that denotes such a value. (By the same token. there's a logical difference between a literal and a value-as just stated, a value is a constant, such as the constant 3 , while a literal is a symbol that denotes such a constant.)
2.3 A THE_ operator is an operator that provides access to some component of some "possible representation," or possrep, of some specified value of some specified type. See Chapter 8 for further discussion. Note: THE operators are effectively provided "automatically" in both Tutorial D and SQL, to a first approximation. However, although the THE_operator concept necessarily exists, SQL doesn't exactly have a term for it; certainly $T H E$ _ operator as such isn't an SQL term. Further details are beyond the scope of this book.
2.4 True in principle; might not be completely true in practice (but to the extent it isn't, we're talking about a confusion over model vs. implementation). Incidentally, the epigraph to the chapter is highly pertinent to the present exercise. Here it is again: "A major purpose of type systems is to avoid embarrassing questions about representations, and to forbid situations in which these questions might come up." In other words, types are a good idea because they raise the level of abstraction (without a proper type system, everything would be nothing but tedious-and error prone-bit twiddling). And here's another nice quote (this one's from Andrew Wright: "On Sapphire and Type-Safe Languages," CACM 46, No. 4, April 2003): "[Types make] program development and debugging easier by making program behavior more understandable."
2.5 A parameter is a formal operand in terms of which some operator is defined. An argument is an actual operand that's substituted for some parameter in some invocation of the operator in question. (People often use these terms as if they were interchangeable; much confusion is caused that way, and you need to be on the lookout for it.) Note: There's also a logical difference between an argument as such and the expression that's used to specify it. For example, consider the expression $(2+3)-1$, which represents an invocation of the arithmetic operator "-". The first argument to that invocation is the value five, but that argument is specified by the expression $2+3$, which represents an invocation of the arithmetic operator " + ". (In fact, of course, every expression represents some operator invocation. Even a simple variable reference- $V$, say-can be regarded as representing an invocation of a certain operator: namely, the operator that returns the current value of the specified variable $V$.)

A database is a repository for data. (Note: In the relational world, we might say, a little more specifically, that a database is a container for relvars. But much more precise definitions are possible; one such can be found in Chapter 5 of this book. See also Appendix A.) A $D B M S$ is a software system for managing databases; it provides data storage, recovery, concurrency, integrity, query/update, and other services.

A foreign key is a subset of the heading of some relvar, values of which must be equal to values of some "target" key in some other relvar (or possibly the same relvar). A pointer is a value (an address, essentially) for
which certain special operators-notably referencing and dereferencing operators-can (and in fact must) be defined. ${ }^{2}$ Note: Brief definitions of the referencing and dereferencing operators were given in a footnote in the body of the chapter.

A generated type is a type obtained by executing some type generator such as ARRAY, RELATION, or (in SQL) CHAR; specific array, relation, and (in SQL) character string types are thus generated types. A nongenerated type is a type that's not a generated type.

A relation is a value; it has a type-a relation type, of course-but it isn't itself a type. A type is a named, finite set of values: viz., all possible values of some particular kind.

Type is a model concept; types have semantics that must be understood by the user. Representation is an implementation concept; representations are supposed to be hidden from the user. In particular (and as noted in the body of the chapter), if $X$ is a value or variable of type $T$, then the operators that apply to $X$ are the operators defined for values and variables of type $T$, not the operators defined for the representation that applies to values and variables of type $T$. For example, just because the representation for type ENO ("employee numbers") happens to be CHAR, say, it doesn't follow that we can concatenate two employee numbers; we can do that only if concatenation is an operator that's defined for values of type ENO. See the answer to Exercise 2.4 above for further discussion.

A system defined (or built in) type is a type that's available for use as soon as the system is installed (it "comes in the same box the system comes in"). A user defined type is a type whose definition and implementation are provided by some suitably skilled user after the system is installed. (To the user of such a type, however-as opposed to the user who actually defines that type-that type should look and feel just like a system defined type.)

A system defined (or built in) operator is an operator that's available for use as soon as the system is installed (it comes in the same box the system comes in). A user defined operator is an operator whose definition and implementation are provided by some suitably skilled user after the system is installed. (To the user of such an operator, however-as opposed to the user who designs and implements that operator-that operator should look and feel just like a system defined operator.) User defined operators can take arguments of either user or system defined types (or a mixture), but system defined operators can obviously take arguments of system defined types only.
2.6 A scalar type is a type that has no user visible components; a nonscalar type is a type that's not a scalar type. Values, variables, operators, and so forth are scalar or nonscalar according as their type is scalar or nonscalar. Be aware, however, that these terms are neither very formal nor very precise, in the final analysis. In particular, we'll meet a couple of important relations in Chapter 3 called TABLE_DUM and TABLE_DEE that are "scalar" by the foregoing definition!-or so it might be argued, at least.
2.7 Coercion is implicit type conversion. It's deprecated because it's error prone (but note that this is primarily a pragmatic issue; whether or not coercions are permitted has little or nothing to do with the relational model as such).
2.8 Because it muddles type and representation.
2.9 A type generator is an operator that returns a type instead of a value (and is invoked at compile time instead of run time). The relational model requires support for two such: namely, TUPLE and RELATION. Points arising:

[^152]- Types generated by the TUPLE and RELATION type generators are nonscalar, but there's no reason in principle why generated types have to be nonscalar. SQL in particular supports several scalar type generators (CHAR, NUMERIC, REF, and many others).
- Type generators are known by many different names in the literature, including type constructors (the SQL term), parameterized types, polymorphic types, type templates, and generic types.
2.10 A relation is in first normal form ( 1 NF ) if and only if every tuple contains a single value, of the appropriate type, in every attribute position; in other words, every relation is in first normal form. Given this fact, you might be forgiven for wondering why we bother to talk about the concept at all (and in particular why it's called "first"). The reason, as I'm sure you know (and as was in fact mentioned in Chapter 1), is that (a) we can extend it to apply to relvars as well as relations, and then (b) we can define a series of "higher" normal forms for relvars that turn out to be important in database design. In other words, 1 NF is the base on which those higher normal forms build. But it really isn't all that important as a notion in itself.

Note: I should add that 1 NF is one of those concepts whose definition has evolved somewhat over the years. It used to be defined to mean that every tuple had to contain a single "atomic" value in every attribute position. As we've come to realize, however (and as I tried to show in the body of the chapter), the concept of data value atomicity actually has no objective meaning. An extensive discussion of such matters can be found in the paper "What First Normal Form Really Means" (see Appendix G).
2.11 The type of $X$ is the type, $T$ say, specified as the type of the result of the operator to be executed last-"the outermost operator"-when $X$ is evaluated. That type is significant because it means $X$ can be used in exactly (that is, in all and only) those positions where a literal of type $T$ can appear.

```
2.12 OPERATOR CUBE ( I INTEGER ) RETURNS INTEGER ;
        RETURN I * I * I ;
    END OPERATOR ;
2.13 OPERATOR AREA_OF_R ( H LENGTH , W LENGTH ) RETURNS AREA ;
        RETURN H *- W ;
    END OPERATOR ;
```

I'm assuming here, not unreasonably, that (a) it's legal to multiply ("*") a value of type LENGTH by another such value, and (b) the result of such a multiplication is a value of type AREA (another user defined type).
2.14 The following relation type is the type of the suppliers relvar S:

RELATION \{ SNO CHAR , SNAME CHAR , STATUS INTEGER , CITY CHAR \}
The suppliers relvar S itself is a variable of this type. And every legal value of that variable-for example, the value shown in Fig. 1.3 in Chapter 1-is a value of this type.
2.15 SQL definitions are given in the answer to Exercise 1.13 earlier in this appendix. Tutorial D definitions:

```
VAR P BASE RELATION
    { PNO CHAR , PNAME CHAR , COLOR CHAR , WEIGHT RATIONAL , CITY CHAR }
    KEY { PNO } ;
```

```
VAR SP BASE RELATION
    { SNO CHAR , PNO CHAR , QTY INTEGER }
    KEY { SNO , PNO }
    FOREIGN KEY { SNO } REFERENCES S
    FOREIGN KEY { PNO } REFERENCES P ;
```

Some differences between the SQL and Tutorial D definitions:

- As noted in the answer to Exercise 1.13 earlier in this appendix, SQL specifies keys and foreign keys, along with table columns (and certain other items too, beyond the scope of the present discussion) all inside the same set of parentheses-a fact that makes it hard to determine exactly what the pertinent type is. (As a matter of fact, SQL doesn't really support the concept of a relation type-or table type-at all. See Chapter 3 for further discussion.)
- The left to right order in which columns are listed matters in SQL. See Chapter 3 for further discussion.
- SQL tables don't have to have keys at all.

The significance of the fact that relvar P , for example, is of a certain relation type is as follows:

- The only values that can ever be assigned to relvar P are relations of that type.
- A reference to relvar P can appear wherever a literal of that type can appear (as in, for example, the expression P JOIN SP), in which case it denotes the relation that happens to be the current value of that relvar at the pertinent time. (In other words, a relvar reference is a valid relational expression in Tutorial $\mathbf{D}$; note, however, that an analogous remark does not apply to SQL, at least not 100 percent.) See Chapters 6 and 12 for further discussion.

One further point: As you can see, I've defined attribute QTY to be of type INTEGER. However, my reason for doing so is partly historical-every DBMS I know supports type INTEGER, while few DBMSs if any support the type that would really be more appropriate in the case at hand: viz., NONNEGATIVE_INTEGER (with the obvious semantics). Of course, we could make NONNEGATIVE_INTEGER a user defined type, but I don't want to get into too much detail regarding user defined types in this book.
2.16 a. Not valid; LOCATION = CITY('London'). b. Valid; BOOLEAN. c. Presumably valid; MONEY (I'm assuming that multiplying a money value by an integer returns another money value). d. Not valid; BUDGET + MONEY(50000). e. Not valid; ENO > ENO('E2'). f. Not valid; NAME(THE_C(ENAME) || THE_C(DNAME)) (I'm assuming that type NAME has a single "possrep component" called C, of type CHAR). g. Not valid; CITY(THE_C(LOCATION) \| 'burg') (I'm assuming that type CITY has a single "possrep component" called C, of type CHAR). Note: I'm also assuming throughout these answers that a given type $T$ always has a selector with the same name. See Chapter 8 for further discussion.
2.17 Such an operation logically means replacing one type by another, not "updating a type" (types aren't variables and hence can't be updated, by definition). Consider the following. First of all, the operation of defining a type doesn't actually create the corresponding set of values; conceptually, those values already exist, and always will exist (think of type INTEGER, for example). All the "define type" operation (the TYPE statement in Tutorial D-see Chapter 8) really does is introduce a name by which that set of values can be referenced. Likewise, dropping a type doesn't actually drop the corresponding values, it just drops the name that was introduced by the corresponding "define type" operation. It follows that "updating a type" really means dropping the type name and
then reintroducing that very same name to refer to a different set of values. Of course, there's nothing to preclude support for some kind of "alter type" shorthand to simplify matters-and SQL does support such an operator, in fact-but invoking such a shorthand shouldn't be thought of as "updating the type."
2.18 The empty type is certainly a valid type; however, it wouldn't make much sense to define a variable to be of such a type, because no value could ever be assigned to such a variable! Despite this fact, the empty type turns out to be crucially important in connection with type inheritance-but that's a topic that's (sadly) beyond the scope of the present book. Refer to the book Databases, Types, and the Relational Model: The Third Manifesto, by Hugh Darwen and myself (see Appendix G), if you want to know more.
2.19 Let $T$ be an SQL type for which " $=$ " is not defined and let $C$ be a column of type $T$. Then $C$ can't be part of a key or foreign key, nor can it be part of the argument to DISTINCT or GROUP BY or ORDER BY, nor can restrictions or joins or unions or intersections or differences be defined in terms of it. And what about implementation constructs such as indexes? There are probably other implications as well.

Second, let $T$ be an SQL type for which the semantics of " $=$ " are user defined (so $T$ is necessarily user defined itself), and let $C$ be a column of type $T$. Then the effects of making $C$ part of a key or foreign key or applying DISTINCT or GROUP BY (etc., etc.) to it will be user defined as well, at best, and unpredictable at worst.
2.20 Here's a trivial example of such violation. Let X be the character string ' AB ' (note the trailing space), let Y be the character string 'AB', and let PAD SPACE apply to the pertinent collation. Then the comparison $\mathrm{X}=\mathrm{Y}$ gives TRUE, and yet the operator invocations CHAR_LENGTH $(\mathrm{X})$ and CHAR_LENGTH $(\mathrm{Y})$ give 3 and 2, respectively. (Note too that even though the comparison $\mathrm{X}=\mathrm{Y}$ gives TRUE, the comparison $\mathrm{X}\|\mathrm{X}=\mathrm{Y}\| \mathrm{Y}$ doesn't!) I leave the detailed implications for you to think about, but it should be clear that problems are likely to surface in connection with DISTINCT, GROUP BY, and ORDER BY operations among others (as well as in connection with keys, foreign keys, and certain implementation constructs, such as indexes).
2.21 Because (a) they're logically unnecessary, (b) they're error prone, (c) end users can't use them, (d) they're clumsy-in particular, they have a direction to them, which other values don't-and (e) they undermine type inheritance. (Details of this last point are beyond the scope of this book.) There are other reasons too. See the paper cited earlier (in a footnote to the answer to Exercise 2.5), "Inclusion Dependencies and Foreign Keys," for further discussion.
2.22 One answer has to do with nulls; if we "set X to null" (which isn't really assigning a value to X , because nulls aren't values, but never mind), the comparison $\mathrm{X}=$ NULL certainly doesn't give TRUE. There are many other examples too, not involving reliance on nulls. E.g., let $X$ be a variable of type $\operatorname{CHAR}(3)$, let $Y$ be the character string 'AB' (no trailing space), and let NO PAD apply to the pertinent collation. Then assigning Y to X will actually set X to the string ' AB ' (one trailing space), and after that assignment the comparison $\mathrm{X}=\mathrm{Y}$ gives FALSE. Again I leave the implications for you to think about.
2.23 No! (Which database does type INTEGER belong to?) In an important sense, the whole subject of types and type management is orthogonal to the subject of databases and database management. We might even imagine the need for a "type administrator," whose job it would be to look after types in a manner analogous to that in which the database administrator looks after databases.
2.24 An expression represents an operator invocation, and it denotes a value; it can be thought of as a rule for computing or determining the value in question. (Incidentally, the arguments to that operator invocation are themselves specified as expressions in turn-though the expressions in question might just be simple literals or
simple variable references.) By contrast, a statement doesn't denote a value; instead, it causes some action to occur, such as assigning a value to some variable or changing the flow of control. In SQL, for example,

$$
X+Y
$$

is an expression, but

```
SET Z = X + Y ;
```

is a statement.
2.25 An RVA is an attribute whose type is some relation type, and whose values are therefore relations of that type (see Chapter 7 for further discussion). A repeating group is an "attribute" of some type $T$ whose values aren't values of type $T-$ note the contradiction in terms here!-but, rather, bags or sets or sequences (or ...) of values of type T. Note: Type $T$ here is often a tuple type (or something approximating a tuple type). In a system that allows repeating groups, for example, a file might be such that each record consists of an ENO field (employee number), an ENAME field (employee name), and a repeating group JOBHIST, in which each entry consists of a JOB field (job title), a FROM field, and a TO field (where FROM and TO are dates).
2.26 "Subquery" is an SQL term meaning, loosely, a SELECT expression enclosed in parentheses. Later chapters will elaborate (especially Chapter 12).
2.27 Regarding SQL row and table types, see Chapter 3. As for type BOOLEAN, yes, "=" does apply; TRUE is equal to TRUE and FALSE is equal to FALSE. In SQL, what's more, " $<$ " applies as well!-FALSE is considered to be less than TRUE (i.e., the comparison "FALSE $<$ TRUE" returns TRUE, in SQL).

## CHAPTER 3

### 3.1 See the body of the chapter.

3.2 Two values of any kind are equal if and only if they're the very same value (meaning they must be of the same type, a fortiori). In particular, therefore, (a) two tuples $t$ and $t^{\prime}$ are equal if and only if they have the same attributes $A 1, A 2, \ldots, A n$ and for all $i(i=1,2, \ldots, n)$, the value $v$ of $A i$ in $t$ is equal to the value $v^{\prime}$ of $A i$ in $t^{\prime} ;$ (b) two relations $r$ and $r^{\prime}$ are equal if and only if they have the same heading and the same body (i.e., their headings are equal and their bodies are equal).
3.3 Tutorial D tuple selector invocations (actually literals):

```
TUPLE { PNO 'P1' , PNAME 'Nut'
    COLOR 'Red' , WEIGHT 12.0 , CITY 'London' }
TUPLE { SNO 'S1' , PNO 'P1' , QTY 300 }
```

SQL analogs ("row value constructor" invocations):

```
ROW ( 'P1' , 'Nut' , 'Red' , 12.0 , 'London' )
```

```
ROW ( 'S1' , 'P1' , 300 )
```

Observe the lack of column names (or field names, to use the SQL term) and the reliance on left to right ordering in these SQL expressions. The keyword ROW can be omitted without changing the meanings.
3.4 The following selector invocation (actually a literal) denotes a relation of two tuples:

```
RELATION { TUPLE { SNO 'S1', PNO 'P1', QTY 300 } ,
    TUPLE { SNO 'S1' , PNO 'P2' , QTY 200 } }
```

SQL analog (a "table value constructor" invocation, involving two "row value constructor" invocations):

```
VALUES ROW ( 'S1', 'P1', 300 ) ,
    ROW ( 'S1' , 'P2' , 200 )
```

Either or both of the two row value constructor invocations here can omit the ROW keyword if desired. By the way, the fact that there are no parentheses enclosing that commalist of row value constructor invocations isn't an error. In fact, the following SQL expression-

```
VALUES ( ROW ( 'S1' , 'P1' , 300 ) ,
```

(which is certainly legal, syntactically speaking)-denotes something entirely different! See the answer to Exercise 3.10 later.
3.5 The list that follows is based on one in my book An Introduction to Database Systems (see Appendix G).

- Each attribute in the heading of a relation involves a type name, but those type names are usually omitted from tables (where by tables I mean tabular pictures of relations).
- Each component of each tuple in the body of a relation involves a type name and an attribute name, but those type and attribute names are usually omitted from tabular pictures.
- Each attribute value in each tuple in the body of a relation is a value of the applicable type, but those values (or literals denoting those values, rather) are usually shown in some abbreviated form-for example, S1 instead of ' S 1 '-in tabular pictures.
- The columns of a table have a left to right ordering, but the attributes of a relation don't. One implication of this point is that (unlike attributes) columns can have duplicate names, or even no names at all. For example, consider the SQL expression

```
SELECT DISTINCT S.CITY , S.STATUS * 2 , P.CITY
FROM S, P
```

What are the column names in the result of this expression?

- The rows of a table have a top to bottom ordering, but the tuples of a relation don't.
- A table might contain duplicate rows, but a relation never contains duplicate tuples.
- Tables (at least in SQL) always have at least one column, while relations are allowed to have no attributes at all (see the section "TABLE_DUM and TABLE_DEE" in the body of the chapter).
- Tables (at least in SQL) are allowed to include nulls, but relations certainly aren't.
- Tables (in the sense of tabular pictures) are "flat" or two-dimensional, but relations are $n$-dimensional.
3.6 One exception is as follows: Since no database relation can have an attribute of any pointer type, no tuple in such a relation can have an attribute of any pointer type either. The other exception is a little harder to state, but what it boils down to is that if tuple $t$ has heading $\{H\}$, then no attribute of $t$ can be defined in terms of any tuple or relation type with that same heading $\{H\}$, at any level of nesting.

Here are Tutorial D expressions denoting (a) a tuple with a tuple valued attribute and (b) a tuple with a relation valued attribute:

```
TUPLE { NAME 'Superman' ,
    ADDR TUPLE { STREET '1600 Pennsylvania Ave.' ,
    CITY 'Washington' , STATE 'DC' , ZIP '20500' } }
TUPLE { SNO 'S2' , PNO_REL RELATION { TUPLE { PNO 'P1' } ,
    TUPLE { PNO 'P2' } } }
```

3.7 For a relation with one RVA, see relation R4 in Fig. 2.2 in Chapter 2; for an equivalent relation with no RVA, see relation R1 in Fig. 2.1 in Chapter 2. As for one with two RVAs, consider the table on the left below. The intended meaning is:

Course CNO can be taught by every teacher TNO in TEACHER (and no other teachers) and uses every textbook XNO in TEXT (and no other textbooks).

The table on the right represents a relation without RVAs that conveys the same information.


| CNO | TNO | XNO |
| :---: | :---: | :---: |
| C1 | T2 | X1 |
| C1 | T2 | X2 |
| C1 | T4 | X1 |
| C1 | T4 | X2 |
| C1 | T5 | X1 |
| C1 | T5 | X2 |
| C2 | T4 | X2 |
| C2 | T4 | X4 |
| C2 | T4 | X5 |

As for a relation with an RVA such that there's no relation without an RVA that represents precisely the same information, one simple example can be obtained from Fig. 2.2 in Chapter 2 by just replacing the PNO_REL value for (say) supplier S2 by an empty relation:


Subsidiary exercise: Why exactly is there no relation without an RVA that represents the same information as the relation just shown?

However, it isn't necessary to invoke the notion of an empty relation in order to come up with an example of a relation with an RVA such that there's no relation without an RVA that represents precisely the same information. (Subsidiary exercise: Justify this remark! If you give up, refer to the discussion of the SIBLING example in Chapter 7.)

Perhaps I should elaborate on what it means for two relations to represent the same information. Basically, relations $r l$ and $r 2$ represent the same information if and only if it's possible to map $r 1$ into $r 2$ and vice versa by means of operations of the relational algebra. ${ }^{3}$ With reference to relations R4 in Fig. 2.2 in Chapter 2 and R1 in Fig. 2.1 in Chapter 2, for example, we have:

```
R4 = R1 GROUP ( { PNO } AS PNO_REL )
R1 = R4 UNGROUP ( PNO_REL )
```

Each relation can thus be defined in terms of the other, and the two therefore do represent the same information. See Chapter 7 for further discussion of the GROUP and UNGROUP operators.
3.8 TABLE_DEE and TABLE_DUM (DEE and DUM for short) are the only relations with no attributes; DEE contains exactly one tuple (the 0 -tuple), DUM contains no tuples at all. SQL doesn't support them because tables in SQL are always required to have at least one column. (In other words, SQL's version of the relational algebra is like an arithmetic that has no zero.) As for why this is so, your guess is as good as mine.
3.9 (Note: You might want to come back and take another look at this answer after reading Chapter 10.) We need the concept of relations in general before we can have the concept of relations of degree zero in particular. The concept of relations in general depends on predicate logic. Predicate logic depends on propositional logic.

[^153]Propositional logic depends on the truth values TRUE and FALSE. So if we tried to replace TRUE and FALSE by DEE and DUM, we would be going round in circles!

Also, it would be a little odd (to say the least) if all boolean expressions suddenly became relational expressions, and host languages thus suddenly all had to support relational data types.

Would it make sense to define a relvar of degree zero? It's hard but not impossible to imagine a situation in which such a relvar might be useful-but that's not the point. Rather, the point is that the system shouldn't include a prohibition against defining such a relvar. If it did, then that fact would constitute a violation of orthogonality, and such violations always come back to bite us eventually.
3.10 The first denotes an SQL table of four rows (three distinct ones, plus a duplicate of one of those three). The second denotes an SQL table of one row, that row consisting of four "field" values all of which are rows in turn. Note that none of the fields involved (in either case) is named.
3.11 The given expression is semantically equivalent to this one:

```
SELECT SNO
FROM S
WHERE STATUS > 20
OR ( STATUS = 20 AND SNO > 'S4' )
OR STATUS IS NULL
OR SNO IS NULL
```

3.12 See the body of the chapter.
3.13 See the body of the chapter.
3.14 EXISTS $(t)$, where $t$ is the SQL analog of the relational expression $r$. Note: Another possibility is (SELECT $\operatorname{COUNT}(*)$ FROM $(t))>0$; however, this possibility is slightly deprecated, for reasons to be explained in Chapter 10.
3.15 The complete syntax for a relation selector invocation in Tutorial D is as follows:

```
RELATION [ <heading> ] { <tuple exp commalist> }
```

Simplifying slightly, a <heading> is a commalist of attribute-name/type-name pairs enclosed in braces; it must be specified if the <tuple exp commalist> is empty, but can be omitted otherwise. By way of example, therefore, the empty suppliers relation can be specified as follows:

```
RELATION { SNO CHAR , SNAME CHAR , STATUS INTEGER , CITY CHAR } { }
```

As an aside, I note that TABLE_DEE and TABLE_DUM can be thought of as shorthand for the relation selector invocations RELATION $\}$ \{TUPLE $\}\}$ and RELATION $\}\}$, respectively.

As for SQL: The SQL analog of a relation selector invocation is (an important special case of) a VALUES expression. The syntax is:

```
VALUES <row exp commalist>
```

As you can see, there's nothing here analogous to the optional <heading> component of a Tutorial D selector invocation. As a consequence, the <row exp commalist> mustn't be empty, and SQL has no direct way of
specifying an empty table. Thus, workarounds are needed. For example, the empty suppliers table might be specified as follows:

```
SELECT * FROM S WHERE FALSE
```

3.16 Yes! However, we would of course want such operators always to produce a valid tuple as a result (i.e., we would want closure for such operations, just as we have closure for relational operations). For tuple union, for example, we would want the input tuples to be such that attributes with the same name have the same value (and are therefore of the same type, a fortiori). By way of example, let $t l$ and $t 2$ be a supplier tuple and a shipment tuple, respectively, and let $t l$ and $t 2$ have the same SNO value. Then the union of $t 1$ and $t 2, t l \mathrm{UNION} t 2$, is a tuple of type TUPLE \{SNO CHAR, SNAME CHAR, STATUS INTEGER, CITY CHAR, PNO CHAR, QTY INTEGER\}, with components as in $t 1$ or $t 2$ or both (as applicable). E.g., if $t 1$ is (S1,Smith, 20,London) and $t 2$ is (S1,P1,300)-to adopt an obvious shorthand notation for tuples-then their union is the tuple (S1,Smith,20,London,P1,300). Note: This operation might not unreasonably be called tuple join instead of tuple union.

Of course, it's not just the usual set operators that might reasonably be adapted to tuples specifically-the same goes for certain of the well known relational operators, too. A particularly important example is provided by the tuple projection operator, which is a straightforward adaptation of the relational projection operator. For example, let $t$ be a supplier tuple; then the projection $t\{$ SNO,CITY $\}$ of $t$ on attributes \{SNO,CITY\} is that subtuple of $t$ that contains just the SNO and CITY components from $t$. Likewise, $t\{\mathrm{CITY}\}$ is that subtuple of $t$ that contains just the CITY component from $t$, and $t\}$ is that subtuple of $t$ that contains no components at all (in other words, it's the 0 -tuple). In fact, it's worth noting explicitly that every tuple has a projection on the empty set of attributes whose value is, precisely, the 0 -tuple.

### 3.17 See the body of the chapter.

3.18 AS is used in SELECT clauses (to introduce column names); CREATE VIEW (ditto); FROM clauses (to introduce range variable names-by contrast, the syntax used to introduce column names in this context doesn't use AS); WITH specifications; and other contexts not discussed in this book.

You were also asked (a) in which cases the keyword was optional; (b) in which cases the AS clause took the form "<something> AS name"; and (c) in which cases it took the form "name AS <something>": No answer provided.

## CHAPTER 4

4.1 To deal with this argument properly would take more space than we have here, but it all boils down to what's sometimes called The Principle of Identity of Indiscernibles (see Appendix A). Let $a$ and $b$ be any two entities-for example, two pennies. Well, if there's absolutely no way whatsoever of distinguishing between $a$ and $b$, then there aren't two entities but only one! Now, it might be true for certain purposes that the two entities can be interchanged, but that fact isn't sufficient to make them indiscernible (there's a logical difference between interchangeability and indiscernibility, in fact, and arguments to the effect that "duplicates occur naturally in the real world" tend to be based on a muddle over this difference). A detailed analysis of this whole issue can be found in the paper "Double Trouble, Double Trouble" (see Appendix G).
4.2 Before we can answer this question, we need to pin down exactly what WHERE and UNION mean in the presence of duplicates. The paper "The Theory of Bags: An Investigative Tutorial" (see Appendix G) goes into details on such matters; here let me just say that if we adopt the SQL definitions, then the law certainly doesn't apply. In fact, it doesn't apply to either UNION ALL or UNION DISTINCT! By way of example, let $T$ be an SQL
table with just one column-C, say-containing just two rows, each of them containing just the value $v$. Then the following expressions produce the indicated results:

```
SELECT C
FROM T
WHERE TRUE
OR TRUE
Result: v*2.
SELECT C
FROM T
WHERE TRUE
UNION DISTINCT
SELECT C
FROM T
WHERE TRUE
Result: v* 1.
SELECT C
FROM T
WHERE TRUE
UNION ALL
SELECT C
FROM T
WHERE TRUE
```

Result: $v * 4$.

Note: If the various (implicit or explicit) ALLs in the foregoing expressions were all replaced by DISTINCT, it would be a different story. What do you conclude?
4.3 Remarks similar to those in the answer to the previous exercise apply here also. Again I'll skip the details; I'll just say for the record that, first, the answer depends, of course, on what definitions we adopt for UNION and INTERSECT for bags as opposed to sets; second, with the SQL definitions, the law doesn't apply. I'll leave development of a counterexample to you.
4.4 As far as I can see, the only way to resolve the ambiguity is by effectively defining a mapping from each of the (multiset) argument tables to a proper set, and likewise defining a mapping of the (multiset) result table-i.e., the desired cartesian product-to a proper set. (The mappings involve attaching a unique identifier to each row.) It seems to me, in fact, that the standard's failed attempt at a definition here serves only to emphasize the point that one of the most fundamental concepts in the entire SQL language (viz., the idea that tables should permit duplicate rows) is fundamentally flawed-and cannot be repaired without, in effect, dispensing with the concept altogether.

### 4.5 I don't think the problem can be fixed.

### 4.6 No answer provided!

4.7 The question was: Do you think nulls occur naturally in the real world? Only you can answer this question-but if your answer is yes, I think you should examine your reasoning very carefully. For example,
consider the statement "Joe's salary is $\$ 50,000$." That statement is either true or false. Now, you might not know whether it's true or false; but your not knowing has nothing to do with whether it actually is true or false. In particular, your not knowing is certainly not the same as saying "Joe's salary is null"! "Joe's salary is $\$ 50,000$ " is a statement about the real world. "Joe's salary is null" is a statement about your knowledge (or lack of knowledge, rather) of the real world. We certainly shouldn't keep a mixture of these two very different kinds of statements in the same relation, or in the same relvar.

Suppose you had to represent the fact that you don't know Joe's salary in some box on some paper form. Would you enter a null, as such, into that form? I don't think so! Rather, you would leave the box blank, or put a question mark, or write "unknown," or something along those lines. And that blank, or question mark, or "unknown"-or whatever-is a value, not a null (recall that the one thing we can be definite about regarding nulls is that they aren't values). Speaking for myself, therefore, no, I don't think nulls do "occur naturally in the real world."
4.8 True (though not in SQL!). Null is a marker that represents the absence of information, while UNKNOWN is a value, just as TRUE and FALSE are values. So there's a logical difference between the two, and to confuse them as SQL does is a logical mistake (I'd like to say it's a big logical mistake, but all logical mistakes are big by definition).

### 4.9 Yes, it does; SQL's analog of MAYBE $p$ is $p$ IS UNKNOWN.

4.10 In 2 VL there are exactly 4 monadic connectives and exactly 16 dyadic connectives, corresponding to the 4 possible monadic truth tables and 16 possible dyadic truth tables. Here are those truth tables (I've indicated the ones that have common names, such as NOT, AND, and OR): ${ }^{4}$


[^154]In 3 VL , by contrast, there are 27 ( 3 to the power 3 ) monadic connectives and 19,683 ( 3 to the power $3^{2}$ ) dyadic connectives. (In general, in fact, $n \mathrm{VL}$ has $n$ to the power $n$ monadic connectives and $n$ to the power $n^{2}$ dyadic connectives.) Many conclusions might be drawn from these facts; one of the most immediate is that 3 VL is vastly more complex than 2VL (much more so, probably, than most people, including those who think nulls are a good thing, realize, or at least admit to).
4.11 Classical 2VL supports (among other things) NOT, AND, and OR and is thus truth functionally complete, because all possible 2VL connectives can be expressed in terms of NOT and either AND or OR (see the answer to Exercise 10.4 later in this appendix for further explanation). And it turns out that SQL's 3VL-under an extremely charitable interpretation of that term!-is also truth functionally complete. The paper "Is SQL's Three-Valued Logic Truth Functionally Complete?" (see Appendix G) discusses this issue in detail.
4.12 It's not a tautology in 3VL, because if $b x$ evaluates to UNKNOWN, the whole expression also evaluates to UNKNOWN. But there does exist an analogous tautology in 3VL: viz., $b x$ OR NOT $b x$ OR MAYBE $b x$. Note: This state of affairs explains why, in SQL, if you execute the query "Get all suppliers in London" and then the query "Get all suppliers not in London," you don't necessarily get (in combination) all suppliers; you have to execute the query "Get all suppliers who may be in London" as well. Note the implications for query rewrite; note too the potential for serious mistakes (on the part of both users and the system, I might add-and there's some history here). To spell the point out: It's very natural to assume that expressions that are tautologies in 2 VL are also tautologies in 3 VL , but such is not necessarily the case.
4.13 It's not a contradiction in 3VL, because if $b x$ evaluates to UNKNOWN, the whole expression also evaluates to UNKNOWN. But there does exist an analogous (slightly tricky!) contradiction in 3VL: viz., $b x$ AND NOT $b x$ AND NOT MAYBE $b x$. Note: As you might expect, this state of affairs has implications similar to those noted in the answer to the previous exercise.
4.14 In 3VL (at least as realized in SQL), $r$ JOIN $r$ isn't necessarily equal to $r$, and INTERSECT isn't a special case of JOIN. Why so? Because in SQL, believe it or not, two nulls don't "compare equal" for join but do "compare equal" for intersection. (I take this state of affairs to be just another of the vast-infinite?-number of absurdities that nulls inevitably seem to lead us into.) However, TIMES is still a special case of JOIN, as it is in 2VL.
4.15 Here are the rules: Let $x$ be an SQL row. Suppose for definiteness and simplicity that $x$ has just two components, $x 1$ and $x 2$ (in left to right order, of course!). Then $x$ IS NULL is defined to be equivalent to $x l$ IS NULL AND $x 2$ IS NULL, and $x$ IS NOT NULL is defined to be equivalent to $x 1$ IS NOT NULL AND $x 2$ IS NOT NULL. For the given row, both of these expressions evaluate to FALSE, and it follows that the row in question is neither null nor nonnull ... What do you conclude from this state of affairs?

By the way: At least one reviewer commented at this point that he'd never thought of a row being null. But rows are values (just as tuples and relations are values), and hence the idea of some row being unknown makes exactly as much sense as, say, the idea of some salary being unknown. Thus, if the concept of representing an unknown value by a "null" makes any sense at all-which of course I don't think it does-then it surely applies to rows (and tables, and any other kind of value you can think of) just as much as it does to scalars. And as this exercise demonstrates, SQL tries to support this position-at least for rows-but fails. (Of course, it ought logically to support it for tables, too, but in that case it doesn't even try. I mean, there's no such thing as a "null table" in SQL.)
4.16 No. Here are the truth tables:

4.17 No. For definiteness, consider the case in which $x$ is an SQL row. Suppose (as in the answer to Exercise 4.15 above) that $x$ has just two components, $x 1$ and $x 2$. Then $x$ IS NOT NULL is defined to be equivalent to $x 1$ IS NOT NULL AND $x 2$ IS NOT NULL, and NOT ( $x$ IS NULL) is defined to be equivalent to $x l$ IS NOT NULL OR $x 2$ IS NOT NULL. What do you conclude from this state of affairs?
4.18 The transformation isn't valid, as you can see by considering what happens if EMP.DNO is null (were you surprised?). The implications, once again, are that users and the system are both likely to make mistakes (and again there's some history here).
4.19 The query means "Get suppliers who are known not to supply part P2" (note that known not, and note also the subtle difference between that phrase and not known); it does not mean "Get suppliers who don't supply part P2." The two formulations aren't equivalent (consider, e.g., the case where the only SP row for part number P2 in table SP has a null supplier number).
4.20 No two of the three statements are equivalent. Statement a. follows the rules of SQL's 3VL; statement b. follows the definition of SQL's UNIQUE operator; and statement c . follows SQL's definition of duplicates. In particular, if $k l$ and $k 2$ are both null, then a. gives UNKNOWN, b. gives FALSE, and c. gives TRUE (!). Here for the record are the rules in question:

In SQL's 3VL, the comparison $k l=k 2$ gives TRUE if $k l$ and $k 2$ are both nonnull and are equal, FALSE if $k 1$ and $k 2$ are both nonnull and are unequal, and UNKNOWN otherwise.

■ With SQL's UNIQUE operator, the comparison $k l=k 2$ gives TRUE if and only if $k l$ and $k 2$ are both nonnull and are equal, and FALSE otherwise. (See Chapter 11 for further explanation.)

In SQL, $k l$ and $k 2$ are duplicates if and only if either (a) they're nonnull and equal or (b) they're both null.
Note: Throughout the foregoing, "equal" refers to SQL's own, somewhat idiosyncratic definition of the " $=$ " operator (see Chapter 2). Subsidiary exercise: Do you think these rules are reasonable? Justify your answer.
4.21 The output from INTERSECT ALL and EXCEPT ALL can indeed contain duplicates, but only if duplicates are present in the input; unlike UNION ALL, therefore, these two operators never "generate" duplicates.
4.22 Yes! (We don't want duplicates in the database, but that doesn't mean we never want duplicates anywhere else. As I said in the body of the chapter, there's a logical difference between logic and rhetoric.)
4.23 A very good question.
4.24 Well, I don't know about you, but I have quite a few comments myself!

- First of all, the phrase "the null value" would be better reduced to just "null" throughout.
- Second, observe that (as noted in Chapter 4) although SQL supports three-valued logic, its BOOLEAN data type has just two values, TRUE and FALSE; "the third truth value" is represented not by a value at all but by null. This state of affairs explains (?) the distinction drawn in the second quote between "boolean values" and "SQL truth values"-as far as SQL is concerned, there are three truth values (TRUE, FALSE, and UNKNOWN) but only two boolean values (TRUE and FALSE).

■ Next: "This [standard] does not make a distinction between the null value of the boolean data type ${ }^{5}$ and the truth value Unknown ... [They] may be used interchangeably to mean exactly the same thing." But, of course, null doesn't always mean "the third truth value," so null and "the truth value Unknown" certainly can't be used "interchangeably" as claimed. In fact, the keyword NULL can't usually be used in place of the keyword UNKNOWN even when UNKNOWN is the sense intended (see c. and f. below).

■ "Unless prohibited by a NOT NULL constraint, the boolean data type also supports the truth value Unknown ...": NOT NULL doesn't apply to data types, it applies to uses of data types (typically as part of a column definition).

- Formal systems (like SQL) in which the truth values are ordered usually define that ordering to be total. In particular, for three-valued logic, the ordering would typically be such that TRUE > UNKNOWN and UNKNOWN > FALSE both return TRUE. SQL, however, defines any comparison involving UNKNOWN (even UNKNOWN = UNKNOWN) to return UNKNOWN.
- Following on from the previous point: TRUE $>$ UNKNOWN and UNKNOWN $>$ FALSE (etc.) are apparently legal SQL expressions-but they're not, according to the standard, legal "boolean value expressions" (despite the fact that they do return a boolean value ... or perhaps I should say, despite the fact that they return "an SQL truth value").

Finally, the six SQL expressions (or would-be expressions):
a. Legal; returns TRUE.
b. Legal; returns null (UNKNOWN).
c. Illegal.
d. Legal; returns TRUE.
e. Legal; returns null (UNKNOWN).
f. Illegal.

### 4.25 No answer provided.

[^155]
## CHAPTER 5

5.1 In some ways a tuple does resemble a record and an attribute a field-but these resemblances are only approximate. A relvar shouldn't be regarded as just a file, but rather as a "file with discipline," as it were. The discipline in question is one that results in a considerable simplification in the structure of the data as seen by the user, and hence in a corresponding simplification in the operators needed to deal with that data, and indeed in the user interface in general. What is that discipline? Well, it's that there's no top to bottom ordering to the records; and no left to right ordering to the fields; and no duplicate records; and no nulls; and no repeating groups; and no pointers; and no anonymous fields (and on and on). Partly as a consequence of these facts, it really is much better to think of a relvar like this: The heading represents some predicate (or some intension), and the body at any given time represents the extension of that predicate at that time.
5.2 Loosely, the specified remark means the UPDATE operation in question "updates the STATUS attribute in tuples for suppliers in London." But tuples (and, a fortiori, attribute values within tuples) are values and simply can't be updated, by definition. Here's a more precise version of the remark:

- Let relation $s$ be the current value of relvar S .
- Let $l s$ be that restriction of $s$ for which the CITY value is London.
- Let $l s^{\prime}$ be that relation that's identical to $l s$ except that the STATUS value in each tuple is the new value as specified in the given UPDATE operation.
- Let $s^{\prime}$ be the relation denoted by the expression ( $s$ MINUS $l s$ ) UNION $l s^{\prime}$.
- Then $s^{\prime}$ is assigned to S .
5.3 Because relational operations are fundamentally set level and SQL's "positioned update" operations are necessarily tuple level (or row level, rather), by definition. Although set level operations for which the set in question is of cardinality one are sometimes-perhaps even frequently-acceptable, they can't always work. In particular, tuple level update operations might work for a while and then cease to work when integrity constraint support is improved.


### 5.4 It's defined in terms of EXCEPT ALL. Consider the SQL DELETE statement:

```
DELETE FROM T WHERE bx ;
```

Let temp denote the result of the expression SELECT * FROM $T$ WHERE $b x$. Note that if row $r$ appears exactly $n$ times in temp, it also appears exactly $n$ times in $T$. Then the effect of the specified DELETE statement is to assign the result of the expression

```
SELECT * FROM T EXCEPT ALL SELECT * FROM temp
```

to table $T$. (Note that EXCEPT DISTINCT would have the additional effect of eliminating duplicates from $T$ that don't appear in temp.)
5.5 The statements aren't equivalent. The source for the first is the table $t 1$ denoted by the specified table subquery; the source for the second is the table $t 2$ containing just the row denoted by the specified row subquery
(i.e., the VALUES argument). If table S does include a row for supplier S 6 , then $t 1$ and $t 2$ are identical. But if table S doesn't include such a row, then $t l$ is empty while $t 2$ contains a row of all nulls.

As for Tutorial D analogs of the two SQL statements: Well, note first of all that in order for such analogs even to exist, it's necessary to assume that table SS doesn't permit duplicates, because "relvars that permit duplicates" aren't supported in Tutorial D (in fact, they're a contradiction in terms). Under this assumption, however, a Tutorial D analog of the first statement is reasonably straightforward:

```
INSERT SS ( S WHERE SNO = 'S6' ) ;
```

As for the second statement, the closest we can get in Tutorial D is:

```
INSERT SS RELATION { TUPLE FROM ( S WHERE SNO = 'S6' ) } ;
```

Recall from Chapter 3 that the expression TUPLE FROM $r x$ extracts the single tuple from the relation denoted by the relational expression $r x$ (that relation must have cardinality one). So if relvar S does contain a (necessarily unique) tuple for supplier S6, the foregoing INSERT will behave more or less as its SQL counterpart. But if relvar S doesn't contain such a tuple, then the INSERT will fail (more precisely, the TUPLE FROM invocation will fail), whereas the SQL analog will as already noted insert a row of all nulls. Subsidiary exercise: Which behavior do you think is more reasonable (or more useful)-Tutorial D's or SQL's?
5.6 The Assignment Principle states that after assignment of the value $v$ to the variable $V$, the comparison $V=v$ must evaluate to TRUE. SQL violates this principle if " $v$ is null"; it also violates it on certain character string assignments; and it certainly also violates it for any type for which the " $=$ " operator isn't defined, including type XML in particular, and possibly certain user defined types as well. Negative consequences: Too many to list here.
5.7 As in the body of the chapter, I assume the availability of certain user defined types in the following definitions. For simplicity, I also choose to overlook the fact that some of the column names I've chosen (which?) are in fact reserved words in SQL.

```
CREATE TABLE TAX BRACKET
    ( LOW MŌNEY NOT NULL ,
        HIGH MONEY NOT NULL ,
        PERCENTAGE INTEGER NOT NULL ,
        UNIQUE ( LOW )
        UNIQUE ( HIGH ) ,
        UNIQUE ( PERCENTAGE ) ) ;
CREATE TABLE ROSTER
    ( DAY DAY OF WEEK NOT NULL ,
        TIME TIME
        GATE GATE - NOT NULL
        PILOT NAME NOT NULL ,
        UNIQUE ( DAY , TIME , GATE ) ,
        UNIQUE ( DAY , TIME , PILOT )' ) ;
```

```
CREATE TABLE MARRIAGE
    ( SPOUSE_A NAME NOT NULL ,
        SPOUSE B NAME NOT NULL,
        DATE O\overline{F}}\mathrm{ MARRIAGE DATE NOT NULL,
        UNIQŪE ( SPOUSE_A , DATE_OF_MARRIAGE ) ,
        UNIQUE ( DATE_O\overline{F}MARRIAGE\overline{E},-}\mathrm{ SPOUSE_B ) ,
        UNIQUE ( SPOU\overline{SE_\overline{B}}, SPOUSE_A ) ) ;
```

5.8 Because keys imply constraints; constraints apply to variables, not values; and relations are values, not variables. (That said, it's certainly possible, and sometimes useful, to think of some subset $k$ of the heading of relation $r$ as if it were "a key for $r$ " if it's unique and irreducible with respect to the tuples of $r$. But thinking this way is strictly incorrect, and potentially confusing, and certainly much less useful than thinking about keys for relvars as opposed to relations.)
5.9 Here's one: Suppose relvar $A$ has a "reducible key" consisting of the disjoint union of $K$ and $X$, say, where $K$ and $X$ are both subsets of the heading of $A$ and $K$ is a genuine key. Then the functional dependency $K \rightarrow X$ holds in relvar $A$. Suppose now that relvar $B$ has a foreign key referencing that "reducible key" in $A$. Then the functional dependency $K \rightarrow X$ holds in $B$ as well. As a result, $B$ probably displays some redundancy; in fact, it's probably not in Boyce/Codd normal form. ${ }^{6}$
5.10 Keys are sets of attributes-in fact, every key is a subset of the pertinent heading-and key values are thus tuples by definition, even when the tuples in question have exactly one attribute. Thus, for example, the key for the parts relvar P is $\{\mathrm{PNO}\}$ and not just PNO , and the key value for the parts tuple for part P1 is TUPLE $\left\{\mathrm{PNO}{ }^{\prime} \mathrm{P} 1\right.$ ' $\}$ and not just ' Pl '.
5.11 Let $m$ be the smallest integer greater than or equal to $n / 2$. $R$ will have the maximum possible number of keys if either (a) every distinct set of $m$ attributes is a key or (b) $n$ is odd and every distinct set of $m-1$ attributes is a key. Either way, it follows that the maximum number of keys in $R$ is $n!/\left(m!^{*}(n-m)!\right)^{7}$ Relvars TAX_BRACKET and MARRIAGE-see the answer to Exercise 5.7 above-are examples of relvars with the maximum possible number of keys; so is any relvar of degree zero. (If $n=0$, the formula becomes $0!/(0!* 0!)$, and $0!$ is 1 .)
5.12 A superkey is a subset of the heading with the uniqueness property; a key is a superkey with the irreducibility property. All keys are superkeys, but "most" superkeys aren't keys.

The concept of a subkey can be useful in studying normalization. Here's a definition: Let $X$ be a subset of the heading of relvar $R$; then $X$ is a subkey for $R$ if and only if there exists some key $K$ for $R$ such that $X$ is a subset of $K$. For example, the following are all of the subkeys for relvar SP: $\{\mathrm{SNO}, \mathrm{PNO}\},\{\mathrm{SNO}\},\{\mathrm{PNO}\}$, and $\}$ (note that the empty set $\}$ is necessarily a subkey for all possible relvars $R$ ). By way of illustration, here's a definition of third normal form that makes use of the subkey concept: Relvar $R$ is in third normal form, 3 NF , if and only if, for every nontrivial functional dependency $X \rightarrow Y$ to which $R$ is subject, $X$ is a superkey or $Y$ is a subkey. (A nontrivial functional dependency is one for which the right side isn't a subset of the left side.)

### 5.13 Sample data:

[^156]EMP

| ENO | MNO |
| :--- | :--- |
| $\overline{\text { E4 }}$ | $=$E2 <br> E3 <br> E2 <br> E1 |
| E1 |  |
| E1 |  |

I'm using the trick here of pretending that a certain employee (namely, employee E1) acts as his or her own manager, which is one way of avoiding the use of nulls in this kind of situation. Another and probably better way is to separate the reporting structure relationships out into a relvar of their own, excluding from that relvar any employee who has no manager:
EMP

| ENO | $\cdots$ |
| :---: | :---: |
| E4 | $\cdots$ |
| E3 | $\cdots$ |
| E2 | $\cdots$ |
| E1 | $\cdots$ |

EM

| ENO | $\cdots$ |
| :---: | :---: |
| E4 | E2 |
| E3 | E2 |
| E2 | E1 |

Subsidiary exercise: What are the predicates for relvar EM and the two versions of relvar EMP here? Thinking carefully about this question should serve to reinforce the suggestion that the second design is preferable.
5.14 Because it doesn't need to, on account of the fact that column correspondences are established in SQL (in this context, at least, though not in all contexts) on the basis of ordinal position rather than name. See the discussion in the body of the chapter.
5.15 Note first that such a situation must represent a one to one relationship, by definition. One obvious case arises if we split some relvar "vertically," as in the following example (suppliers):

```
VAR SNT BASE RELATION
    { SNO CHAR , SNAME CHAR , STATUS INTEGER }
    KEY { SNO }
    FOREIGN KEY { SNO } REFERENCES SC ;
VAR SC BASE RELATION
    { SNO CHAR , CITY CHAR }
    KEY { SNO }
    FOREIGN KEY { SNO } REFERENCES SNT ;
```

One implication is that we probably need a mechanism for updating two or more relvars at the same time, and probably a mechanism for defining two or more relvars at the same time as well. See the discussion of multiple assignment in Chapter 8.
5.16 Tutorial D definitions (in accordance with the paper "Toward an Industrial Strength Dialect of Tutorial D"-see Appendix G-I assume here that Tutorial D supports the self-explanatory referential actions CASCADE and NO CASCADE):

```
VAR P BASE RELATION { PNO ... , ... } KEY { PNO } ;
VAR PP BASE RELATION { MAJOR_PNO . .. , MINOR_PNO . . , QTY . .. }
    KEY { MAJOR_PNO , MINOR_\overline{PNO }}
    FOREIGN KEY' { MAJOR PNO- } REFERENCES P
    RENAME { PNO}\mathrm{ AS MAJOR_PNO } ON DELETE CASCADE
    FOREIGN KEY { MINOR_PNO } REFERENCES P
    RENAME { PNO
```

With these definitions, deleting a part $p$ will cascade to delete parts that are components of $p$ but not parts of which $p$ is a component.

SQL definitions:

```
CREATE TABLE P ( PNO ... , ... , UNIQUE ( PNO ) ) ;
CREATE TABLE PP ( MAJOR_PNO ... , MINOR_PNO ... , QTY ... ,
    UNIQU\overline{E ( MAJOR_PNO , MINOR_PNO') ,}
    FOREIGN KEY ( M
                                ON DELETE CASCADE ,
    FOREIGN KEY ( MINOR_PNO ) REFERENCES P
                        ON DELETE RESTRICT ) ;
```

Regarding the specification ON DELETE RESTRICT here, see the answer to the next exercise.
Note: In this example, the two foreign keys in table PP both refer to the same key in table P. Now, in the body of the chapter, I said that in such a case "you might want to ensure ... that one of the foreign keys has the same column names as [the target key], even though the other one doesn't (and can't)." As you can see, however, I haven't adopted my own suggestion in the case at hand; instead, I've opted for a more symmetric design, in which each of the foreign key columns has a name consisting of the corresponding target column name prefixed with a kind of role name (MAJOR_ and MINOR_, respectively).
5.17 It's obviously not possible to give a definitive answer to this exercise. I'll just mention the referential actions supported by the standard, which are NO ACTION (the default), CASCADE, RESTRICT, SET DEFAULT, and SET NULL. Subsidiary exercise: What do you think the difference is (if any) between NO ACTION and RESTRICT? Does it make sense? Is it useful?
5.18 Loosely, a predicate is a truth valued function, and a proposition is a predicate with an empty set of parameters. See the body of the chapter for some examples, and Chapter 10 for more examples and an extended discussion of these concepts in general.
5.19 Relvar P: Part PNO is used in the enterprise, is named PNAME, has color COLOR and weight WEIGHT, and is stored in city CITY. Relvar SP: Supplier SNO supplies part PNO in quantity QTY.
5.20 The intension of relvar $R$ is the intended interpretation of $R$; the extension of relvar $R$ at a given time is the set of tuples appearing in $R$ at that time. In other words, the intension corresponds to the heading and the extension to the body.

### 5.21 No answer provided.

5.22 The Closed World Assumption says (loosely) that everything stated or implied by the database is true and everything else is false. And The Open World Assumption-yes, there is such a thing-says that everything stated or implied by the database is true and everything else is unknown. (Loosely speaking, in other words, The Closed World Assumption says tuple $t$ appears in relvar $R$ if and only if $t$ satisfies the predicate for $R$; The Open World Assumption says tuple $t$ appears in relvar $R$ only if $t$ satisfies the predicate for $R$.)

What are the implications of the foregoing? Well, first let's agree to abbreviate Closed World Assumption and Open World Assumption to CWA and OWA, respectively. Now consider the query "Is supplier S6 under contract?" Of course, the system has no understanding of what it means for a "supplier" to be "under contract," and so we have to formulate the query a little more precisely, thus: "Does there exist a tuple for supplier S6 in relvar S?" Given our usual sample data values, the answer is no, and under the CWA that no is interpreted as meaning supplier S6 isn't under contract. Under the OWA, however, that same no is interpreted as meaning it's unknown whether supplier S6 is under contract. Now consider the inverse query "Is it not the case that supplier S6 is under contract?"-more precisely, "Is it not the case that there exists a tuple for supplier S6 in relvar S?" The answer is yes, which is interpreted as meaning supplier S 6 isn't under contract under the CWA but as meaning, again, that it's unknown whether supplier S6 is under contract under the OWA. In this example, therefore, yes and no apparently mean the same thing, under the OWA! The net of this discussion is that the OWA can't properly deal with negation, and further that it's likely to lead to a requirement for three-valued logic, and hence that it's strongly deprecated for these very reasons. The paper "The Closed World Assumption" (see Appendix G) gives more information. See also Appendix C.
5.23 To say relvar $R$ has an empty key is to say $R$ can never contain more than one tuple. Why? Because every tuple has the same value for the empty set of attributes-namely, the empty tuple (see the answer to Exercise 3.16, elsewhere in this appendix); thus, if $R$ had an empty key, and if $R$ were to contain two or more tuples, we would have a key uniqueness violation on our hands. And, yes, constraining $R$ never to contain more than one tuple could certainly be useful. I'll leave finding an example of such a situation as a subsidiary exercise.

### 5.24 See the answer to Exercise 5.18 above.

5.25 The question certainly makes sense, insofar as every relvar has an associated predicate. However, just what the predicate is for some given relvar is in the mind of the definer of that relvar (and in the user's mind too, I trust). For example, if I define a relvar C as follows-

```
VAR C BASE RELATION { CITY CHAR } KEY { CITY } ;
```

-the corresponding predicate might be almost anything! It might, for example, be CITY is a city in California; or CITY is a city in which at least one supplier is located; or CITY is a city that's the capital of some country; ${ }^{8}$ and so on. In the same way, the predicate for a relvar of degree zero-

```
VAR Z BASE RELATION { } KEY { } ;
```

-might also be "almost anything," except that (since the relvar has no attributes and the corresponding predicate therefore has no parameters) the predicate in question must in fact degenerate to a proposition. That proposition will evaluate to TRUE when the value of Z is TABLE_DEE and FALSE when the value is TABLE_DUM.

[^157]By the way, observe that relvar Z has an empty key. It's obvious that every degree zero relvar must have an empty key; however, you shouldn't conclude that degree zero relvars are the only ones with empty keys (see the answer to Exercise 5.23 above).
5.26 Of course not. In fact, "most" relations aren't values of some relvar. As a trivial example, the relation denoted by $\mathrm{S}\{\mathrm{CITY}\}$, the projection of the current value of relvar S on $\{\mathrm{CITY}\}$, isn't a value of any relvar in the suppliers-and-parts database. Note, therefore, that throughout this book, when I talk about some relation, I don't necessarily mean a relation that's the value of some relvar.
5.27 There are two cases to consider: (a) The relation depicted is a sample value for some relvar $R$; (b) the relation depicted is a sample value for some relational expression $r x$, where $r x$ is something other than a simple relvar reference (recall that a relvar reference is basically just the pertinent relvar name). In the first case, double underlining simply indicates that a primary key $P K$ has been declared for $R$ and the pertinent attribute is part of $P K$. In the second case, you can think of $r x$ as the defining expression for some temporary relvar $R$ (think of it as a view defining expression and $R$ as the corresponding view, if you like); then double underlining indicates that a primary key $P K$ could in principle be declared for $R$ and the pertinent attribute is part of $P K$.

## CHAPTER 6

First of all, here are answers to a couple of exercises that were stated inline in the body of the chapter. The first asked what the difference was, given our usual sample data, between the expressions P JOIN (S $\{\mathrm{CITY}\}$ ) and ( P JOIN S) \{CITY\}. Answer: The first yields full part details (PNO, PNAME, COLOR, WEIGHT, and CITY) for parts in the same city as at least one supplier, the second yields just CITY values for those same parts (speaking a trifle loosely in both cases).

The second exercise asked what the difference was between an equijoin and a natural join. Answer: Let the relations to be joined be $r l$ and $r 2$, and assume for simplicity that $r l$ and $r 2$ have just one common attribute, $A$. Before we can perform the equijoin, then, we need to do some renaming. For definiteness, suppose we apply the renaming to $r 2$, to yield $r 3=r^{2}$ RENAME $\{A$ AS $B\}$. Then the equijoin is defined to be equal to ( $r 1$ TIMES $r 3$ ) WHERE $A=B$. Note in particular that $A$ and $B$ are both attributes of the result, and every tuple in that result will have the same value for those two attributes. Projecting attribute $B$ away from that result yields the natural join $r l$ JOIN $r 2$.
6.1 a. The result has duplicate column names (as well as left to right column ordering). b. The result has left to right column ordering. c. The result has an unnamed column (as well as left to right column ordering). d. The result has duplicate rows (even though the SELECT clause explicitly specifies $\boldsymbol{S}$.SNO, not SP.SNO, and SNO values are unique in table S). e. Compile time error: S NATURAL JOIN P has no column called S.CITY. ${ }^{9}$ f. Nothing wrong (though it would be nice if CORRESPONDING were specified). g . The result has duplicate rows and left to right column ordering; it also has no SNO column, a fact that might come as a surprise. ${ }^{10} \mathrm{~h}$. The result has duplicate column names (as well as left to right column ordering). i. Compile time error: AS specification not allowed

[^158](because the expression "(S NATURAL JOIN P)" is neither a table name nor a table subquery-see Chapter 12). j . The result has duplicate column names (as well as left to right column ordering).
6.2 No! In particular, certain relational divides that you might expect to fail don't. Here are some examples (which won't make much sense until you've read the relevant section of Chapter 7):

- Let relation $z$ be of type RELATION \{PNO CHAR \} and let its body be empty. Then the expression

```
SP { SNO , PNO } DIVIDEBY z { PNO }
```

reduces to the projection $\mathrm{SP}\{\mathrm{SNO}\}$ of SP on SNO .

- Let $z$ be either TABLE_DEE or TABLE_DUM. Then the expression

```
r DIVIDEBY z
```

reduces to $r \operatorname{JOIN} z$. In other words, if $z$ is TABLE_DEE, the result is just $r$; if $z$ is TABLE_DUM, the result is the empty relation of the same type as $r$.

- Let relations $r$ and $s$ be of the same type. Then the expression

```
r DIVIDEBY s
```

gives TABLE_DEE if $r$ is nonempty and every tuple of $s$ appears in $r$, TABLE_DUM otherwise.

- Finally, $r$ DIVIDEBY $r$ gives TABLE_DUM if $r$ is empty, TABLE_DEE otherwise.
6.3 The joining attributes are SNO, PNO, and CITY (each of which is a common attribute for exactly two of the relations to be joined, as it happens). The result predicate is: Supplier SNO is under contract, is named SNAME, has status STATUS, and is located in city CITY; part PNO is used in the enterprise, is named PNAME, has color COLOR and weight WEIGHT, and is stored in city CITY; and supplier SNO supplies part PNO in quantity QTY. Note that both appearances of SNO in this predicate refer to the same parameter, as do both appearances of PNO and both appearances of CITY. Given our usual sample values, the result looks like this:

| SNO | SNAME | STATUS | CITY | PNO | QTY | PNAME | COLOR | WEIGHT |
| :--- | :--- | ---: | :--- | :--- | :--- | :--- | :--- | :--- |
| S1 | Smith | 20 | London | P1 | 300 | Nut | Red | 12.0 |
| S1 | Smith | 20 | London | P4 | 200 | Screw | Red | 14.0 |
| S1 | Smith | 20 | London | P6 | 100 | Cog | Red | 19.0 |
| S2 | Jones | 10 | Paris | P2 | 400 | Bolt | Green | 17.0 |
| S3 | Blake | 30 | Paris | P2 | 200 | Bolt | Green | 17.0 |
| S4 | Clark | 20 | London | P4 | 200 | Screw | Red | 14.0 |

The simplest SQL formulation is just
S NATURAL JOIN SP NATURAL JOIN P
(though it might be necessary to prefix this expression with "SELECT * FROM," depending on context-see Chapter 12).
6.4 In 2-dimensional cartesian geometry, the points $(x, 0)$ and $(0, y)$ are the projections of the point $(x, y)$ on the $X$ axis and the $Y$ axis, respectively; equivalently, $(x)$ and $(y)$ are the projections into certain 1-dimensional spaces of the point $(x, y)$ in 2 -dimensional space. These notions are readily generalizable to $n$ dimensions (recall from Chapter 3 that relations are indeed $n$-dimensional).
6.5 Throughout these answers, I show SQL expressions that aren't necessarily direct transliterations of their algebraic counterparts but are, rather, "more natural" formulations of the query in SQL terms.
a. SQL analog:

```
SELECT DISTINCT CITY
FROM S NATURAL JOIN SP
WHERE PNO = 'P2'
```

Predicate: City CITY is such that some supplier who supplies part P2 is located there.

```
CITY
London
Paris
```

b. SQL analog:

```
SELECT *
FROM P
WHERE PNO NOT IN
    ( SELECT PNO
        FROM SP
        WHERE SNO = 'S2' )
```

Predicate: Part PNO is used in the enterprise, is named PNAME, has color COLOR and weight WEIGHT, is stored in city CITY, and isn't supplied by supplier S2.

| PNO | PNAME | COLOR | WEIGHT | CITY |
| :--- | :--- | :--- | ---: | :--- |
| P3 | Screw | Blue | 17.0 | Oslo |
| P4 | Screw | Red | 14.0 | London |
| P5 | Cam | Blue | 12.0 | Paris |
| P6 | Cog | Red | 19.0 | London |

c. SQL analog:

```
SELECT CITY
FROM S
EXCEPT CORRESPONDING
SELECT CITY
FROM P
```

Predicate: City CITY is such that some supplier is located there but no part is stored there.

| $\overline{\text { CITY }}$ |
| :---: |
| Athens |

d. SQL analog:

```
SELECT SNO , PNO
FROM S NATURAL JOIN P
```

Note: There's no need to do the preliminary projections (of S on $\{\mathrm{SNO}, \mathrm{CITY}\}$ and P on $\{\mathrm{PNO}, \mathrm{CITY}\}$ ) in the Tutorial D version, either. Do you think the optimizer might ignore them?

Predicate: Supplier $S N O$ and part PNO are colocated.

| SNO | PNO |
| :--- | :--- |
| S1 | P1 |
| S1 | P4 |
| S1 | P6 |
| S2 | P2 |
| S2 | P5 |
| S3 | P2 |
| S3 | P5 |
| S4 | P1 |
| S4 | P4 |
| S4 | P6 |

e. SQL analog:

```
SELECT S.CITY AS SC , P.CITY AS PC
FROM S , P
```

Predicate: Some supplier is located in city SC and some part is stored in city PC.

| SC | PC |
| :--- | :--- |
| London | London |
| London | Paris |
| London | Oslo |
| Paris | London |
| Paris | Paris |
| Paris | Oslo |
| Athens | London |
| Athens | Paris |
| Athens | Oslo |

6.6 Intersection and product are both special cases of join, so we can ignore them here. The fact that union and join are commutative is immediate from the fact that the definitions are symmetric in the two relations concerned. I now show that union is associative. Let $t$ be a tuple. Using " $\equiv$ " to stand for "if and only if" (or "is equivalent to") and " $\epsilon$ " to stand for "appears in," we have:

$$
\begin{aligned}
t \in r \text { UNION }(S \text { UNION } u) & \equiv t \in r \text { OR } t \in(s \text { UNION } u) \\
& \equiv t \in r \text { OR }(t \in S \text { OR } t \in u) \\
& \equiv(t \in r \text { OR } \in \in S) \text { OR } t \in u \\
& \equiv t \in(r \in \text { UNION } s) \text { OR } t \in u \\
& \equiv t \in(r \text { UNION } s) \text { UNION } u
\end{aligned}
$$

Note the appeal in the third line to the associativity of OR. The proof that join is associative is analogous. As for SQL, well, let's first of all ignore nulls and duplicate rows (what happens if we don't?). Then:

- SELECT A, B FROM T1 UNION CORRESPONDING SELECT B, A FROM T2 and SELECT B, A FROM T2 UNION CORRESPONDING SELECT A, B FROM T1 aren't equivalent, because they produce results with different left to right column orderings. Thus, union in general isn't commutative in SQL (and the same goes for intersection).
- T1 JOIN T2 and T2 JOIN T1 aren't equivalent (in general), because they produce results with different left to right column orderings. Thus, join in general isn't commutative in SQL (and the same goes for product).

The operators are, however, all associative.
6.7 RENAME is the only one-and even that one's debatable! See the answer to Exercise 7.3 later in this appendix.
6.8 The product of a single table $t$ is defined to be just $t$. But the question of what the product of $t 1$ and $t 2$ is if $t 1$ and $t 2$ both contain duplicate rows is a tricky one! See the answer to Exercise 4.4 earlier in this appendix for further discussion.
6.9 Tutorial D on the left, SQL on the right, as usual (the solutions aren't unique, in general; note too that the Tutorial D solutions in particular could often be improved by using operators to be described in Chapter 7) :
a. $\quad \mathrm{SP}$

```
SELECT * FROM SP
or
TABLE SP /* see Chapter 12 */
```

b. ( SP WHERE PNO = 'P1' )

```
SELECT SNO
FROM SP
WHERE PNO = 'P1'
```

c. S WHERE STATUS $\geq 15$
AND STATUS $\leq 25$

```
SELECT *
FROM S
WHERE STATUS BETWEEN 15 AND 25
```

d. ( ( S JOIN SP )

```
SELECT DISTINCT PNO
FROM SP , S
WHERE SP.SNO = S.SNO
AND S.CITY = 'London'
```

e. $P$ \{ PNO \} MINUS
( ( S JOIN SP )
WHERE CITY = 'London' )
\{ PNO \}

```
SELECT PNO
FROM P
EXCEPT CORRESPONDING
SELECT PNO
FROM SP , S
WHERE SP.SNO = S.SNO
AND S.CITY = 'London'
```

f. WITH ( Z := SP \{ SNO , PNO \} ) :
( ( Z RENAME \{ PNO AS X \} )
JOIN
( Z RENAME \{ PNO AS Y \} ) )
\{ X , Y \}
g. ( S WHERE STATUS < STATUS FROM ( TUPLE FROM ( $S$ WHERE SNO $='$ S1' ) ) \{ SNO \}

```
SELECT DISTINCT XX.PNO AS X ,
                                    YY.PNO AS Y
    FROM SP AS XX , SP AS YY
    WHERE XX.SNO = YY.SNO
SELECT SNO
FROM S
WHERE STATUS <
    ( SELECT STATUS
                                    FROM
                        WHERE SNO = 'S1' )
```

Note: The expression STATUS FROM (TUPLE FROM ...) in the Tutorial D solution here extracts the STATUS value from the single tuple in the relation that's the TUPLE FROM argument (that relation must have cardinality one). By contrast, the SQL solution effectively does a double coercion: First, it coerces a table of one row to that row; second, it coerces that row to the single scalar value it contains.
h. WITH ( RX : = S WHERE
CITY = 'London' ,
RY $:=\underset{\{\text { SP RENAME }}{\text { \{ PNO AS } Y \text { \} ) }:}$
( ( P WHERE
( RY WHERE Y = PNO ) ) \{ SNO \}
$\supseteq \operatorname{RX}\left\{\begin{array}{l}\text { SNO }\} \\ \{\text { PNO }\}\end{array}\right)$

```
SELECT PNO
FROM P
WHERE NOT EXISTS (
SELECT * FROM S
WHERE CITY = 'London'
AND NOT EXISTS (
SELECT *
FROM SP
WHERE SP.SNO = S.SNO
AND SP.PNO = P.PNO ) )
```

Note the use of a relational comparison in the Tutorial D expression here. The SQL version uses EXISTS (see Chapter 10). A more elegant Tutorial D solution can be found as the answer to Exercise 7.9e later in this appendix.

```
i. ( S { SNO } JOIN P { PNO } )
    MINUS SP { SNO , PNO }
j. WITH ( RX := SP WHERE
    RY := SP 年 RENAME
        { SNO AS Y } ) :
    S WHERE ( RY WHERE Y = SNO )
    { PNO } \supseteq RX { PNO }
```

```
SELECT SNO , PNO
```

SELECT SNO , PNO
FROM S , P
FROM S , P
EXCEPT CORRESPONDING
EXCEPT CORRESPONDING
SELECT SNO , PNO
SELECT SNO , PNO
FROM SP
FROM SP
SELECT SNO
SELECT SNO
FROM S
FROM S
WHERE NOT EXISTS (
WHERE NOT EXISTS (
SELECT *
SELECT *
FROM SP AS SPX
FROM SP AS SPX
WHERE SNO = 'S2'
WHERE SNO = 'S2'
AND NOT EXISTS (
AND NOT EXISTS (
SELECT *
SELECT *
FROM SP AS SPY
FROM SP AS SPY
WHERE SPY.SNO = S.SNO
WHERE SPY.SNO = S.SNO
AND SPY.PNO = SPX.PNO ) )

```
AND SPY.PNO = SPX.PNO ) )
```

A more elegant Tutorial D solution can be found as the answer to Exercise 7.9f later in this appendix.
6.10 It's intuitively obvious that all three statements are true. No further answer provided.
6.11 Union isn't idempotent in SQL, because the expression SELECT * FROM $T$ UNION CORRESPONDING SELECT * FROM $T$ isn't identically equal to SELECT * FROM $T$. That's because if $T$ contains any duplicates, they'll be eliminated from the result of the union. (And what happens if $T$ contains any nulls? Good question!)

Join is idempotent, and therefore so are intersection and cartesian product-in all cases, in the relational model but not in SQL, "thanks" again to duplicates and nulls.
6.12 As explained in the body of the chapter, the expression $r\}$ denotes the projection of $r$ on no attributes; it returns TABLE_DUM if $r$ is empty and TABLE_DEE otherwise. The answer to the question "What's the corresponding predicate?" depends on what the predicate for $r$ is. For example, the predicate for $\mathrm{SP}\}$ is (a trifle loosely): There exists a supplier SNO, there exists a part PNO, and there exists a quantity QTY such that supplier number SNO supplies part PNO in quantity QTY. Note that this predicate is in fact a proposition; if SP is empty (in which case SP $\}$ is TABLE_DUM) it evaluates to FALSE, otherwise (in which case SP $\}$ is TABLE_DEE) it evaluates to TRUE.

The expression $r$ \{ALL BUT\} denotes the projection of $r$ on all of its attributes (in other words, it denotes the identity projection of $r$ ); it returns $r$. The corresponding predicate is identical to that for $r$.
6.13 So far as I know, DB2 and Ingres both perform this kind of optimization (DB2 refers to it as "predicate transitive closure"). Other products might do so too.
6.14 The expression means "Get suppliers who supply all purple parts." Of course, the point is that (given our usual sample data values) there aren't any purple parts. The expression correctly returns a relation identical to the current value of relvar S (i.e., all five suppliers, loosely speaking). For further explanation-in particular, for justification of the fact that this is indeed the correct answer-see Chapter 11.
6.15 For S \{CITY\} D_UNION P \{CITY $\}$, a rough equivalent in SQL might look like this:

```
SELECT CITY
FROM ( SELECT CITY FROM S
        UNION CORRESPONDING
        SELECT CITY FROM P ) AS TEMP
WHERE NOT EXISTS
    ( SELECT CITY FROM S
        INTERSECT CORRESPONDING
        SELECT CITY FROM P )
```

This SQL expression isn't precisely equivalent to the original, however. To be specific, if supplier cities and part cities aren't disjoint, then the SQL expression won't fail at run time but will simply return an empty result. Note: The CORRESPONDING specifications could safely be omitted in this example-why, exactly?-but it's easier, and shouldn't hurt, always to specify CORRESPONDING, even when it's logically unnecessary. As for the AS TEMP specification, it's required-on the subquery in the FROM clause but not on the subquery in the WHERE clause (where in fact it would be illegal!)-for reasons explained in Chapter 7.

Turning now to S $\{$ CITY \} I_MINUS P $\{$ CITY $\}$, a rough equivalent in SQL might look like this:

```
SELECT CITY
FROM ( SELECT CITY FROM S
        EXCEPT CORRESPONDING
        SELECT CITY FROM P ) AS TEMP
WHERE NOT EXISTS
    ( SELECT CITY FROM P
        EXCEPT CORRESPONDING
        SELECT CITY FROM S )
```

Again, however, this SQL expression isn't precisely equivalent to the original, however. To be specific, if part cities aren't a subset of supplier cities, then the SQL expression won't fail at run time but will simply return an empty result.
6.16 Relations $r 1$ and $r 2$ are joinable if and only if attributes with the same name are of the same type (equivalently, if and only if the set theory union of their headings is a legal heading). That's the dyadic case. Extending the definition to the $n$-adic case is easy: Relations $r 1, r 2, \ldots, r n(n>0)$ are joinable if and only if, for all $i$, $j(1 \leq i \leq n, 1 \leq j \leq n)$, relations $r i$ and $r j$ are joinable.
6.17 It's possible to define $n$-adic versions of JOIN, UNION, and D_UNION because the operators (a) are all both commutative and associative and (b) all have a corresponding identity value.

SQL does effectively support (analogs of) $n$-adic join and union, though not for $n<2$. For join, the syntax is:

```
[ SELECT * FROM ] t1 NATURAL JOIN t2
    NATURAL JOIN t3
    NATURAL JOIN tn
```

For union, the syntax is:

```
SELECT * FROM t1 UNION CORRESPONDING SELECT * FROM t2
UNION CORRESPONDING SELECT * FROM t3
UNION CORRESPONDING SELECT * FROM tn
```

An $n$-adic version of MINUS or I_MINUS makes no sense because MINUS and I_MINUS are neither commutative nor associative, nor do they have a corresponding identity value.
6.18 For a brief justification, see the answer to Exercise 6.12 above. A longer one follows. Consider the projection $\mathrm{S}\{\mathrm{SNO}\}$ of (the relation that's current value of) the suppliers relvar S on $\{\mathrm{SNO}\}$. Let's refer to the result of this projection as $r$; given our usual sample data values, $r$ contains five tuples. Now consider the projection of that relation $r$ on the empty set of attributes, $r\}$. As we saw in the answer to Exercise 3.16 earlier in this appendix, projecting any tuple on no attributes at all yields an empty tuple; thus, every tuple in $r$ produces an empty tuple when $r$ is projected on no attributes. But all empty tuples are duplicates of one another; thus, projecting the 5 -tuple relation $r$ on no attributes yields a relation with no attributes and one (empty) tuple, or in other words TABLE_DEE. Now recall that every relvar has an associated predicate. For relvar S, that predicate looks like this:

Supplier SNO is under contract, is named SNAME, has status STATUS, and is located in city CITY.
For the projection $r=\mathrm{S}\{\mathrm{SNO}\}$, it looks like this:
There exists some name SNAME, there exists some status STATUS, and there exists some city CITY such that supplier SNO is under contract, is named SNAME, has status STATUS, and is located in city CITY.

And for the projection $r\}$, it looks like this:
There exists some supplier number SNO, there exists some name SNAME, there exists some status STATUS, and there exists some city CITY such that supplier SNO is under contract, is named SNAME, has status STATUS, and is located in city CITY.

Observe now that this last predicate is in fact a proposition: It evaluates to TRUE or FALSE, unequivocally. In the case at hand, $r\}$ is TABLE_DEE and the predicate (proposition) evaluates to TRUE. But suppose no suppliers at all were represented in the database at this time. Then $\mathrm{S}\{\mathrm{SNO}\}$ would yield an empty relation $r, r\{ \}$ would be TABLE_DUM, and the predicate (proposition) in question would evaluate to FALSE.
6.19 The expression returns the current value of table S -unless table P is currently empty, in which case it returns an empty result.

## CHAPTER 7

Here first are answers to certain exercises that were stated inline in the body of the chapter. In one, we were given relvars as follows-

| S | SNO |  | /* suppliers |
| :---: | :---: | :---: | :---: |
| SP | SNO | PNO \} | /* supplier supplies part |
| PJ | PNO | JNO \} | /* part is used in project |
| J | JNO |  | /* projects |

-and we were asked for a SQL formulation of the query "Get all (sno,jno) pairs such that sno appears in S, jno appears in J , and supplier sno supplies all parts used in project jno." A possible formulation is as follows:

```
SELECT SX.SNO , JX.JNO
FROM S AS SX , J AS JX
WHERE NOT EXISTS
    ( SELECT *
        FROM P AS PX
        WHERE EXISTS
            ( SELECT *
                FROM PJ AS PJX
                WHERE PJX.PNO = PX.PNO
                AND PJX.JNO = JX.JNO )
        AND NOT EXISTS
            ( SELECT *
                FROM SP AS SPX
                WHERE SPX.PNO = PX.PNO
                    AND SPX.SNO = SX.SNO ) )
```

Note: For a detailed discussion of how to tackle complicated queries like this one in SQL, see Chapter 11. Another inline exercise asked what happens if (a) $r 1$ and $r 2$ are relations with no attribute names in common, (b) $r 2$ is empty, (c) we form the product $r 1$ TIMES $r 2$, and finally (d) we divide that product by $r 2$. Answer: It should be clear that the product is empty, and hence the final result is empty too (it has the same heading as $r l$, but of course it isn't equal to $r l$, in general). Do note, however, that dividing by an empty relation isn't an error (it's not like dividing by zero in arithmetic).

Another inline exercise asked why the following Tutorial D and SQL expressions weren't quite equivalent:

```
S WHERE SUM ( !!SP , QTY ) < 1000
SELECT S.*
FROM S , SP
WHERE S.SNO = SP.SNO
GROUP BY S.SNO , S.SNAME , S.STATUS , S.CITY
HAVING SUM ( SP.QTY ) < 1000
```

The difference is that the Tutorial $\mathbf{D}$ expression will return a result that includes suppliers (like supplier S5, given our usual sample data values) who supply no parts at all, but the SQL expression won't. Subsidiary exercise: What accounts for the discrepancy?
7.1 Throughout these answers, I show SQL expressions that aren't necessarily direct transliterations of their algebraic counterparts but are, rather, "more natural" formulations of the query in SQL terms. The solutions aren't necessarily unique. Note: This latter remark applies to many of the code solutions throughout the remainder of this appendix, and I won't bother to make it again.
a. SQL analog:

```
SELECT *
FROM S
WHERE SNO IN
    ( SELECT SNO
        FROM SP
        WHERE PNO = 'P2' )
```

Predicate: Supplier SNO is under contract, is named SNAME, has status STATUS, is located in city CITY, and supplies part P2.

| SNO | SNAME | STATUS | CITY |
| :---: | :---: | ---: | :--- |
| S1 | Smith | 20 | London |
| S2 | Jones | 10 | Paris |
| S3 | Blake | 30 | Paris |
| S4 | Clark | 20 | London |

b. SQL analog:

```
SELECT *
FROM S
WHERE SNO NOT IN
    ( SELECT SNO
        FROM SP
        WHERE PNO = 'P2' )
```

Predicate: Supplier SNO is under contract, is named SNAME, has status STATUS, is located in city CITY, and doesn't supply part P2.

| SNO | SNAME | STATUS | CITY |
| :---: | :---: | ---: | :--- |
| S5 | Adams | 30 | Athens |

c. SQL analog:

```
SELECT *
FROM P AS PX
WHERE NOT EXISTS
    ( SELECT *
        FROM S AS SX
        WHERE NOT EXISTS
            ( SELECT *
                FROM SP AS SPX
                WHERE SPX.SNO = SX.SNO
                AND SPX.PNO = PX.PNO ) )
```

Predicate: Part PNO is used in the enterprise, is named PNAME, has color COLOR and weight WEIGHT, is stored in city CITY, and is supplied by all suppliers.

| PNO | PNAME | COLOR | WEIGHT | CITY |
| :---: | :---: | :---: | :---: | :--- |
|  |  |  |  |  |

d. SQL analog:

```
SELECT *
FROM P
WHERE ( SELECT COALESCE ( SUM ( QTY ) , 0 )
    FROM SP
    WHERE SP.PNO = P.PNO ) < 500
```

Predicate: Part PNO is used in the enterprise, is named PNAME, has color COLOR and weight WEIGHT, is stored in city CITY, and is supplied in a total quantity, taken over all suppliers, that's less than 500.

| PNO | PNAME | COLOR | WEIGHT | CITY |
| :--- | :--- | :--- | ---: | :--- |
| P3 | Screw | Blue | 17.0 | Oslo |
| P6 | Cog | Red | 19.0 | London |

e. SQL analog:

```
SELECT *
FROM P
WHERE CITY IN
    ( SELECT CITY
    FROM S )
```

Predicate: Part PNO is used in the enterprise, is named PNAME, has color COLOR and weight WEIGHT, is stored in city CITY, and is located in the same city as some supplier.

| PNO | PNAME | COLOR | WEIGHT | CITY |
| :--- | :--- | :--- | ---: | :--- |
| P1 | Nut | Red | 12.0 | London |
| P2 | Bolt | Green | 17.0 | Paris |
| P4 | Screw | Red | 14.0 | London |
| P5 | Cam | Blue | 12.0 | Paris |
| P6 | Cog | Red | 19.0 | London |

f. SQL analog:

```
SELECT S.* , 'Supplier' AS TAG
FROM S
```

Predicate: Supplier SNO is under contract, is named SNAME, has status STATUS, is located in city CITY, and has a TAG of 'Supplier'.

| SNO | SNAME | STATUS | CITY | TAG |
| :---: | :---: | ---: | :--- | :--- |
| S1 | Smith | 20 | London | Supplier |
| S2 | Jones | 10 | Paris | Supplier |
| S3 | Blake | 30 | Paris | Supplier |
| S4 | Clark | 20 | London | Supplier |
| S5 | Adams | 30 | Athens | Supplier |

g. SQL analog:

```
SELECT DISTINCT SNO, S.* , 3 * STATUS AS TRIPLE_STATUS
FROM S NATURAL JOIN SP
WHERE PNO = 'P2'
```

Predicate: Supplier SNO is under contract, is named SNAME, has status STATUS, is located in city CITY, supplies part P2, and has TRIPLE_STATUS equal to three times the value of STATUS.

| SNO | SNAME | STATUS | CITY | TRIPLE_STATUS |
| :---: | :---: | ---: | :--- | :--- |
| S1 | Smith | 20 | London | 60 |
| S2 | Jones | 10 | Paris | 30 |
| S3 | Blake | 30 | Paris | 90 |
| S4 | Clark | 20 | London | 60 |

h. SQL analog:

```
SELECT PNO , PNAME, COLOR , WEIGHT , CITY , SNO , QTY
FROM P NATURAL JOIN SP
```

Predicate: Part PNO is used in the enterprise, is named PNAME, has color COLOR and weight WEIGHT, is stored in city CITY, is supplied by supplier SNO in quantity QTY, and that shipment (of PNO by SNO) has total weight SHIPWT equal to WEIGHT times QTY.

| PNO | PNAME | COLOR | WEIGHT | CITY | SNO | QTY | SHIPWT |
| :--- | :--- | :--- | ---: | :--- | :--- | :--- | :--- |
| P1 | Nut | Red | 12.0 | London | S1 | 300 | 3600.0 |
| P1 | Nut | Red | 12.0 | London | S2 | 300 | 3600.0 |
| P2 | Bolt | Green | 17.0 | Paris | S1 | 200 | 3400.0 |
| P2 | Bolt | Green | 17.0 | Paris | S2 | 400 | 6800.0 |
| P2 | Bolt | Green | 17.0 | Paris | S3 | 200 | 3400.0 |
| P2 | Bolt | Green | 17.0 | Paris | S4 | 200 | 3400.0 |
| P3 | Screw | Blue | 17.0 | Oslo | S1 | 400 | 6800.0 |
| P4 | Screw | Red | 14.0 | London | S1 | 200 | 2800.0 |
| P4 | Screw | Red | 14.0 | London | S4 | 300 | 4200.0 |
| P5 | Cam | Blue | 12.0 | Paris | S1 | 100 | 1200.0 |
| P5 | Cam | Blue | 12.0 | Paris | S4 | 400 | 4800.0 |
| P6 | Cog | Red | 19.0 | London | S1 | 100 | 1900.0 |

i. SQL analog:

```
SELECT P.* , WEIGHT * 454 AS GMWT , WEIGHT * 16 AS OZWT
FROM P
```

Predicate: Part PNO is used in the enterprise, is named PNAME, has color COLOR, weight WEIGHT, weight in grams GMWT (= 454 times WEIGHT), and weight in ounces OZWT (= 16 times WEIGHT).

| PNO | PNAME | COLOR | WEIGHT | CITY | GMWT | OZWT |
| :--- | :--- | :--- | ---: | :--- | :--- | :--- |
| P1 | Nut | Red | 12.0 | London | 5448.0 | 192.0 |
| P2 | Bolt | Green | 17.0 | Paris | 7718.0 | 204.0 |
| P3 | Screw | Blue | 17.0 | Oslo | 7718.0 | 204.0 |
| P4 | Screw | Red | 14.0 | London | 6356.0 | 168.0 |
| P5 | Cam | Blue | 12.0 | Paris | 5448.0 | 192.0 |
| P6 | Cog | Red | 19.0 | London | 8626.0 | 228.0 |

j. SQL analog:

```
SELECT P.* , ( SELECT COUNT ( SNO )
    FROM SP
    WHERE SP.PNO = P.PNO ) AS SCT
FROM P
```

Predicate: Part PNO is used in the enterprise, is named PNAME, has color COLOR, weight WEIGHT, and city CITY, and is supplied by SCT suppliers.

| PNO | PNAME | COLOR | WEIGHT | CITY | SCT |
| :--- | :--- | :--- | ---: | :--- | ---: |
| P1 | Nut | Red | 12.0 | London | 2 |
| P2 | Bolt | Green | 17.0 | Paris | 4 |
| P3 | Screw | Blue | 17.0 | Oslo | 1 |
| P4 | Screw | Red | 14.0 | London | 2 |
| P5 | Cam | Blue | 12.0 | Paris | 2 |
| P6 | Cog | Red | 19.0 | London | 1 |

k. SQL analog:

```
SELECT S.* , ( SELECT COUNT ( PNO )
                        FROM SP
    WHERE SP.SNO = S.SNO ) AS NP
FROM S
```

Predicate: Supplier SNO is under contract, is named SNAME, has status STATUS, is located in city CITY, and supplies NP parts.

| SNO | SNAME | STATUS | CITY | NP |
| :---: | :---: | ---: | :--- | :---: |
| S1 | Smith | 20 | London | 6 |
| S2 | Jones | 10 | Paris | 2 |
| S3 | Blake | 30 | Paris | 1 |
| S4 | Clark | 20 | London | 3 |
| S5 | Adams | 30 | Athens | 0 |

1. SQL analog:
```
SELECT CITY , SUM ( STATUS ) AS SUM STATUS
FROM S
GROUP BY CITY
```

Predicate: The sum of status values for suppliers in city CITY is SUM_STATUS.

| CITY | SUM_STATUS |
| :--- | ---: |
| London | 40 |
| Paris | 40 |
| Athens | 30 |

m. SQL analog:

```
SELECT COUNT ( SNO ) AS N
FROM S
WHERE CITY = 'London'
```

Predicate: There are $N$ suppliers in London.

| $N$ |
| :---: |
| 2 |

The lack of double underlining here is not a mistake.
n. SQL analog:

```
SELECT 'S7' AS SNO , PNO , QTY * 0.5 AS QTY
FROM SP
WHERE SNO = 'S1'
```

Predicate: SNO is $S 7$ and supplier $S 1$ supplies part PNO in quantity twice QTY.

| SNO | PNO | QTY |
| :---: | :---: | :---: |
| S7 | P1 | 150 |
| S7 | P2 | 100 |
| S7 | P3 | 200 |
| S7 | P4 | 100 |
| S7 | P5 | 50 |
| S7 | P6 | 50 |

7.2 The expressions $r 1$ MATCHING $r 2$ and $r 2$ MATCHING $r 1$ are equivalent if and only if $r 1$ and $r 2$ are of the same type, in which case both expressions reduce to just $\operatorname{JOIN}\{r 1, r 2\}$ (and this latter expression reduces in turn to INTERSECT $\{r 1, r 2\}$ ).
7.3 RENAME isn't primitive because (for example) the expressions

```
S RENAME { CITY AS SCITY }
```

and

```
( EXTEND S : { SCITY := CITY } ) { ALL BUT CITY }
```

are equivalent. Note: Possible appearances to the contrary notwithstanding, EXTEND isn't primitive either-it can be defined in terms of join (at least in principle), as is shown in the book Databases, Types, and the Relational Model: The Third Manifesto, by Hugh Darwen and myself (see Appendix G).

```
7.4 EXTEND S { SNO } : { NP := COUNT ( !!SP ) }
```

7.5 You can determine which of the expressions are equivalent to which by inspecting the following results of evaluating them. Note that the "summary" SUM(1), evaluated over $n$ tuples, is equal to $n$. (Even if $n$ is zero! SQL, of course, would say the result is null in such a case.)
a. $\quad r$ empty:
r has n tuples
r has n tuples
( $n>0$ ) :


b. r empty: | CT |
| :---: |
| 0 |


c. $\quad r$ empty:


d. $\quad r$ empty: | CT |
| :---: |
| 0 |

$r$ has $n$ tuples
( $n>0$ ) :


In other words, the result is a relation of degree one in every case. If $r$ is nonempty, all four expressions are equivalent; otherwise $a$. and $c$. are equivalent, and $b$. and d. are equivalent. SQL analogs:

```
a. SELECT COUNT ( * ) AS CT
    FROM r
    EXCEPT CORRESPONDING
    SELECT O AS CT
    FROM r
b. SELECT COUNT ( * ) AS CT
    FROM r
```

c. Same as a.
d. Same as b.
7.6 They return, respectively, the empty relation and the universal relation (of the applicable type in each case). Note: The universal relation of type RELATION $\{H\}$ is the relation of that type that contains all possible tuples of type TUPLE $\{H\}$. The implementation might reasonably want to outlaw invocations of INTERSECT on an empty argument (at least if those invocations really need the result to be materialized).

Just to remind you, SQL's analogs of the UNION and INTERSECT aggregate operators are called FUSION and INTERSECTION, respectively. If their arguments are empty, they both return null. Otherwise, they return a result as follows (these are direct quotes from the standard; $T$ is the table over which the aggregation is being done). First FUSION:
[The] result is the multiset $M$ such that for each value $V$ in the element type, including the null value [sic], the number of elements of $M$ that are identical to $V$ is the sum of the number of identical copies of $V$ in the multisets that are the values of the column in each row of $T$.
(The "element type" is the type of the elements of the multisets in the argument column.) Now INTERSECTION:
[The] result is a multiset $M$ such that for each value $V$ in the element type, including the null value, the number of duplicates of $V$ in $M$ is the minimum of the number of identical copies of $V$ in the multisets that are the values of the column in each row of $T$.

Note the asymmetry, incidentally: In SQL, INTERSECTION (and INTERSECT) are defined in terms of MIN, but FUSION (and UNION) are defined in terms not of MAX but of SUM (?).
7.7 The predicate can be stated in many different ways, of course. Here's one reasonably straightforward formulation: Supplier SNO supplies part PNO if and only if part PNO is mentioned in relation PNO_REL. That "and only if" is important, by the way (right?).
7.8 Relation $r$ has the same cardinality as SP and the same heading, except that it has one additional attribute, X , which is relation valued. The relations that are values of X have degree zero; furthermore, each is TABLE_DEE, not TABLE_DUM, because every tuple $s p$ in SP effectively includes the 0 -tuple as its value for that subtuple of $s p$ that corresponds to the empty set of attributes. Thus, each tuple in $r$ effectively consists of the corresponding tuple from SP extended with the X value TABLE_DEE, and the original GROUP expression is logically equivalent to the following:

```
EXTEND SP : { X := TABLE_DEE }
```

The expression $r$ UNGROUP (X) yields the original SP relation again.
7.9 Tutorial D on the left, SQL on the right as usual:
a. $\mathrm{N}:=\begin{aligned} & \text { COUNT ( } \mathrm{SP} \\ & \text { WHERE } \mathrm{SNO}= \\ & \text { 'S1' ) ; }\end{aligned}$

```
SET N = ( SELECT COUNT ( * )
                                    FROM SP
    WHERE SNO = 'S1' ) ;
```

b. ( S WHERE CITY = MIN ( S , CITY ) ) \{ SNO \}

```
SELECT *
FROM S
WHERE CITY =
    ( SELECT MIN ( CITY )
```

c. $S$ \{ CITY \} WHERE COUNT ( !!S ) > 1

```
SELECT DISTINCT CITY
FROM S AS SX
WHERE ( SELECT COUNT ( * )
    FROM S AS SY
    WHERE SY.CITY = SX.CITY ) > 1
```

d. $S$ \{ CITY \} XUNION

SELECT CITY
FROM S WHERE CITY NOT IN ( SELECT CITY FROM P )
UNION CORRESPONDING SELECT CITY FROM P WHERE CITY NOT IN ( $\underset{\text { FROM }}{\text { SELECT }} \underset{\text { SITY }}{\text { S }}$
e. ( P WHERE ( !!SP ) \{ SNO \}
( S WHERE CITY $=$ 'London' ) \{ SNO \} ) \{ PNO \}

```
SELECT PNO
FROM P
WHERE NOT EXISTS (
SELECT * FROM S
WHERE CITY = 'London'
AND NOT EXISTS (
SELECT *
FROM SP
WHERE SP.SNO = S.SNO
AND SP.PNO = P.PNO ) )
```

f. S WHERE ( !!SP ) \{ PNO \} $\supseteq$ ( SP WHERE SNO = 'S2' ) \{ PNO \}

```
SELECT SNO
FROM S
WHERE NOT EXISTS (
SELECT *
FROM SP AS SPX
WHERE SNO = 'S2'
AND NOT EXISTS (
SELECT *
FROM SP AS SPY
WHERE SPY.SNO = S.SNO
AND SPY.PNO = SPX.PNO ) )
```

7.10 It's the same as TCLOSE ( $p p$ ). In other words, transitive closure is idempotent. Note: I'm extending the definition of idempotence slightly here. In Chapter 6, I said (in effect) that a dyadic operator $O p$ is idempotent if and only if $x O p x=x$ for all $x$; now I'm saying (in effect) that a monadic operator $O p$ is idempotent if and only if $O p(O p(x))=O p(x)$ for all $x$.
7.11 It denotes a relation looking like this (in outline):

| SNO | SNAME | STATUS | CITY | PNO_REL |
| :---: | :---: | :---: | :---: | :---: |
| S 1 | Smith | 20 | London | PNO |
|  |  |  |  | P1 P2 |
|  |  |  |  | $\dot{P} \dot{6}$ |
| S 2 | Jones | 10 | Paris | PNO |
|  |  |  |  | P1 P2 |


| $\cdots$ | $\cdots \cdots$ | $\cdots$ | $\cdots \cdots$ | $\cdots$ |
| :---: | :---: | :---: | :---: | :---: |
|  | Adams | 30 | Athens | PNO <br> $=$ |

The expression is logically equivalent to the following:

```
( S JOIN SP { SNO , PNO } ) GROUP ( { PNO } AS PNO_REL )
```

Attribute PNO_REL is an RVA. Note, incidentally, that if $r$ is the foregoing relation, then the expression

```
( r UNGROUP ( PNO_REL ) ) { ALL BUT PNO }
```

will not return our usual suppliers relation. To be precise, it will return a relation that differs from our usual suppliers relation only in that it'll have no tuple for supplier S5.
7.12 The first is straightforward: It inserts a new tuple, with supplier number S6, name Lopez, status 30, city Madrid, and PNO_REL value a relation containing just one tuple, containing in turn the PNO value P5. As for the second, I think it would be helpful to repeat from Appendix D the Tutorial D grammar for <relation assign> (the names of the syntactic categories are meant to be self-explanatory):

```
<relation assign>
    ::= <relvar name> := <relation exp>
    | <insert> | <d_insert> | <delete> | <i_delete> | <update>
```

```
<insert>
    ::= INSERT <relvar name> <relation exp>
<d_insert>
    ::= D_INSERT <relvar name> <relation exp>
<delete>
    ::= DELETE <relvar name> <relation exp>
        | DELETE <relvar name> [ WHERE <boolean exp> ]
<i_delete>
    ::= I_DELETE <relvar name> <relation exp>
<update>
    ::= UPDATE <relvar name> [ WHERE <boolean exp> ] :
    { <attribute assign commalist> }
```

And an <attribute assign>, if the attribute in question is relation valued, is basically just a <relation assign> (except that the pertinent <attribute name> appears in place of the target <relvar name> in that <relation assign>), and that's where we came in. Thus, in the exercise, what the second update does is replace the tuple for supplier S2 by another in which the PNO_REL value additionally includes a tuple for part P5.

### 7.13 Query a. is easy:

```
WITH ( X := ( SSP RENAME { SNO AS XNO } ) { XNO , PNO_REL }
    Y := ( SSP RENAME { SNO AS YNO } ) { YNO , PNO_REL } ) :
( X JOIN Y ) { XNO , YNO }
```

Note that the join here is being done on an RVA (and so is implicitly performing relational comparisons).
Query b., by contrast, is not so straightforward. Query a. was easy because SSP "nests parts within suppliers," as it were; for Query b. we would really like to have suppliers nested within parts instead. So let's do that: ${ }^{11}$

```
WITH ( PPS := ( SSP UNGROUP ( PNO_REL ) ) GROUP ( { SNO } AS SNO_REL ) ,
    X := ( PPS RENAME { PNO A
    Y := ( PPS RENAME { PNO AS YNO } ) { YNO , SNO-REL } ) :
( X JOIN Y ) { XNO , YNO }
7.14 WITH ( R1 := P RENAME { WEIGHT AS WT } ,
    R2 := EXTEND P : { N HEAVIER :=
                                CŌUNT ( R1 WHERE WT > WEIGHT ) } ) :
( R2 WHERE N_HEAVIER < 2 ) { ALL BUT N_HEAVIER }
SELECT *
FROM P AS PX
WHERE ( SELECT COUNT ( * )
        FROM P AS PY
    WHERE PX.WEIGHT < PY.WEIGHT ) < 2
```

[^159]Both formulations return parts P2, P3, and P6 (i.e., a result of cardinality three, even though the specified quota was two). Quota queries can also return a result of cardinality less than the specified quota (e.g., consider the query "Get the ten heaviest parts").

Note: Quota queries are quite common in practice. In our book Databases, Types, and the Relational Model: The Third Manifesto (see Appendix G), therefore, Hugh Darwen and I suggest a shorthand for expressing them, according to which the foregoing query might be expressed thus in Tutorial D:

```
( ( RANK P BY ( DESC WEIGHT AS W ) ) WHERE W \leq 2 ) { ALL BUT W }
```

SQL has something similar.
7.15 This formulation does the trick:

```
SUMMARIZE SP { SNO, QTY } PER ( S { SNO } ) : { SDQ := SUM ( QTY ) }
```

But the following formulation, using EXTEND and image relations, is surely to be preferred:

```
EXTEND S { SNO } : { SDQ := SUM ( !!SP { QTY } }
```

Here for interest is an SQL analog:

```
SELECT SNO ,
    ( SELECT COALESCE ( SUM ( DISTINCT QTY ) , O ) AS SDQ
FROM S
7.16 EXTEND S : { NP := COUNT ( !!SP ) , NJ := COUNT ( !!SJ ) }
JOIN {S , SUMMARIZE SP PER ( S { SNO } ) : {NP := COUNT ( PNO ) } ,
SELECT SNO , ( SELECT COUNT ( PNO )
        FROM SP
        WHERE SP.SNO = S.SNO ) AS NP ,
    ( SELECT COUNT ( JNO )
        FROM SJ
        WHERE SJ.SNO = S.SNO ) AS NJ
FROM S
```

7.17 For a given supplier number, sno say, the expression !!SP denotes a relation with heading \{PNO,QTY\} and body consisting of those (pno,qty) pairs that correspond in SP to that supplier number sno. Call that relation ir (for image relation). By definition, then, for that supplier number sno, the expression !!(!!SP) is shorthand for the following:

```
( ( ir ) MATCHING RELATION { TUPLE { } } ) { ALL BUT }
```

This expression in turn is equivalent to:

```
( ( ir ) MATCHING TABLE_DEE ) { PNO , QTY }
```

And this expression reduces to just $i r$. Thus, "!!" is idempotent (i.e., !!(!! $r$ ) is equivalent to !! $r$ for all $r$ ), and the overall expression

```
S WHERE ( !!(!!SP) ) { PNO } = P { PNO }
```

is equivalent to:

```
S WHERE ( !!SP ) { PNO } = P { PNO }
```

("Get suppliers who supply all parts").
7.18 No, there's no logical difference.
7.19 S JOIN SP isn't a semijoin; S MATCHING SP isn't a join (it's a projection of a join). The expressions $r 1$ JOIN $r 2$ and $r 1$ MATCHING $r 2$ are equivalent if and only if relations $r 1$ and $r 2$ are of the same type (when the final projection becomes an identity projection, and the expression overall degenerates to $r$ I INTERSECT $r 2$ ).
7.20 If $r 1$ and $r 2$ are of the same type and $t 1$ is a tuple in $r 1$, the expression !! $r 2$ (for $t l$ ) denotes a relation of degree zero-TABLE_DEE if $t 1$ appears in $r 2$, TABLE_DUM otherwise. And if $r 1$ and $r 2$ are the same relation ( $r$, say), ! $!r 2$ becomes $!!r$, and it denotes TABLE_DEE for every tuple in $r$.
7.21 They're the same unless table $S$ is empty, in which case the first yields a one-column, one-row table containing a zero and the second yields a one-column, one-row table "containing a null."
7.22 In SQL, typically in a cursor definition; in Tutorial D (where ORDER BY is spelled just ORDER), in a special operator ("LOAD"), not discussed further in this book, that retrieves a specified relation into an array (of tuples).

## CHAPTER 8

8.1 A type constraint is a definition of the set of values that constitute a given type. The type constraint for type $T$ is checked whenever some selector for type $T$ is invoked; if the check fails, the selector invocation fails on a type constraint violation. Subsidiary exercise: What do you think should happen if the type constraint for type $T$ evaluates to FALSE at the time type $T$ is defined? (Answer: This state of affairs isn't necessarily an error, but the type in question will be empty. See the answer to Exercise 2.18 elsewhere in this appendix.)

A database constraint is a constraint on the values that can appear in a given database. Database constraints are checked "at semicolons"-more specifically, at the end of any statement that assigns a value to any of the pertinent relvars. If the check fails, the assignment fails on a database constraint violation. Note: Database constraints must also be checked when they're defined. If that check fails, the constraint definition must be rejected.
8.2 The Golden Rule states (in effect) that no update operation must ever cause any database constraint to evaluate to FALSE, and hence a fortiori that no update operation must ever cause any relvar constraint to evaluate to FALSE either. However, a (total) relvar constraint might evaluate to FALSE, not because some single relvar constraint is violated, but rather because some multirelvar constraint is violated. The point is hardly significant, however, given that-as mentioned in the body of the chapter and explained in more detail in Chapter 9-which relvar constraints are single relvar and which multirelvar is somewhat arbitrary anyway.
8.3 Assertion is SQL's term for a constraint specified via CREATE ASSERTION. An attribute constraint is a specification to the effect that a certain attribute is of a certain type. A base table constraint is an SQL constraint that's specified as part of a base table definition (and not as part of a column definition within such a base table definition). A column constraint is an SQL constraint that's specified as part of a column definition within a base table definition. A multirelvar constraint is a database constraint that mentions two or more distinct relvars. A referential constraint is a constraint to the effect that if $B$ references $A$, then $A$ must exist. A relvar constraint for relvar $R$ is a database constraint that mentions $R$. A row constraint is an SQL constraint with the property that it can be checked for a given row by examining just that row in isolation. A single relvar constraint is a database constraint that mentions just one relvar. A state constraint is a database constraint that isn't a transition constraint. "The" (total) database constraint for database $D B$ is the logical AND of all of the relvar constraints for relvars in $D B$ and TRUE. "The" (total) relvar constraint for relvar $R$ is the logical AND of all of the database constraints that mention $R$ and TRUE. A transition constraint is a constraint on the legal transitions a database can make from one "state" (i.e., value) to another. A tuple constraint is a relvar constraint with the property that it can be checked for a given tuple by examining just that tuple in isolation. Which of these categories if any do (a) key constraints, (b) foreign key constraints, fall into? No answers provided.
8.4 See the body of the chapter.
8.5 a. The integer 345. b. The value of QTY (where, let's assume, QTY is a variable of type QTY).
8.6 See the body of the chapter.
8.7 TYPE CITY POSSREP $\left\{\begin{array}{l}\text { C CHAR CONSTRAINT } C\end{array}=\right.$ 'London'
OR $C=$ 'Paris'
OR $C=$ 'Rome'
OR $C=$ 'Athens'
OR $C=$ 'Oslo'
OR $C=$ 'Stockholm'
OR $C=$ 'Madrid'
OR $C=$ 'Amsterdam' \};

Now we can define the CITY attribute in relvars $S$ and $P$ to be of type CITY instead of just type CHAR.
8.8 By definition, there's no way to impose a constraint in SQL that's exactly equivalent to the one given in the previous answer, even if we define an explicit type, because SQL doesn't support type constraints as such. But we could impose the constraint that supplier cities in particular are limited to the same eight values by means of a suitable database constraint, and similarly for part cities. For example, we could define a base table as follows:

```
CREATE TABLE C ( CITY VARCHAR(20) , UNIQUE ( CITY ) ) ;
```

We could then "populate" this table with the eight city values:

```
INSERT INTO C ( CITY ) VALUES 'London', ,
    'Paris' ,
    'Rome' ,
    'Athens' ,
    'Oslo' ,
    'Stockholm',
    'Madrid' ,
    'Amsterdam' ;
```

Now we could define some foreign keys:

```
CREATE TABLE S (... ,
    FOREIGN KEY ( CITY ) REFERENCES C ( CITY ) ) ;
CREATE TABLE P ( ... ,
    FOREIGN KEY ( CITY ) REFERENCES C ( CITY ) ) ;
```

This approach has the advantage that it makes it easier to change the set of valid cities, if the requirement should arise.

Another approach would be to define an appropriate set of base table (or column) constraints as part of the definitions of base tables S and P. Note: SQL's "domains"-see Chapter 2-could help with this approach (if they're supported, of course!), because they could allow the pertinent constraint to be written just once and shared by all pertinent columns. For example (in outline):

```
CREATE DOMAIN CITY AS VARCHAR(20)
    CONSTRAINT ... CHECK ( VALUE IN ( 'London' ,
                                'Paris' ,
                                'Rome' ,
                                'Athens' ,
                                'Oslo' ,
                                'Stockholm' ,
                                'Madrid' ,
    `Amsterdam' ) ) ;
```

Now we can define the CITY columns in tables S and P to be of "domain CITY" instead of type VARCHAR(20), and they'll then "automatically" be subject to the required constraint.

Another approach would be to use an appropriate set of CREATE ASSERTION statements. Yet another would be to define some appropriate triggered procedures.

All of these approaches are somewhat tedious, with the first perhaps being the least unsatisfactory.

## 8.9

TYPE SNO POSSREP
\{ C CHAR CONSTRAINT
CHAR_LENGTH ( C ) $\geq 2$ AND CHAR_LENGTH ( C ) 5
AND $\overline{S u} U B S T R(C, 1,1)=' S '$
AND CAST_AS_INTEGER ( SUBSTR ( C , 2 ) ) $\geq 0$ AND CAST_AS_INTEGER ( SUBSTR ( C , 2 ) ) 9999 \} ;

I'm assuming that operators CHAR_LENGTH, SUBSTR, and CAST_AS_INTEGER are available and have the obvious semantics.

```
8.10 TYPE LINESEG POSSREP { BEGIN POINT , END POINT } ;
```

I'm assuming the existence of a user defined type called POINT as defined in the body of the chapter. Note, incidentally, that an SQL analog of the foregoing type definition wouldn't be able to use BEGIN and END as names of the corresponding attributes-attributes being (most unfortunately!) SQL's term for components of what it calls a "structured type"-because BEGIN and END are reserved words in SQL. (It would, however, be able to use the delimited identifiers "BEGIN" and "END" for the purpose. A delimited identifier in SQL is an arbitrary string of characters-including, possibly, the string of characters that forms an SQL reserved word-enclosed in what SQL calls double quotes, or in other words conventional quotation marks.)
8.11 Type POINT is an example, but there are many others-for example, you might like to think about type PARALLELOGRAM, which can "possibly be represented" in numerous different ways (how many can you think of?). As for type constraints for such a type: Conceptually, each possrep specification must include a type constraint; however, those constraints must all be logically equivalent. For example:

```
TYPE POINT
    POSSREP CARTESIAN { X RATIONAL , Y RATIONAL
    CONSTRAINT SQRT ( X ** 2 + Y ** 2 ) \leq 100.0 }
    POSSREP POLAR { R RATIONAL , THETA RATIONAL
        CONSTRAINT R \leq 100.0 } ;
```

Whether some shorthand could be provided that would effectively allow us to specify the constraint just once instead of once per possrep is a separate issue, beyond the scope of this book.
8.12 A line segment can possibly be represented by its begin and end points or by its midpoint, length, and slope (angle of inclination).
8.13 I'll give answers in terms of the INSERT, DELETE, and UPDATE shorthands, not relational assignment as such:

CX1: INSERT into S, UPDATE of STATUS in S
CX2: INSERT into S, UPDATE of CITY or STATUS in S
CX3: INSERT into S, UPDATE of SNO in S
CX4: INSERT into S, UPDATE of SNO or CITY in S
CX5: UPDATE of STATUS in S, INSERT into SP, UPDATE of SNO or PNO in SP (I'm assuming here that constraint CX6, the foreign key constraint from SP to S , is being enforced)

CX6: DELETE from S, UPDATE of SNO in S, INSERT into SP, UPDATE of SNO in SP
CX7: INSERT into LS or NLS, UPDATE of SNO in LS or NLS
CX8: INSERT into S or P , UPDATE of SNO or CITY in S, UPDATE of PNO or CITY in P
CX9: UPDATE of SNO or STATUS in S
8.14 This exercise is a little unfair, since you aren't supposed to be an expert in Tutorial D! Be that as it may, the answer is yes for KEY and FOREIGN KEY constraints, no for other constraints. Note: There's no particular reason why the answer shouldn't be yes in the latter case too, if it were thought desirable; however, any temptation to intermingle (and thereby muddle, à la SQL) specification of the pertinent relation type and specification of such constraints should be firmly resisted. Also, we'd have to be careful over what it might mean for such a "base relvar" constraint if the base relvar to whose definition it's attached happens to be empty (see the answer to Exercise 8.16 below).
8.15 (The following answer is a little simplified but captures the essence of what's going on.) Let $c$ be a base table constraint on table $T$; then the CREATE ASSERTION counterpart to $c$ is logically of the form FORALL $r$
(c)-or, in terms a little closer to concrete SQL syntax, NOT EXISTS $r$ (NOT $c$ )-where $r$ stands for a row in $T$. In other words, the logically necessary universal quantification is implicit in a base table constraint but has to be explicit in an assertion. See Chapter 10 for further explanation.
8.16 The formal reason has to do with the definition of FORALL when the applicable "range" is an empty set; again, see Chapter 10 for further explanation. Tutorial D has nothing directly analogous to base table constraints in general and thus doesn't display analogous behavior.

```
8.17 CREATE TABLE S
    ( ... ,
        CONSTRAINT CX5
            CHECK ( STATUS >= 20 OR SNO NOT IN ( SELECT SNO
                                    FROM SP
                                    WHERE PNO = 'P6' ) ) ) ;
```

```
CREATE TABLE P
```

CREATE TABLE P
( ... ,
( ... ,
CONSTRAINT CX5
CONSTRAINT CX5
CHECK ( NOT EXISTS ( SELECT *
CHECK ( NOT EXISTS ( SELECT *
FROM S NATURAL JOIN SP
FROM S NATURAL JOIN SP
WHERE STATUS < 20
WHERE STATUS < 20
AND PNO = 'P6' ) ) ) ;

```
                        AND PNO = 'P6' ) ) ) ;
```

Observe that in this latter formulation, the constraint specification makes no reference to the base table whose definition it forms part of. Thus, the same specification could form part of the definition of absolutely any base table whatsoever. (It's essentially identical to the CREATE ASSERTION version, anyway.)
8.18 The boolean expression in constraint CX1 is a simple restriction condition; the one in constraint CX5 is more complex. One implication is that a tuple presented for insertion into S can be checked against constraint CX1 without even looking at any of the values currently existing in the database, whereas the same is not true for constraint CX5.
8.19 Yes, of course it's possible; constraint CX3 does the trick. But note that, in general, neither a constraint like CX3 nor an explicit KEY specification can guarantee that the specified attribute combination satisfies the irreducibility requirement on candidate keys-though it would at least be possible to impose a syntax rule to the effect that if two distinct keys are specified for the same relvar, then neither is allowed to be a proper subset of the other. Such a rule would help, but it still wouldn't do the whole job. ${ }^{12}$
8.20

```
CREATE ASSERTION CX8 CHECK
    ( ( SELECT COUNT ( * )
        FROM ( SELECT CITY
        FROM S
        WHERE SNO = 'S1'
        UNION CORRESPONDING
        SELECT CITY
        FROM P
        WHERE PNO = 'P1' ) AS POINTLESS ) < 2 ) ;
```

[^160]Note the need for an AS clause to accompany the subquery in the outer FROM clause here, even though the name it introduces is never referenced. See the discussion in the section on EXTEND in Chapter 7 if you need to refresh your memory regarding this point.
8.21 Space reasons make it too difficult to show Tutorial D and SQL formulations side by side here, so in each case I'll show the former first and the latter second. I omit details of which operations might cause the constraints to be violated.

```
a. CONSTRAINT CXA IS EMPTY
    ( P WHERE COLO\overline{R}= 'Red' AND WEIGHT \geq 50.0 ) ;
    Or:
    CONSTRAINT CXA
    AND ( P , COLOR f 'Red' OR WEIGHT < 50 ) ;
    CREATE ASSERTION CXA CHECK ( NOT EXISTS (
    SELECT *
    FROM P
    WHERE COLOR = 'Red'
    AND WEIGHT >= 50.0 ) ) ;
Or:
    CREATE ASSERTION CXA CHECK (
    ( SELECT COALESCE ( EVERY ( COLOR <> 'Red' OR
                                WEIGHT < 50 ) ,
                TRUE )
        FROM S ) = TRUE ) ;
b. CONSTRAINT CXB IS_EMPTY (
        ( S WHERE CITY = 'London' )
            WHERE TUPLE { PNO 'P2' } & (!!SP) { PNO } ) ;
    CREATE ASSERTION CXB CHECK (
        NOT EXISTS ( SELECT * FROM S
                        WHERE CITY = 'London'
                        AND SNO NOT IN
            ( SELECT SNO FROM SP
                WHERE PNO = 'P2' ) ) ) ;
c. CONSTRAINT COUNT ( S ) = COUNT ( S { CITY } ) ;
    CREATE ASSERTION CXC CHECK ( UNIQUE ( SELECT CITY FROM S ) ) ;
d. CONSTRAINT CXD COUNT ( S WHERE CITY = 'Athens' ) < 2 ;
    CREATE ASSERTION CXD CHECK
    ( UNIQUE ( SELECT * FROM S WHERE CITY = 'Athens' ) ) ;
e. CONSTRAINT CXE IS_NOT_EMPTY ( S WHERE CITY = 'London' ) ;
CREATE ASSERTION CXE CHECK
    ( EXISTS ( SELECT * FROM S WHERE CITY = 'London' ) ) ;
```

```
f. CONSTRAINT CXF IS_NOT_EMPTY ( P WHERE COLOR = 'Red'
    AND WEIGHT < 50.0 ) ;
    CREATE ASSERTION CXF CHECK
        ( EXISTS ( SELECT * FROM P
        WHERE COLOR = 'Red'
        AND WEIGHT < 50.0 ) ) ;
g. CONSTRAINT CXG AVGX ( S , STATUS , 10 ) \geq 10 ;
CREATE ASSERTION CXG CHECK
    ( CASE
        WHEN NOT EXISTS ( SELECT * FROM S ) THEN TRUE
        ELSE ( SELECT AVG ( STATUS ) FROM S ) \geq 10
        END ) ;
```

Note: The foregoing formulations allow relvar S to be empty without violating the required constraint. But suppose the SQL formulation were simplified thus:

```
CREATE ASSERTION CXG CHECK
    ( ( SELECT AVG ( STATUS ) FROM S ) \geq 10 ) ;
```

Now if relvar S is empty, the AVG invocation returns null, and the comparison "null $\geq 10$ " returns UNKNOWN. Now, we saw in Chapter 4 that (to quote) "queries in SQL retrieve data for which the expression in the WHERE clause evaluates to TRUE, not to FALSE and not to UNKNOWN"; in other words, UNKNOWN effectively gets coerced to FALSE in the context of a query. But if the same thing happens in the context of a constraint like the one under discussion, the effect is that the constraint is considered to be satisfied. In such a context, in other words, UNKNOWN is coerced to TRUE instead of FALSE!

To pursue the point a moment longer, suppose (a) we execute a CREATE ASSERTION saying that shipment quantities must be greater than zero $(\mathrm{QTY}>0)$, and then (b) we execute the following sequence of SQL statements:

```
INSERT INTO SP ( SNO , PNO , QTY ) VALUES ( 'S5' , 'P6' , NULL ) ;
SELECT * FROM SP WHERE QTY > 0 ;
```

The INSERT will succeed-in the constraint, the expression QTY $>0$ will evaluate to UNKNOWN, which will be coerced to TRUE-but the inserted row won't appear in the result of the SELECT. (In fact, knowing that shipment quantities are supposed to be greater than zero, the user would be within his or her rights to expect that SELECT to be logically equivalent to just SELECT * FROM SP.) At the very least, therefore, the user will see a violation of The Assignment Principle in this example. I regard this state of affairs as yet another of the vast-infinite?-number of weirdnesses that nulls inevitably seem to give rise to.
h. CONSTRAINT CXH IS_EMPTY ( SP WHERE QTY > 2 * AVGX ( SP , QTY , 0 ) ) ;

## CREATE ASSERTION CXH CHECK

( CASE
WHEN NOT EXISTS ( SELECT * FROM SP ) THEN TRUE ELSE NOT EXISTS ( SELECT * FROM SP WHERE QTY > $2 *(\underset{\text { FROM }}{\operatorname{SELECT}} \underset{\mathrm{SP}}{\mathrm{AVG}})(\mathrm{QTY})$
END ) ;
i. CONSTRAINT CXI CASE WHEN COUNT ( S ) < 2 THEN TRUE ELSE IS EMPTY ( JOIN
\{ ( S WHERE STATUS $=\operatorname{MAX}(\mathrm{S}\{\operatorname{STATUS}\})$ ) \{ CITY \} , ( S WHERE STATUS $=$ MIN ( $S$ \{ STATUS \} ) ) \{ CITY \} \} ) END CASE ;

CREATE ASSERTION CXI CHECK ( CASE
WHEN ( SELECT COUNT ( * ) FROM S ) < 2 THEN TRUE ELSE NOT EXISTS
( SELECT * FROM S AS X , S AS Y WHERE X.STATUS $=(\operatorname{SELECT}$ MAX ( STATUS ) FROM $S$ ) AND Y.STATUS $=(\operatorname{SELECT} \operatorname{MIN}(S T A T U S)$ FROM $S$ ) AND X.CITY = Y.CITY ) END ) ;
j. CONSTRAINT CXJ $P$ \{ CITY \} $\subseteq \mathrm{S}$ \{ CITY \} ;

CREATE ASSERTION CXJ CHECK ( NOT EXISTS
( SELECT * FROM P WHERE NOT EXISTS
( SELECT * FROM S WHERE S.CITY = P.CITY ) ) ) ;
k. CONSTRAINT CXK IS_EMPTY (
( EXTEND $\mathrm{P}^{-}:\{\mathrm{SC}:=(\mathrm{l}!\mathrm{SP})$ JOIN S$)$ \{ CITY \} \} ) WHERE TUPLE \{ CITY CITY \} $\notin S C$ ) ;

CREATE ASSERTION CXK CHECK ( NOT EXISTS
( SELECT * FROM P WHERE NOT EXISTS
( SELECT * FROM S WHERE S.CITY = P.CITY AND EXISTS
( SELECT * FROM SP WHERE S.SNO = SP.SNO AND P.PNO = SP.PNO ) ) ) ; ;

1. The interesting thing about this exercise (or one of the interesting things, at any rate) is that it's ambiguous. It might mean every individual London supplier must supply more different kinds of part than every individual Paris supplier; or it might mean the number of different kinds of parts supplied by London suppliers considered en masse must be greater than the number of different kinds of parts supplied by Paris suppliers considered en masse; and there might be other interpretations, too. The following formulations assume the second of these interpretations, but the whole question of ambiguity is revisited in Chapter 11.

CONSTRAINT CXL
COUNT ( ( ( S WHERE CITY = 'London' ) JOIN SP ) \{ PNO \} ) >
COUNT ( ( ( S WHERE CITY = 'Paris' ) JOIN SP ) \{ PNO \} ) ;

```
CREATE ASSERTION CXL CHECK (
    ( SELECT COUNT ( DISTINCT PNO ) FROM S NATURAL JOIN SP
        WHERE CITY = 'London' ) >
    ( SELECT COUNT ( DISTINCT PNO ) FROM S NATURAL JOIN SP
                WHERE CITY = 'Paris' ) ) ;
```

m. CONSTRAINT CXM
SUM ( ( ( S WHERE CITY = 'London' ) JOIN SP ) , QTY ) >
SUM ( ( S WHERE CITY = 'Paris' ) JOIN SP ) , QTY ) ;
CREATE ASSERTION CXM CHECK (
( SELECT COALESCE ( SUM ( QTY ) , O ) FROM S NATURAL JOIN SP
WHERE CITY = 'London' ) >
( SELECT COALESCE ( SUM ( QTY ) , 0 ) FROM S NATURAL JOIN SP
WHERE CITY = 'Paris' ) ) ;
n. CONSTRAINT CXN IS EMPTY
( ( SP JOIN P ) WHERE QTY * WEIGHT > 20000.0 ) ;
CREATE ASSERTION CXN CHECK
( NOT EXISTS ( SELECT * FROM SP NATURAL JOIN P
WHERE QTY * WEIGHT > 20000.0 ) ) ;
8.22 Constraint CX22a certainly suffices (it's directly analogous to the formulation I gave for CX4 in the body of the chapter). As for constraint CX22b: Well, let's see if we can prove it does the job. First of all, to simplify the discussion, let's agree to ignore supplier names, since they're irrelevant to the matter at hand. Then we need to show, first, that if the FD $\{$ CITY $\} \rightarrow\{$ STATUS $\}$ holds, then $S$ is equal to the join of its projections on $\{$ SNO,CITY $\}$ and $\{$ CITY,STATUS $\}$; second, if S is equal to the join of its projections on $\{$ SNO,CITY $\}$ and $\{$ CITY,STATUS $\}$, then the FD $\{\mathrm{SNO}\} \rightarrow\{$ CITY $\}$ holds. Denote $\mathrm{S}\{$ SNO,CITY $\}$ and S $\{$ CITY,STATUS $\}$ by SC and CT, respectively, and denote JOIN $\{\mathrm{SC}, \mathrm{CT}\}$ by J. Adopting an obvious shorthand notation for tuples, then, we have for the first part of the proof:

■ Let $(s, c, t) \in \mathrm{S}$. Then $(s, c) \in \mathrm{SC}$ and $(c, t) \in \mathrm{CT}$, and so $(s, c, t) \in \mathrm{J}$; so $\mathrm{S} \subseteq \mathrm{J}$.

- Let $(s, c, t) \in \mathrm{J}$. Then $(s, c) \in \mathrm{SC}$; hence $\left(s, c, t^{\prime}\right) \in \mathrm{S}$ for some $t^{\prime}$. But $t=t^{\prime}$ thanks to the $\mathrm{FD}, \mathrm{so}(s, c, t) \in \mathrm{S}$ and hence $\mathrm{J} \subseteq \mathrm{S}$. It follows that $\mathrm{S}=\mathrm{J}$.

Turning to the second part:

- Let both $(s, c, t)$ and $\left(s^{\prime}, c, t^{\prime}\right) \in \mathrm{S}$. Then $(s, c) \in \mathrm{SC}$ and $\left(c, t^{\prime}\right) \in \mathrm{CT}$, so $\left(s, c, t^{\prime}\right) \in \mathrm{J}$; hence $\left(s, c, t^{\prime}\right) \in \mathrm{S}$. But $\{\mathrm{SNO}\}$ is a key for S and so $t=t^{\prime}$ (because certainly $(s, c, t) \in \mathrm{S}$ ); hence the FD $\{$ CITY $\} \rightarrow\{$ STATUS $\}$ holds.

It follows that constraint CX22b does indeed represent the FD, as required. Note carefully, however, that it does so only because we were able to appeal (in the second part of the proof) to the fact that $\{\mathrm{SNO}\}$ is a key for relvar S ; it would not correctly represent the desired FD, absent that key constraint.
8.23 It guarantees that the constraint is satisfied by an empty database (i.e., one containing no relvars).
8.24 Suppose we were to define a relvar SC with attributes SNO and CITY and predicate Supplier SNO has no office in city CITY. Suppose further that supplier S1 has an office in just ten cities. Then The Closed World

Assumption would imply that relvar SC must have $n-10$ tuples for supplier S1, where $n$ is the total number of valid cities (possibly in the entire world)!
8.25 We need a multiple assignment (if we are to do the delete in a single statement as requested). Let the supplier number of the specified supplier be $x$. Then:

DELETE $S$ WHERE SNO $=x$, DELETE SP WHERE SNO $=x$;
The individual assignments (DELETEs) can be specified in either order.
8.26 These constraints can't be expressed declaratively in either SQL or Tutorial D, since neither of those languages currently has any direct support for transition constraints. Triggered procedures can be used, but details of triggered procedures are beyond the scope of this book. However, here are possible formulations using the "primed relvar name" convention discussed briefly in the section "Miscellaneous Issues":
a. CONSTRAINT CXA IS EMPTY
( P WHERE SUM ( !!SP , QTY ) > SUM ( !!SP' , QTY ) ) ;
b. CONSTRAINT CXB

IS_EMPTY ( ( ( $S^{\prime}$ WHERE CITY = 'Athens' ) \{ SNO \} ) JOIN S ) WHERE CITY $\neq$ 'Athens' AND CITY $\neq$ 'London' AND CITY $\neq$ 'Paris' )
AND IS_EMPTY ( ( ( $\mathrm{S}^{\prime}$ WHERE CITY $=$ 'London' ) \{ SNO \} ) JOIN S ) WHERE CITY $\neq$ 'London' AND CITY $\neq$ 'Paris' ) ;
c. CONSTRAINT CXC IS EMPTY
( S WHERE SUM $\overline{( }!!S P$, QTY ) < 0.5 * SUM ( !!SP' , QTY ) ) ;
The qualification "in a single update" is important because we aren't trying to outlaw the possibility-and in fact we can't-of reducing the total shipment quantity by, say, one third in one update and then another third in another.

### 8.27 No answer provided.

8.28 SQL fails to support type constraints for a rather complicated reason having to do with type inheritance. The specifics are beyond the scope of this book; if you're interested, you can find a detailed discussion in the book Databases, Types, and the Relational Model: The Third Manifesto, by Hugh Darwen and myself (see Appendix G). As for consequences, one is that when you define a type in SQL, you can't even specify the values that make up that type!-except for the a priori constraint imposed by the representation-and so, absent further controls, you can wind up with incorrect data in the database (even nonsensical data, like a shoe size of 1000 , or even -1000 ).
8.29 In principle they all apply-though Tutorial D in particular deliberately provides no way of specifying constraints, other than a priori ones, for either nonscalar or system defined types.
8.30 The generic expansion of an arbitrary UPDATE in terms of DELETE and INSERT can be inferred by straightforward generalization from the following simple, albeit somewhat abstract, example. Let relvar $R$ have just two attributes, $X$ and $Y$, and consider the following UPDATE on $R$ :

```
UPDATE R WHERE }X=x : { Y:= y } 
```

Let the current ("old") value of $R$ be $r$. Define $d$ and $i$ as follows:

```
d = {t:t { R AND t.X = x }
i}={\mp@subsup{t}{}{\prime}:\operatorname{EXISTS}t\ind(t.X=\mp@subsup{t}{}{\prime}.X) AND t'.Y = y 
```

Then the original UPDATE is logically equivalent to the following assignment:

```
R := r MINUS ( d UNION i ) ;
```

Or equivalently to the following multiple assignment:

```
DELETE R d, INSERT R i ;
```


## CHAPTER 9

9.1 VAR NON COLOCATED VIRTUAL

```
    ( ( S { SNO } JOIN P { PNO } ) NOT MATCHING ( S JOIN P ) )
        KEY { SNO , PNO } ;
```

CREATE VIEW NON COLOCATED AS
( SELECT SN̄O , PNO
FROM $\mathrm{S}, \mathrm{P}$
WHERE S.CITY <> P.CITY )
/* UNIQUE ( SNO , PNO ) */ ;
9.2 Substituting the view definition for the view reference in the outer FROM clause, we obtain:

```
SELECT DISTINCT STATUS , QTY
FROM ( SELECT SNO , SNAME , STATUS , PNO , QTY
        FROM S NATURAL JOIN SP
        WHERE CITY = 'London' ) AS Temp
WHERE PNO IN
    ( SELECT PNO
        FROM P
        WHERE CITY <> 'London' )
```

This simplifies (potentially!) to:

```
SELECT DISTINCT STATUS , QTY
FROM S NATURAL JOIN SP
WHERE CITY = 'London'
AND PNO IN
    ( SELECT PNO
        FROM P
        WHERE CITY <> 'London' )
```

9.3 The sole key is $\{\mathrm{SNO}, \mathrm{PNO}\}$. The predicate is: Supplier SNO is under contract, is named SNAME, has status STATUS, is located in London, and supplies part PNO in quantity QTY.
9.4 Note that a . and b . are expressions, the rest are statements.
a. ( P WHERE WEIGHT > 14.0 ) WHERE COLOR $=$ 'Green'

This expression can be simplified to:

```
P WHERE WEIGHT > 14.0 AND COLOR = 'Green'
```

The simplification is worth making, too, because the first formulation implies two passes over the data while the second implies just one.
b. ( EXTEND ( P WHERE WEIGHT > 14.0 ) :
\{ W := WEIGHT + 5.3 \} ) \{ PNO , W \}
c. INSERT ( P WHERE WEIGHT > 14.0 )

RELATION \{ TUPLE \{ PNO 'P9' , PNAME 'Screw' , WEIGHT 15.0, COLOR 'Purple' , CITY 'Rome' \} \} ;

Observe that this INSERT is logically equivalent to a relational assignment in which the target is specified as something other than a simple relvar reference. The ability to update views implies that such assignments must indeed be legitimate, both syntactically and semantically, although the corresponding syntax is currently not supported in Tutorial D (neither for assignment in general nor for INSERT in particular). Note: Similar but not identical remarks apply to parts d. and e. below.
d. DELETE ( P WHERE WEIGHT > 14.0 ) WHERE WEIGHT < 9.0 ;

This syntax is currently illegal, although oddly enough the following (which is obviously logically equivalent to that just shown) is legal:

DELETE P WHERE WEIGHT > 14.0 AND WEIGHT < 9.0 ;

Of course, this DELETE is actually a "no op," because WEIGHT > 14.0 AND WEIGHT $<9.0$ is a logical contradiction. Do you think the optimizer would be able to recognize this fact?
e. UPDATE ( P WHERE WEIGHT > 14.0 ) WHERE WEIGHT $=18.0$ :

```
                        { COLOR := 'White' } ;
```

Again this syntax is currently illegal, but the following is legal:

```
UPDATE P WHERE WEIGHT > 14.0 AND WEIGHT = 18.0 :
    { COLOR := 'White' } ;
```

Do you think the optimizer would be able to recognize the fact that the restriction condition WEIGHT $>14.0$ here can be ignored?
9.5 Here first is an SQL version of the view definition from Exercise 9.4:

```
CREATE VIEW HP AS
    ( SELECT PNO , PNAME , COLOR , WEIGHT , CITY
        FROM P
        WHERE WEIGHT > 14.0 )
        /* UNIQUE ( PNO ) * / ;
```

For parts a.-e., I first show an SQL analog of the Tutorial D formulation, followed by the expanded form:
a. SELECT HP.PNO , HP.PNAME , HP.COLOR , HP.WEIGHT , HP.CITY

FROM HP
WHERE HP.COLOR = 'Green'
SELECT HP.PNO, HP.PNAME , HP.COLOR , HP.WEIGHT , HP.CITY
FROM ( SELECT PNO , PNAME , COLOR, WEIGHT , CITY
FROM P
WHERE WEIGHT > 14.0 ) AS HP
WHERE HP.COLOR = 'Green'

I leave further simplification, here and in subsequent parts, as a subsidiary exercise (barring explicit statements to the contrary).
b. SELECT PNO, WEIGHT + 5.3 AS W FROM HP

SELECT HP.PNO , HP.WEIGHT + 5.3 AS W
FROM ( SELECT P.PNO , P.PNAME , P.COLOR , P.WEIGHT , P.CITY FROM P WHERE P.WEIGHT > 14.0 ) AS HP
c. INSERT INTO HP ( PNO , PNAME , WEIGHT , COLOR , CITY ) VALUES ( 'P9' , 'Screw' , 15.0 , 'Purple' , 'Rome' ) ;

INSERT INTO ( SELECT P.PNO , P.PNAME , P.WEIGHT , P.COLOR , P.CITY FROM P WHERE P.WEIGHT > 14.0 ) AS HP
VALUES ( 'P9' , 'Screw' , 15.0 , 'Purple' , 'Rome' ) ;
The remarks regarding Tutorial D in the solution to Exercise 9.4c apply here also, mutatis mutandis.
d. DELETE FROM ( SELECT P.PNO , P.PNAME , P.COLOR , P.WEIGHT , P.COLOR FROM P WHERE P.WEIGHT > 14.0 ) AS HP
WHERE HP.WEIGHT < 9.0 ;
This transformed version isn't valid SQL syntax, but a valid equivalent is a little easier to find:
DELETE FROM P WHERE WEIGHT > 14.0 AND WEIGHT < 9.0 ;
(As noted in the answer to Exercise 9.4d, this DELETE is actually a "no op.")
e. UPDATE ( SELECT P.PNO , P.PNAME , P.COLOR , P.WEIGHT , P.COLOR FROM P WHERE P.WEIGHT > 14.0 ) AS HP
SET COLOR = 'White'
WHERE HP.WEIGHT $=18.0$;

Syntactically valid equivalent:

```
UPDATE P
SET COLOR = 'White'
WHERE WEIGHT = 18.0 AND WEIGHT > 14.0 ;
```

9.6 Here are some:

- If users are to operate on views instead of base relvars, it's clear that those views should look to the user as much like base relvars as possible. In accordance with The Principle of Interchangeability, in fact, the user shouldn't have to know they're views at all but should be able to treat them as if they were base relvars. And just as the user of a base relvar needs to know what keys that base relvar has, so the user of a view needs to know what keys that view has. Explicitly declaring those keys is one way to make that information available.
- The DBMS might be unable to infer keys for itself (this is almost certainly the case, in general, with SQL products on the market today). Explicit declarations are thus likely to be the only means available (to the DBA, that is) of informing the DBMS, as well as the user, of the existence of such keys.
- Even if the DBMS were able to infer keys for itself, explicit declarations would at least enable the system to check that its inferences and the DBA's explicit specifications were consistent.
- The DBA might have some knowledge that the DBMS doesn't, and might thus be able to improve on the DBMS's inferences.
- As shown in the body of the chapter, such a facility could provide a simple and convenient way of stating certain important constraints that could otherwise be stated only in circumlocutory fashion.

Subsidiary exercise: Which if any of the foregoing points do you think apply not just to key constraints in particular but to integrity constraints in general?
9.7 One example is as follows: The suppliers relvar is equal to the join of its projections on \{SNO,SNAME\}, \{SNO,STATUS\}, and \{SNO,CITY\}-just so long as appropriate constraints are in force, that is (what are those constraints exactly?). So we could make those projections base relvars and make the join a view.
9.8 Here are some pertinent observations. First, the replacement process itself involves several steps, which might be summarized as follows:

```
/* define the new base relvars: */
VAR LS BASE RELATION
    { SNO CHAR , SNAME CHAR , STATUS INTEGER , CITY CHAR }
    KEY { SNO } ;
VAR NLS BASE RELATION
    { SNO CHAR , SNAME CHAR , STATUS INTEGER , CITY CHAR }
    KEY { SNO } ;
```

```
/* copy the data to the new base relvars: */
LS := ( S WHERE CITY = 'London' ) ;
NLS := ( S WHERE CITY f 'London' ) ;
/* drop the old relvar: */
DROP VAR S ;
/* create the desired view: */
VAR S VIRTUAL ( LS D_UNION NLS ) KEY { SNO } ;
```

Now we must do something about the foreign key in relvar SP that references the old base relvar S. Clearly, it would be best if that foreign key could now be taken as referring to the view S instead; ${ }^{13}$ if this is impossible (as it typically is in today's products), then we might want to define another base relvar as follows:

```
VAR SS BASE RELATION { SNO CHAR } KEY { SNO } ;
```

And copy the data (obviously this must be done before dropping base relvar S ):

```
SS := S { SNO } ;
```

Now we need to add the following foreign key specification to the definitions of relvars LS and NLS:

```
FOREIGN KEY { SNO } REFERENCES SS
```

Finally, we must change the specification for the foreign key $\{\mathrm{SNO}\}$ in relvar SP to refer to SS instead of S .
9.9 a. No answer provided-except to note that if it's difficult to answer the question for some product, then that very fact is part of the point of the exercise in the first place. b. Same as for part a., but more so. c. Ditto.
9.10 For the distinction, see the body of the chapter. SQL doesn't support snapshots at the time of writing. (It does support CREATE TABLE AS—see the last part of the answer to Exercise 1.16, earlier in this appendix—which allows a base table to be initialized when it's created, but CREATE TABLE AS has no REFRESH option.)
9.11 "Materialized view" is a deprecated term for a snapshot. The term is deprecated because it muddies concepts that are logically distinct and ought to be kept distinct-by definition, views simply aren't materialized, so far as the model is concerned-and it's leading us into a situation in which we no longer have a clear term for a concept we did have a clear term for, originally. It should be firmly resisted. (I realize I've probably already lost this battle, but I'm an eternal optimist.) In fact, I'm tempted to go further; it seems to me that people who advocate use of the term

[^161]"materialized view" are betraying their lack of understanding of the relational model in particular and the distinction between model and implementation in general.
9.12 First, here's a definition of Design b. in terms of Design a. (pertinent constraints included):

```
VAR SSP VIRTUAL ( S JOIN SP )
    KEY { SNO , PNO } ;
VAR XSS VIRTUAL ( S NOT MATCHING SP )
    KEY { SNO } ;
CONSTRAINT B_FROM_A IS_EMPTY ( SSP JOIN XSS ) ;
```

(Constraint B_FROM_A and the specified key constraints are together what we would have to tell the user if we wanted the user to think of relvars SSP and XSS as base relvars, not views.) And here's a definition of Design a. in terms of Design b.:

```
VAR S VIRTUAL ( XSS D_UNION SSP { ALL BUT PNO , QTY } )
    KEY { SNO } ;
VAR SP VIRTUAL ( SSP { SNO , PNO , QTY } )
    KEY { SNO , PNO } ;
CONSTRAINT A_EROM_B IS_EMPTY ( SP NOT MATCHING S ) ;
```

Given these constraints, the designs are information equivalent. But Design a. is superior, because the relvars in that design are in fifth normal form. By contrast, relvar SSP in Design b. isn't even in second normal form; as a consequence, it displays redundancy and is thereby subject to certain "update anomalies." Consider also what happens with Design b. if some supplier ceases to supply any parts, or used not to supply any but now does. Further discussion of the problems with Design b. is beyond the scope of this book; I just note that (as the example suggests) database design disciplines like normalization can help in the task of choosing "the best" design from a set of designs that are information equivalent.

Incidentally, I note in passing that-given that $\{\mathrm{SNO}\}$ is a key for relvar S-constraint A_FROM_B here shows another way of formulating a referential constraint. In practice, of course, it would be simpler just to include the following foreign key specification as part of the definition of relvar SP:

FOREIGN KEY \{ SNO \} REFERENCES S
9.13 The following discussion relies on the fact that (as Appendix A explains in more detail) databases are really variables-i.e., we really need to draw a distinction between database values and database variables, analogous to that between relation values and relation variables. Let DBD1 and DBD2 be (logical) database designs; let DB1 and $D B 2$ be database variables conforming to $D B D 1$ and $D B D 2$, respectively; and let $d b 1$ and $d b 2$ be the current values of $D B 1$ and $D B 2$, respectively. Further, let there exist mappings $M 12$ and $M 21-$ i.e., sequences of relational algebra operations, loosely speaking-that transform $d b 1$ into $d b 2$ and $d b 2$ into $d b 1$, respectively. Then $d b 1$ and $d b 2$ are information equivalent, meaning that for every expression involving only relations from $d b 1$, there's an expression involving only relations from $d b 2$ that yields the same result (and vice versa).

Now let database variables $D B 1$ and $D B 2$ be such that for every possible value $d b 1$ of $D B 1$ there exists an information equivalent value $d b 2$ of $D B 2$ (and vice versa). Then $D B 1$ and $D B 2$ per se are information equivalent, as are the corresponding designs $D B D 1$ and $D B D 2$.

Now let database variables $D B 1$ and $D B 2$, as well as their current values $d b 1$ and $d b 2$, be information equivalent. Let $U 1$ be an update on $D B 1$ that transforms $d b 1$ into $d b 1^{\prime}$. Then there must exist an update $U 2$ on $D B 2$ that transforms $d b 2$ into $d b 2^{\prime}$, such that $d b 1^{\prime}$ and $d b 2^{\prime}$ are information equivalent. Note that the remarks of this paragraph apply in particular to the case in which $D B 1$ consists only of base relvars and $D B 2$ consists only of views of relvars in DBI.

Finally, let database variables $D B 1$ and $D B 2$, as well as their current values $d b 1$ and $d b 2$, not be information equivalent. Then there must exist an expression involving only relations from $d b l$ with no counterpart involving only relations from $d b 2$ (or vice versa), and there must exist an update on $D B 1$ with no counterpart on $D B 2$ (or vice versa)-speaking somewhat loosely in both cases. Again note that the remarks of this paragraph apply in particular to the case in which $D B 1$ consists only of base relvars and $D B 2$ consists only of views of relvars in $D B 1$.
9.14 (You might want to review the section "The Reliance on Attribute Names" in Chapter 6 before reading this answer.) Yes, views should indeed have been sufficient to solve the logical data independence problem. But the trouble with views as conventionally understood is that a view definition specifies both the application's perception of some portion of the database and the mapping between that perception and the database "as it really is." In order to achieve the kind of data independence I'm talking about here, those two specifications need to be kept separate (and the mapping specification in particular should be hidden from the user).

## CHAPTER 10

First of all, in the body of the chapter I asked which of the following natural language sentences were legal propositions. My own answers are as follows (but some of them might be open to debate):

- Bach is the greatest musician who ever lived. Yes.

What's the time? No. It doesn't make sense to ask "Is it true that $p$ ?" where $p$ is "What's the time?"

- Supplier S2 is located in some city, $x$. My own answer here is yes, but you might argue reasonably argue that it should be no. In fact, the example is a good illustration of the ambiguity problem discussed earlier in the chapter, my answer is based on the assumption that the phrase "some city, $x$ " could be abbreviated to just "some city" without changing the overall meaning of the sentence, ${ }^{14}$ but you might reasonably argue the opposite. Compare and contrast "We both have the same favorite author, $x$ " (see below).
- Some countries have a female president. Yes.
- All politicians are corrupt. Yes.

■ Supplier S1 is located in Paris. Yes (though it's false, given our usual sample values).

- We both have the same favorite author, $x$. No. Here I'm assuming that the phrase "the same favorite author, $x$ " couldn't be abbreviated to just "the same favorite author" without changing the overall meaning of the sentence (but you could reasonably argue the opposite). If my assumption is correct, then $x$ is a variable (better: a parameter), and until we know what argument is to be substituted for that parameter-i.e., until we

[^162]know what that $x$ stands for, as it were-we can't assign a truth value to the sentence. But when we do know-i.e., when we substitute Jane Austen, say, for $x$-then the sentence does become a proposition.

- Nothing is heavier than lead. Yes.
- It will rain tomorrow. No. It doesn't make sense to ask "Is it true that it will rain tomorrow?"-at least, not if you're expecting an answer (yes or no) that's guaranteed to be unequivocally correct. Alternatively, we might say that tomorrow here is a variable, and we can't assign a truth value to the sentence until we know what that variable stands for.
- Supplier S6's city is unknown. Yes. See Appendix C for further discussion of propositions like this one.
10.1 See the answer to Exercise 4.10 earlier in this appendix.
10.2 This is one of De Morgan's laws. It's proved in Chapter 11.
10.3 Consider the following truth table:

| $p$ | $q$ | NOT $p$ <br> $(=x)$ | $p$ OR $q$ <br> $(=y)$ | $x$ AND $y$ | $(x$ AND $y)$ IMPLIES $q$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $T$ | T | F | T | F |  |
| T | F | F | T | F | T |
| F | T | T | T | T | T |
| F | F | T | F | F | T |

Since the final column contains $T$ (true) in every position, the given expression is indeed a tautology.
10.4 First of all, it's easy to see that we don't need both OR and AND, because

```
p AND q \equiv NOT ( NOT ( p ) OR NOT (q) )
```

and

```
p OR q \equivNOT (NOT ( p ) AND NOT (q ) )
```

These equivalences are easily established by means of truth tables, as in the answer to Exercise 10.3 above (in fact, the first is the contrapositive of the equivalence-one of De Morgan's laws-from Exercise 10.2). What they show is that OR and AND can each be defined in terms of the other, together with NOT. It follows that we can freely use both OR and AND in what follows.

Now consider the connectives involving a single proposition $p$. Let $c(p)$ be the connective under consideration. Then the possibilities are as follows:

```
c(p) \equiv p OR NOT ( p ) /* always TRUE */
c(p) \equiv p AND NOT ( p ) /* always FALSE */
c(p) \equiv p /* identity */
c(p) \equivNOT ( p ) /* NOT */
```

Now consider the connectives involving two propositions $p$ and $q$. Let $c(p, q)$ be the connective under consideration. Then the possibilities are as follows:

```
c(p,q) \equiv p OR NOT ( p ) OR q OR NOT ( q )
c(p,q) \equiv p AND NOT ( p ) AND q AND NOT ( q )
c(p,q) \equiv p
c(p,q) \equiv NOT ( p )
c(p,q) \equiv q
c(p,q) \equiv NOT ( q )
c(p,q) \equiv pOR q
c(p,q) \equiv p AND q
c(p,q) \equiv p OR NOT ( q )
c(p,q) \equiv p AND NOT ( q )
c(p,q) \equivNOT ( p ) OR q
c(p,q) \equivNOT ( p ) AND q
c(p,q) \equivNOT ( p ) OR NOT ( q )
c(p,q) \equivNOT ( p ) AND NOT (q)
c(p,q) \equiv(NOT ( p ) OR q ) AND ( NOT ( q ) OR p )
c(p,q) \equiv(NOT ( p ) AND q ) OR (NOT ( q ) AND p )
```

As a subsidiary exercise, and in order to convince yourself that the foregoing definitions do indeed cover all of the possibilities, you might like to construct the corresponding truth tables (and compare them with those given in the answer to Exercise 4.10 earlier in this appendix).

Turning to part (b) of the exercise: Actually there are two such primitives, NOR and NAND, often denoted by a down arrow, " $\downarrow$ " (the Peirce arrow) and a vertical bar, "|" (the Sheffer stroke), respectively. Here are the truth tables:

| NOR | T | F | NAND | T | F |
| :---: | :---: | :---: | :---: | :---: | :---: |
| T | F | F | T | F | T |
| F | F | T | F | T | T |

As these tables suggest, $p \downarrow q$ (" $p$ NOR $q$ ") is equivalent to NOT ( $p$ OR $q$ ) and $p \mid q$ (" $p$ NAND $q "$ ) is equivalent to NOT ( $p$ AND $q$ ). In what follows, I'll concentrate on NOR (I'll leave NAND to you). Observe that this connective can be thought of as "neither nor" ("neither the first operand nor the second is true"). I now show how to define NOT, OR, and AND in terms of this operator:


For example, let's take a closer look at the case of $p$ AND $q$ ( $\mathrm{I}^{\prime} l l$ leave the $\operatorname{NOT}(p)$ and $p$ OR $q$ cases to you):

| $p$ | $q$ | $p \downarrow_{p}$ | $q \downarrow q$ | $\left(p \downarrow_{p)} \downarrow(q \downarrow q)\right.$ |
| :---: | :---: | :---: | :---: | :---: |
| T | T | F | F | T |
| T | F | F | T | F |
| F | T | T | F | F |
| F | F | T | T | F |

This truth table shows that $(p \downarrow p) \downarrow(q \downarrow q)$ is equivalent to $p$ AND $q$, because its first, second, and final columns are identical to the corresponding columns in the truth table for AND:

| $p$ | $q$ | $p$ AND $q$ |
| :--- | :--- | :--- |
| $T$ | $T$ | T |
| T | F | F |
| F | T | F |
| F | F | F |

Since we've already seen that all of the other connectives can be expressed in terms of NOT, OR, and AND, the overall conclusion follows.
10.5 (a) "The sun is a star" and "the moon is a star" are both propositions, though the first is true and the second false. (b) The sun satisfies the predicate, the moon doesn't.
10.6 This exercise points up the question of ambiguity once more. If the predicate means $x$ has exactly two moons, then clearly Jupiter doesn't satisfy it. But if it means $x$ has at least two moons (and maybe more), then Jupiter does satisfy it. Once again, then, we see how logic forces us-or does its best to force us, at any rate-into thinking clearly and saying exactly what we mean.
10.7 A parameter can be replaced by any argument whatsoever, just so long as it's of the right type. A designator isn't, and in fact can't be, replaced by anything at all; instead-just like a variable reference in a programming language, in fact-it simply "designates," or denotes, the value of the pertinent variable at the pertinent time (i.e., when the constraint is checked, in the case at hand).
10.8 "Get names of suppliers such that there exists a shipment-a single shipment, that is-that links them to every part." The query probably isn't very sensible; note that it will return either (a) all supplier names if the cardinality of relvar P is less than two or (b) an empty result otherwise.
10.9 The following SQL expressions are deliberately meant to be as close to their relational counterparts (as given in the body of the chapter) as possible. See Chapter 11 for further discussion.

Example 1: Get all pairs of supplier numbers such that the suppliers concerned are colocated.

```
SELECT SX.SNO AS SA , SY.SNO AS SB
FROM S AS SX , S AS SY
WHERE SX.CITY = SY.CITY
AND SX.SNO < SY.SNO
```

Example 2: Get supplier names for suppliers who supply at least one red part.

```
SELECT DISTINCT SX.SNAME
FROM S AS SX
WHERE EXISTS
        ( SELECT *
        FROM SP AS SPX
        WHERE EXISTS
    ( SELECT *
        FROM P AS PX
        WHERE PX.COLOR = 'Red'
        AND SX.SNO = SPX.SNO
        AND SPX.PNO = PX.PNO ) )
```

Example 3: Get supplier names for suppliers who supply at least one part supplied by supplier S2.

```
SELECT DISTINCT SX.SNAME
FROM S AS SX
WHERE EXISTS
    ( SELECT *
        FROM SP AS SPX
        WHERE EXISTS
            ( SELECT *
                FROM SP AS SPY
                WHERE SX.SNO = SPX.SNO
                AND SPX.PNO = SPY.PNO
                AND SPY.SNO = 'S2' ) )
```

Example 4: Get supplier names for suppliers who don't supply part P2.

```
SELECT DISTINCT SX.SNAME
FROM S AS SX
WHERE NOT EXISTS
    ( SELECT *
        FROM SP AS SPX
        WHERE SPX.SNO = SX.SNO
        AND SPX.PNO = 'P2' )
```

Example 5: For each shipment, get full shipment details, including total shipment weight.

```
SELECT SPX.* , PX.WEIGHT * SPX.QTY AS SHIPWT
FROM P AS PX, SP AS SPX
WHERE PX.PNO = SPX.PNO
```

Example 6: For each part, get the part number and the total shipment quantity.

```
SELECT PX.PNO , ( SELECT COALESCE ( SUM ( SPX.QTY ) , 0 )
                                    FROM SP AS SPX
                                    WHERE SPX.PNO = PX.PNO ) AS TOTQ
FROM P AS PX
```

Example 7: Get part cities that store more than five red parts.

```
SELECT DISTINCT PX.CITY
FROM P AS PX
WHERE ( SELECT COUNT ( * )
        FROM P AS PY
        WHERE PY.CITY = PX.CITY
        AND PY.COLOR = 'Red' ) > 5
```

10.10 The following truth table shows (how, exactly?) that AND is associative; the proof for OR is analogous.

| $p$ | 9 | $r$ | $p$ AND $q$ | ( $p$ AND $q$ ) AND $r$ | $q$ AND $r$ | $p$ AND | ( $q$ AND r) |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| T | T | T | T | T | T |  | T |
| T | T | F | F | F | F |  | F |
| T | F | T | F | F | F |  | F |
| T | F | F | F | F | F |  | F |
| F | T | T | F | F | F |  | F |
| F | T | F | F | F | F |  | F |
| F | F | T | F | F | F |  | F |
| F | F | F | F | F | F |  | F |

Note: The formal name for AND is conjunction, and its operands (i.e., the terms being ANDed together) are called conjuncts. Similarly, the formal name for OR is disjunction, and its operands are called disjuncts. Further, since (a) AND and OR are commutative as well as associative and (b) they both possess an identity value, we can define $n$-adic versions, as follows:

- AND $\{p 1, p 2, \ldots, p n\}$ is defined to be equivalent to

```
( p1 ) AND ( p2 ) AND ... AND ( pn ) AND TRUE
```

If none of the $p$ 's involves any ANDs, this expression overall is said to be in conjunctive normal form (CNF).

- OR $\{p 1, p 2, \ldots, p n\}$ is defined to be equivalent to

```
( p1 ) OR ( p2 ) OR ... OR ( pn ) OR FALSE
```

If none of the $p$ 's involves any ORs, this expression overall is said to be in disjunctive normal form (DNF).
10.11 a. Not valid (suppose $x$ ranges over an empty set and $q$ is TRUE; then EXISTS $x(q)$ is FALSE). b. Not valid (suppose $x$ ranges over an empty set and $q$ is FALSE; then FORALL $x(q)$ is TRUE). c. Valid. d. Valid. e. Not valid (suppose $x$ ranges over an empty set; then FORALL $x(p(x)$ ) is TRUE but EXISTS $x(p(x))$ is FALSE, and TRUE $\Rightarrow$ FALSE is FALSE). f. Not valid (suppose $x$ ranges over an empty set; then EXISTS $x$ (TRUE) is FALSE). g. Not valid (suppose $x$ ranges over an empty set; then FORALL $x$ (FALSE) is TRUE). h. Valid. i. Not valid (e.g., the fact that exactly one integer is equal to zero doesn't imply that all integers are equal to zero). j. Not valid (e.g., the fact that all days are 24 hours long and the fact there exists at least one day that's 24 hours long don't together imply that exactly one day is 24 hours long). k. Valid. Note: (Valid!) equivalences and implications like those under discussion here (and in the next exercise) can be used as a basis for a set of calculus expression transformation rules, much like the algebraic expression transformation rules discussed in Chapter 6. See Chapter 11 for further discussion.
10.12 a. Valid. b. Valid. c. Valid. d. Valid. e. Not valid (e.g., saying that for every integer $y$ there exists a greater integer $x$ isn't the same as saying there exists an integer $x$ that's greater than all integers $y$ ). f. Valid.
10.13 I give solutions only where there's some significant point to be made regarding the solution in question. For cross reference purposes, I show the numbers of the original exercises in italics. Exercises from Chapter 6:
6.12 The following relational calculus expressions denote TABLE_DEE and TABLE_DUM, respectively:

```
{ } WHERE TRUE
```

```
{ } WHERE FALSE
```

And this expression denotes the projection of the current value of relvar S on no attributes:
\{ \} WHERE EXISTS ( SX )

The relational calculus isn't usually considered as having a direct counterpart to Tutorial D's $r$ \{ALL BUT $\ldots\}$, but there's no reason in principle why it shouldn't.
6.15 The relational calculus isn't usually considered as having a direct counterpart to Tutorial D's D_UNION or I_MINUS, but there's no reason in principle why it shouldn't.
10.13 (cont.) Exercises from Chapter 7:
7.1
d. \{ PX \} WHERE SUM ( SPX WHERE SPX.PNO = PX.PNO, QTY ) < 500
e. \{ PX \} WHERE EXISTS ( SX WHERE SX.CITY = PX.CITY )
j. \{ PX , SCT := COUNT ( SPX WHERE SPX.PNO = PX.PNO ) \}
7.8 A relational calculus analog of the Tutorial D expression SP GROUP ( $\}$ AS X) is:
$\{\operatorname{SPX}, X:=\{ \}\}$
7.11 \{ SX , PNO_REL $:=$ \{ SPX.PNO \} WHERE SPX.SNO = SX.SNO \}
7.12 In practice we need analogs of the conventional INSERT, DELETE, and UPDATE (and relational assignment) operators that are in keeping with a calculus style rather than an algebraic one (and this observation is true regardless of whether we're talking about relvars with RVAs, as in the present context, or without them). Further details are beyond the scope of the present book, but in any case are straightforward. No further answer provided.
10.13 (cont.) Exercises from Chapter 8: No answers provided.
10.13 (cont.) Exercises from Chapter 9: No answers provided.
10.14 Recall from the body of the chapter that the set over which a range variable ranges is always the body of some relation-usually but not always the relation that's the current value of some relvar (emphasis added). In this example, the range variable ranges over what is, in effect, a union:

```
RANGEVAR CX RANGES OVER { SX.CITY } , { PX.CITY } ;
{ CX } WHERE TRUE
```

Note that the definition of range variable CX makes use of range variables SX and PX, which I assume to have been previously defined.
10.15 In order to show that SQL is relationally complete, it's sufficient to show that (a) there exist SQL expressions for each of the algebraic operators restrict, project, product, union, and difference (because as we saw in Chapter 6, all of the other algebraic operators discussed in this book can be defined in terms of these five), ${ }^{15}$ and (b) the operands to those SQL expressions can be arbitrarily complex SQL expressions in turn. So let's try.

First of all, as we know, SQL effectively does support the relational algebra RENAME operator, thanks to the availability of the optional AS specification on items in the SELECT clause. ${ }^{16}$ We can therefore ensure that tables do all have proper column names, and hence that the operands to product, union, and difference in particular satisfy the requirements of the algebra with respect to column naming. Furthermore-provided those column naming requirements are indeed satisfied-the SQL column name inheritance rules in fact coincide with those of the algebra as described in Chapter 6.

Here then are SQL expressions corresponding approximately to the five primitive operators mentioned above:


Moreover, (a) R,R1, and $R 2$ in the SQL expressions shown above are all table expressions (in the sense in which I've been using this latter term throughout this book up to this point), and (b) if we take any of those expressions and enclose it in parentheses, what results is a table expression in turn. ${ }^{17}$ It follows that SQL is indeed relationally complete. Or is it? Unfortunately, the answer is no. The reason is that there's a slight (?) glitch in the foregoing argument-SQL fails to support projection on no columns at all, because it doesn't support empty

[^163]commalists in the SELECT clause. As a consequence, it doesn't support TABLE_DEE or TABLE_DUM, and therefore it isn't relationally complete after all ... but it "nearly" is.
10.16 Let $T P$ and $D C$ denote the propositions "the database contains only true propositions" and "the database is consistent," respectively. Then the first bullet item says:

```
IF TP THEN DC
```

The second says:

```
IF NOT ( DC ) THEN NOT ( TP )
```

And it's easy to see that these two propositions are indeed equivalent (in fact, each is the contrapositive of the other).
10.17 No, it isn't-though textbooks on logic usually claim the opposite, and in practice it's "usually" achievable. Here's an example of a relational calculus expression where the predicate in the WHERE clause can't be replaced by a PNF equivalent:

```
SX WHERE EXISTS SPX ( SPX.SNO = SX.SNO ) OR SX.CITY = 'Athens'
```

The paper "A Remark on Prenex Normal Form" (see Appendix G) explains the situation in detail.

## CHAPTER 11

11.1 First of all, you were asked several times in the body of the chapter whether it was necessary to worry about the possibility that the tables involved might include duplicate rows or nulls or both. But I categorically refuse-and so, I would like to suggest politely, should you-to waste any more time worrying about such matters. Avoid duplicates, avoid nulls, and then the transformations will all work just fine (and so will many other things, too).

That said, let me now give solutions to a couple of the more significant inline exercises:
(From the end of the section on Example 7.) Here's an SQL formulation of the query "Get suppliers SX such that for all parts PX and PY, if PX.CITY $\neq$ PY.CITY, then SX doesn't supply both of them." (How does this formulation differ from the one shown in the body of the chapter?)

```
SELECT SX.*
FROM S AS SX
WHERE NOT EXISTS
    ( SELECT *
        FROM P AS PX
        WHERE EXISTS
            ( SELECT *
                FROM P AS PY
                WHERE PX.CITY <> PY.CITY
                AND EXISTS
                    ( SELECT *
                    FROM SP AS SPX
                    WHERE SPX.SNO = SX.SNO
                    AND SPX.PNO = PX.PNO )
```

```
AND EXISTS
    ( SELECT *
    FROM SP AS SPX
    WHERE SPX.SNO = SX.SNO
    AND SPX.PNO = PY.PNO ) ) )
```

(From the end of the section on Example 12.) You were asked to give SQL formulations (a) using GROUP BY and HAVING, (b) not using GROUP BY and HAVING, for the following queries:

■ Get supplier numbers for suppliers who supply $N$ different parts for some $N>3$.
■ Get supplier numbers for suppliers who supply $N$ different parts for some $N<4$.
Here are GROUP BY and HAVING formulations:

```
SELECT SNO
FROM SP
GROUP BY SNO
HAVING COUNT ( * ) > 3
SELECT SNO
FROM SP
GROUP BY SNO
HAVING COUNT ( * ) < 4
UNION CORRESPONDING
SELECT SNO
FROM S
WHERE SNO NOT IN
    ( SELECT SNO
        FROM SP )
```

And here are non GROUP BY, non HAVING formulations:

```
SELECT SNO
FROM S
WHERE ( SELECT COUNT ( * )
                                FROM SP
                                WHERE SP.SNO = S.SNO ) > 3
SELECT SNO
FROM S
WHERE ( SELECT COUNT ( * )
    FROM SP
    WHERE SP.SNO = S.SNO ) < 4
```

You were also asked: What do you conclude from this exercise? Well, one thing I conclude is that we need to be very circumspect in our use of GROUP BY and HAVING. Observe in particular that the natural language queries were symmetric, which the GROUP BY / HAVING formulations aren't. By contrast, the non GROUP BY, non HAVING formulations are symmetric.

### 11.2 No answer provided.

### 11.3 No answer provided (obviously).

11.4 First of all, the exercise asked if you think the GROUP BY / HAVING expressions are easier to understand than the relational calculus expression (or the direct SQL transliteration of that expression). Only you can answer this question, of course, but I'm pretty sure the answer for most people would have to be no. Second, the exercise also asked if those GROUP BY / HAVING expressions accurately represent the desired query. Answer: The third one does; by contrast, the first returns all employee numbers in EMP and the second returns no employee numbers at all. Third, the exercise also asked what happens in each case if there aren't exactly three shortest employees. I'll leave this one to you!
11.5 I'm certainly not going to give anything like a complete answer to this exercise, but I will at least observe that the following equivalences allow certain algebraic expressions to be converted into calculus ones and vice versa:

```
■ r WHERE bx1 AND bx2 \equiv ( r WHERE bx1 ) JOIN ( r WHERE bx2 )
■ r WHERE bx1 OR bx2 \equiv ( r WHERE bx1 ) UNION ( r WHERE bx2 )
■ r WHERE NOT ( bx ) \equiv r MINUS ( r WHERE bx )
```

Other transformations were discussed in passing throughout the body of the book (from Chapter 6 on).
11.6 Well, I certainly don't see why not.

## CHAPTER 12

12.1 A NATURAL JOIN B : Illegal

A Intersect B : Illegal
SELECT * FROM A NATURAL JOIN B : Legal
SELECT * FROM A INTERSECT B : Illegal
SELECT * FROM ( A NATURAL JOIN B ) : Legal
SELECT * FROM ( A INTERSECT B ) : Illegal
SELECT * FROM ( SELECT * FROM A INTERSECT SELECT * FROM B ) : Illegal
SELECT * FROM ( A NATURAL JOIN B ) AS C : Illegal
SELECT * FROM ( A INTERSECT B ) AS C : Illegal
TABLE A NATURAL JOIN TABLE $B$ : Illegal

```
TABLE A INTERSECT TABLE B : Legal
SELECT * FROM A INTERSECT SELECT * FROM B : Legal
( SELECT * FROM A ) INTERSECT ( SELECT * FROM B ) : Legal
( SELECT * FROM A ) AS AA INTERSECT ( SELECT * FROM B ) AS BB : Illegal
```

You were also asked what you conclude from this exercise. One thing I conclude is that the rules are very difficult to remember (to say the least). In particular, SQL expressions involving INTERSECT can't always be transformed straightforwardly into their JOIN counterparts. I remark also that if we replace INTERSECT by NATURAL JOIN in the last two expressions, then the legal one becomes illegal and vice versa! That's because, believe it or not, the expressions

```
( SELECT * FROM A )
```

and

```
( SELECT * FROM B )
```

are considered to be subqueries in the context of NATURAL JOIN but not that of INTERSECT. (In other words, a subquery is a SELECT expression enclosed in parentheses, loosely speaking, but a SELECT expression enclosed in parentheses isn't necessarily a subquery.)
12.2 The effects are as follows: The second expression was previously illegal but becomes legal; the third, fifth, eleventh, twelfth, and thirteen were legal but become illegal; and the others were all illegal anyway and remain so. What do you conclude from this exercise?
12.3 It gives FALSE. Note, therefore (to spell the point out), it's possible in SQL for two values to be "equal" and yet not "like" each other! (Lewis Carroll, where are you?)

### 12.4 The first gives:

| STATUS |
| ---: |
| $=$ |
| 20 |
| 30 |

(The point here is that BETWEEN is inclusive, not exclusive, and so 10 and 30 are both included in the result. Does this state of affairs accord with your own intuitive understanding of the meaning of between?) The second gives:

| CITY |
| :--- |
| $\overline{\text { London }}$ |

And the third gives:


London isn't included in the result. The reason is that the expression

```
y BETWEEN }x\mathrm{ AND }
```

is shorthand for

```
x <= y AND y <= z
```

The problem here is that the natural language expression " $y$ is between $x$ and $z$ " is symmetric in $x$ and $z$ (i.e., switching $x$ and $z$ has no effect on the meaning), while the same is not true for the SQL expression " $y$ BETWEEN $x$ AND $z$." In a nutshell, BETWEEN in SQL doesn't mean the same as between in natural language.
12.5 First of all, observe that both comparand expressions are subqueries, and they therefore evaluate to tables. Now, those tables both have exactly one column, a fact that can be determined at compile time. What's more, given our usual sample values, they also both have exactly one row; the subqueries are therefore scalar subqueries, and the overall comparison is thus legal (a double coercion occurs on both sides, and the net effect is that two scalar values are compared). But suppose the WHERE clause in the second subquery had specified 12.0 instead of 14.0 . Given our usual sample values, the comparison overall would then no longer be legal (it would fail at run time), because the second subquery would now be a table subquery instead of a scalar one.

### 12.6 No answer provided.

12.7 No answer provided.
12.8 No answer provided.
12.9 No answer provided.

## APPENDIX C

C. 1 Here's an SQL version of constraint EQD2 (only; constraint EQD3 is essentially similar, of course).

```
CREATE ASSERTION EQD2 CHECK
    ( NOT EXISTS ( SELECT SNO
                FROM ST
                WHERE SNO IN ( SELECT SNO SROM SN ( )
```

            AND
    ```
        NOT EXISTS ( SELECT SNO
                                FROM SUT
                                WHERE SNO IN ( SELECT SNO
                        FROM ST ) )
    AND
    NOT EXISTS ( SELECT SNO
    FROM SN
    WHERE SNO NOT IN ( SELECT SNO
                                FROM ST
                                UNION CORRESPONDING
                        SELECT SNO
                        FROM SUT ) )
    AND
    NOT EXISTS ( SELECT SNO
        FROM ( SELECT SNO
        FROM ST
                UNION CORRESPONDING
                SELECT SNO
                FROM SUT ) AS POINTLESS
    WHERE SNO NOT IN ( SELECT SNO
                                FROM SN ) ) ) ;
C. 2 WITH T1 AS ( SELECT SNO, STATUS,
    CAST ( STATUS AS CHAR ( 3 ) ) AS XSTATUS
    FROM ST ) ,
    T2 AS ( SELECT SNO , XSTATUS
        FROM T1 ) ,
    T3 AS ( SELECT SNO , ' d/k' AS XSTATUS
        FROM SUT ) ,
    T4 AS ( SELECT SNO , XSTATUS
        FROM T1
        UNION CORRESPONDING
        SELECT SNO , XSTATUS
        FROM T3 ),
    T5 AS ( SELECT SNO , CITY AS XCITY
        FROM SC ) ,
    T6 AS ( SELECT SNO ,' ' }d/k'\mp@code{AS XCITY
        FROM SUC )
    T7 AS ( SELECT SNO ,' 'n/a' AS XCITY
        FROM SNC ) ,
    T8 AS ( SELECT SNO , XCITY
        FROM T5
        UNION CORRESPONDING
        SELECT SNO , XCITY
        FROM T6
        UNION CORRESPONDING
        SELECT SNO , XCITY
        FROM T7 ) ,
    S AS ( SELECT SNO , SNAME , XSTATUS , XCITY
        FROM SN NATURAL JOIN T4 NATURAL JOIN T8 )
SELECT SNO , SNAME , XSTATUS , XCITY
FROM S
```

C. 3 Because CORRESPONDING means "match on column names" and the single column in the table produced by the expression VALUES('S1') doesn't have a name.

## Appendix G

## Suggestions for Further Reading

As the title says, this appendix gives some suggestions for further reading. Let me immediately apologize for the fact that so many of the publications listed are ones for which I'm either the author or a coauthor ... The publications are listed in alphabetical order by author and chronological order within author. Note: This book isn't concerned with specific SQL products, and I therefore don't mention any specific product publications in this appendix. But many such publications exist, and you'll probably want to refer to one or more of them as well if you want to apply the ideas discussed in the present book to some individual project or product.

1. Surajit Chaudhuri and Gerhard Weikum: "Rethinking Database System Architecture: Towards a SelfTuning RISC-style Database System," Proc. 26th Int. Conf. on Very Large Data Bases, Cairo, Egypt (September 2000).

Among other things, this paper strongly endorses one of the messages of the present book: viz., that (as I put it in the preface) "SQL is complicated, confusing, and error prone (much more so, I venture to suggest, than its apologists would have you believe)." Here's an extract from the introduction to the paper:

SQL is painful. A big headache that comes with a database system is the SQL language. It is the union of all conceivable features (many of which are rarely used or should be discouraged to use anyway) and is way too complex for the typical application developer. Its core, say selection-projection-join queries and aggregation, is extremely useful, but we doubt that there is wide and wise use of all the bells and whistles. Understanding semantics of SQL (not even of SQL-92), covering all combinations of nested (and correlated) subqueries, null values, triggers, etc. is a nightmare. Teaching SQL typically focuses on the core, and leaves the featurism as a "learning-on-the-job" life experience.
2. Donald D. Chamberlin and Raymond F. Boyce: "SEQUEL: A Structured English Query Language," Proc. 1974 ACM SIGMOD Workshop on Data Description, Access, and Control, Ann Arbor, Mich. (May 1974).

This is the paper that first introduced the SQL language (or SEQUEL-Structured English QUEry Language-as it was originally called; the name was subsequently changed for legal reasons). There are some interesting differences between SEQUEL as described in this paper and SQL as generally understood today. Here are some of them:

- There were no nulls.

■ Although the SELECT clause was supported, the "SELECT *" form didn't exist. Thus, for example, to obtain all suppliers in London, you just wrote S WHERE CITY = 'London'-and to obtain all suppliers, you just wrote S.

- Duplicates were eliminated by default (though not in "set functions").
- The FROM clause always contained exactly one table name. In other words, what I called in Chapter 6 "the only [form of join] supported in SQL as originally defined" wasn't supported at all in SEQUEL as originally defined!
- The right comparand in a comparison in a WHERE clause was allowed to be a subquery (though the term subquery didn't exist-the construct was called a mapping instead), in which case the comparison was in fact an ANY or ALL comparison. ANY was the default, and could only be specified implicitly. IN syntax as such was not supported; "=" (meaning, by default, "=ANY") was used instead.
- Set comparison operators (inclusion, etc.) were supported.
- There was no GROUP BY clause as such; instead, GROUP BY could be specified as an option on the FROM clause.
- There was no HAVING clause; "set functions" could be invoked in the WHERE clause instead.
- There were no correlation names. Instead, "blocks" (apparently another term for mappings in the sense explained above) could be labeled, and block labels could be used as dot qualifiers.

■ Expressions such as QTY / AVG(QTY)-i.e., expressions involving "set function" invocations, as well as simple column references and the like-were legal in the SELECT clause, and presumably in the WHERE clause also.

- "Mappings" could be combined by means of intersection, union, and difference (and these operators were denoted by conventional mathematical symbols instead of English keywords).

The paper also discusses several perceived differences between SEQUEL and the relational calculus, claiming in every case an advantage for SEQUEL over the calculus. However, the differences and claims in question don't really stand up to careful analysis.
3. E. F. Codd: "Derivability, Redundancy, and Consistency of Relations Stored in Large Data Banks," IBM Research Report RJ599 (August 19th, 1969); "A Relational Model of Data for Large Shared Data Banks," CACM 13, No. 6 (June 1970). Note: The first of these papers was reprinted in ACM SIGMOD Record 38, No. 1 (March 2009); the second was reprinted in Milestones of Research-Selected Papers 1958-1982 (CACM 25th Anniversary Issue), CACM 26, No. 1 (January 1983) and elsewhere.

The 1969 paper was Codd's very first paper on the relational model; essentially, it's a preliminary version of the 1970 paper, with a few interesting differences (the main one being that the 1969 paper permitted relation valued attributes while the 1970 one didn't). That 1970 paper was the first widely available paper on the subject. It's usually credited with being the seminal paper in the field, though that characterization is a little unfair to its 1969 predecessor. I would like to suggest, politely, that every database professional should read one or both of these papers every year.
4. E. F. Codd: "Relational Completeness of Data Base Sublanguages," in Randall J. Rustin (ed.), Data Base Systems, Courant Computer Science Symposia Series 6. Englewood Cliffs, N.J.: Prentice Hall (1972).

This is the paper in which Codd first formally defined the original relational algebra and relational calculus. Not an easy read, but it repays careful study.
5. E. F. Codd and C. J. Date: "Much Ado about Nothing," in C. J. Date, Relational Database Writings 19911994. Reading, Mass.: Addison-Wesley (1995).

Codd was perhaps the foremost advocate of nulls and three-valued logic as a basis for dealing with missing information (a curious state of affairs, you might think, given that nulls violate Codd's own Information Principle). This article contains the text of a debate between Codd and myself on the subject. It includes the following delightful remark: "Database management would be easier if missing values didn't exist" (Codd). Note: I include this particular reference, out of a huge number of available publications on the topic, because it does at least touch on most of the arguments on both sides of the issue.
6. Hugh Darwen: "The Role of Functional Dependence in Query Decomposition," in C. J. Date and Hugh Darwen, Relational Database Writings 1989-1991. Reading, Mass.: Addison-Wesley (1992).

This paper gives a set of inference rules by which functional dependencies (FDs) satisfied by the relation $r$ denoted by an arbitrary relational expression can be inferred from those holding for the relvar(s) referenced in the expression in question. The set of FDs thus inferred can then be inspected to determine the key constraints satisfied by $r$, thus providing a basis for the key inference rules mentioned in passing in Chapter 4 of the present book.
7. Hugh Darwen: "What a Database Really Is: Predicates and Propositions," in C. J. Date, Relational Database Writings 1994-1997. Reading, Mass.: Addison-Wesley (1998).

A very readable tutorial on relvar predicates and related matters.
8. Hugh Darwen: "The Decomposition Approach," in reference [39]. Note: This paper is based on an earlier presentation titled "How to Handle Missing Information Without Using Nulls," the slides for which can be found at www.thethirdmanifesto.com (May 9th, 2003; revised May 16th, 2005).

Appendix B of the present book is based on this paper.
9. C. J. Date: "Fifty Ways to Quote Your Query," www.dbpd.com (July 1998).

A discussion of redundancy in the SQL language.
10. C. J. Date: "Composite Keys," in C. J. Date and Hugh Darwen, Relational Database Writings 1989-1991. Reading, Mass.: Addison-Wesley (1992).

Among other things, this paper includes a discussion of the pros and cons of surrogate keys.
11. C. J. Date: An Introduction to Database Systems (8th edition). Boston, Mass.: Addison-Wesley (2004).

A college level text on all aspects of database management. SQL discussions are at the SQL:1999 level, with a few comments on SQL:2003; in particular, they include a detailed discussion of SQL's "object/relational" features (REF types, reference values, and so on), explaining why they violate relational principles. Note: Other textbooks covering similar territory are references [42], [52], and [53].
12. C. J. Date: The Relational Database Dictionary, Extended Edition. Berkeley, Calif.: Apress (2008).

Many of the definitions given in the body of the present book are based on ones in this reference.
13. C. J. Date: "Double Trouble, Double Trouble," in reference [20].

An extensive and detailed treatment of the problems caused by duplicates. The discussion of duplicates in Chapter 4 of the present book is based in large part on an example from this paper.
14. C. J. Date: "What First Normal Form Really Means," in reference [20].

First normal form has been the subject of much misunderstanding over the years. This paper is an attempt to set the record straight-even to be definitive, as far as possible. The crux of the argument, as indicated in Chapter 2 of the present book, is that the concept of atomicity (in terms of which first normal form was originally defined) has no absolute meaning.
15. C. J. Date: "A Sweet Disorder," in reference [20].

Relations don't have a left to right ordering to their attributes, but SQL tables do have such an ordering to their columns. This paper explores some of the consequences of this state of affairs, which turn out to be much less trivial than many seem to think. (Many of the recommendations in the present book have to do with techniques for behaving as if the state of affairs in question didn't exist after all.)
16. C. J. Date: "On the Notion of Logical Difference," "On the Logical Difference Between Model and Implementation," and "On the Logical Differences Between Types, Values, and Variables," all in reference [20].

The titles say it all.
17. C. J. Date: "Two Remarks on SQL's UNION," in reference [20].

This short paper describes some of the weirdnesses that arise in connection with SQL's UNION operator (and by implication its INTERSECT and EXCEPT operators as well) from (a) coercions and (b) duplicate rows.
18. C. J. Date: "A Cure for Madness," in reference [20].

A detailed examination of the fact that, very counterintuitively, the SQL expressions

```
SELECT sic FROM ( SELECT * FROM t WHERE p ) WHERE q
```

and

SELECT sic FROM $t$ WHERE $p$ AND $q$
aren't always logically equivalent-even though they ought to be, and even though at least one current SQL product does sometimes transform the former into the latter. Note: For simplicity I choose to ignore the fact that the standard would actually require the subquery in the FROM clause in the first of the foregoing expressions to be accompanied by an AS specification.
19. C. J. Date: "Why Three- and Four-Valued Logic Don't Work," in reference [20].

As noted in the body of the present book, SQL's null support is based on three-valued logic. Actually its implementation of that logic is seriously flawed-but even if it weren't, it would still be advisable not to use it, and this paper explains why.
20. C. J. Date: Date on Database: Writings 2000-2006. Berkeley, Calif.: Apress (2006).
21. C. J. Date: "The Logic of View Updating," in reference [24].

This paper offers evidence in support of the claim that views are always logically updatable, modulo possible violations of either The Assignment Principle or The Golden Rule. See also the annotation to reference [25].
22. C. J. Date: "The Closed World Assumption," in reference [24].

The Closed World Assumption is seldom articulated, and yet it forms the basis of almost everything we do when we use a database. This paper examines that assumption in detail; in particular, it shows why it's preferred to its rival, The Open World Assumption (on which the "semantic web" is based, incidentally-or so it has been claimed).
23. C. J. Date: "The Theory of Bags: An Investigative Tutorial," in reference [24].

Among other things, this paper discusses what happens to operators like union when their operands are bags instead of sets.
24. C. J. Date: Logic and Databases: The Roots of Relational Theory. Victoria, BC: Trafford Publishing (2007). See www.trafford.com/07-0690.
25. C. J. Date: "How to Update Views," in reference [39].

The problem of (a) updating base relvars appropriately in order to support updates on views is, abstractly, the same problem as (b) the problem of updating stored data appropriately in order to support updates on base relvars. They just show up at different points in the overall system architecture, that's all. It follows that we must solve this problem, for otherwise we have to give up on the goal of data independence. (Note, therefore, that logical and physical data independence are really the same problem, too; they differ only in that they too show up at different points in the overall architecture.) This paper elaborates on the idea, briefly discussed in Chapter 9, that a fruitful way to think about view updating in general is to consider what would happen if the view in question were defined as a base relvar instead, living alongside (as it were) the base relvar(s) in terms of which it's defined, with constraints interrelating the two.

Note: Unfortunately, certain details of the foregoing paper are slightly incorrect, though not seriously so. My most recent thoughts on the topic can be found in a slide presentation with the title "The View Updating Problem: Notes toward a Proposed Solution." I plan to publish a paper based on this presentation as soon as possible.
26. C. J. Date: "Inclusion Dependencies and Foreign Keys," in reference [39].

An alternative title for this paper might be "Rethinking Foreign Keys"; it demonstrates among other things that the foreign key notion encompasses far more than it's usually given credit for. It also includes a detailed discussion of the logical differences between foreign keys and pointers. (As noted in passing in Chapter 2 of the present book, some writers have claimed that foreign keys are nothing more than traditional pointers in sheep's clothing, but such is not the case.)
27. C. J. Date: "Image Relations," in reference [39].
28. C. J. Date: " $N$-adic vs. Dyadic Operators: An Investigation," in reference [39].

Tutorial D supports $n$-adic versions of several relational operators-union, join, and so on-that are more usually considered to be dyadic operators merely. This paper examines the twin questions of (a) what makes it possible to define an $n$-adic version of some dyadic operator and (b) how such $n$-adic versions can sensibly be defined.
29. C. J. Date: "A Remark on Prenex Normal Form," in reference [39].
30. C. J. Date: "Is SQL's Three-Valued Logic Truth Functionally Complete?", in reference [39].

Among other things, this paper includes a comprehensive description of SQL's support for nulls and three-valued logic.
31. C. J. Date: Normal Forms and All That Jazz: A Database Professional's Guide to Database Design Theory
(to appear).

Design theory is the scientific foundation for database design, just as the relational model is the scientific foundation for database technology in general. This book, a companion to the present book, is a tutorial on database design theory (normalization, orthogonality, and related matters) for database professionals.
32. C. J. Date: Go Faster! The TransRelational ${ }^{\text {TM }}$ Approach to DBMS Implementation (to appear).

A detailed description of The TransRelational ${ }^{\text {TM }}$ Model, a novel implementation technology mentioned briefly in Appendix A of the present book. Note: A short (and very incomplete) introduction to that technology can also be found in Appendix A of reference [11].
33. C. J. Date and Hugh Darwen: $A$ Guide to the SQL Standard (4th edition). Reading, Mass.: Addison-Wesley (1997).

This book provides thorough coverage of the SQL standard as of early 1997. Numerous features have been added to the standard since that time (including the so called object/relational features (see reference [11]), but they're mostly irrelevant so far as the goal of using SQL relationally is concerned. In my not unbiased opinion, therefore, the book is a good source for more detail on just about every aspect of SQL-at least in its standard incarnation-touched on in the body of the present book.
34. C. J. Date and Hugh Darwen: Databases, Types, and the Relational Model: The Third Manifesto (3rd edition). Boston, Mass.: Addison-Wesley (2006).

This book introduces and explains The Third Manifesto, a detailed proposal for the future of data and database management systems. It includes a precise though somewhat formal definition of the relational model; it also includes a detailed proposal for the necessary supporting type theory (including a comprehensive model of type inheritance).
35. C. J. Date and Hugh Darwen: "Multiple Assignment," in reference [20].
36. C. J. Date and Hugh Darwen: "The Third Manifesto," in reference [39].

This paper (Chapter 1 of reference [39]) presents the very latest version of The Third Manifesto. It consists for the most part of a revised version of the pertinent chapter from reference [34].
37. C. J. Date and Hugh Darwen: "Tutorial D," in reference [39].

This paper provides a comprehensive description of the most recent version of Tutorial $\mathbf{D}$ (which is the version used in examples in the present book). Note: The website www.thethirdmanifesto.com gives information regarding a variety of existing Tutorial D implementations, as well as other projects related to proposals of The Third Manifesto.
38. C. J. Date and Hugh Darwen: "Toward an Industrial Strength Dialect of Tutorial D," in reference [39].

A proposal for upgrading Tutorial $\mathbf{D}$ to make it more suitable for commercial implementation. Certain of the ideas from this proposal (including in particular image relations and foreign key support) have been assumed in the body of this book.
39. C. J. Date and Hugh Darwen: Database Explorations: Essays on The Third Manifesto and Related Matters. Bloomington, Ind.: Trafford Publishing (2010). See www.trafford.com/Bookstore/.
40. C. J. Date, Hugh Darwen, and Nikos A. Lorentzos: Temporal Data and the Relational Model. San Francisco, Calif.: Morgan Kaufmann (2003).

Some indication of what this book covers can be found in Appendix A of the present book.
41. C. J. Date and David McGoveran: "Why Relational DBMS Logic Must Not Be Many-Valued," in reference [24].

This paper presents a series of logical arguments in support of the position that database languages should be based (like the relational model, but unlike SQL) on two-valued logic.
42. Ramez Elmasri and Shamkant Navathe: Fundamentals of Database Systems (4th edition). Boston, Mass.: Addison-Wesley (2004).
43. Stéphane Faroult with Peter Robson: The Art of SQL. Sebastopol, Calif.: O’Reilly Media Inc. (2006).

A practitioner's guide on how to use SQL to get good performance in currently available products. The following lightly edited list of subtitles from the book's twelve chapters gives some idea of the scope:

1. Designing Databases for Performance
2. Accessing Databases Efficiently
3. Indexing
4. Understanding SQL Statements
5. Understanding Physical Implementation
6. Classic SQL Patterns
7. Dealing with Hierarchic Data
8. Difficult Cases
9. Concurrency
10. Large Data Volumes
11. Response Times
12. Monitoring Performance

The book doesn't deviate much from relational principles in its suggestions and recommendations-in fact, it explicitly advocates adherence to those principles, for the most part. But it also recognizes that today's optimizers are less than perfect; thus, it gives guidance on how to choose the specific SQL formulation for a given problem, out of many logically equivalent formulations, that's likely to perform best (and it explains why). It also describes a few coding tricks that can help performance, such as using MIN to determine that all entries in a yes/no column are yes (instead of doing an explicit existence test for no). On the question of hints to the optimizer (which many products do support), it includes the following wise words: "The trouble with hints is that they are more imperative than their name suggests, and every hint is a gamble on the future-a bet that circumstances, volumes, database algorithms, hardware
and the rest will evolve in such a way that [the] forced execution path will forever remain, if not absolutely the best, at least acceptable ... Remember that you should heavily document anything that forces the hand of the DBMS."
44. Patrick Hall, Peter Hitchcock, and Stephen Todd: "An Algebra of Relations for Machine Computation," Conf. Record of the 2nd ACM Symposium on Principles of Programming Languages, Palo Alto, Calif. (January 1975).

This paper is perhaps a little "difficult," but I think it's important. Tutorial D and the version of the relational algebra I've described in this book both have their roots in this paper.
45. G. D. Held, M. R. Stonebraker, and E. Wong: "INGRES—A Relational Data Base System," Proc. NCC 44, Anaheim, Calif. Montvale, N.J.: AFIPS Press (May 1975).

There were two major relational prototypes under development in the mid to late 1970s-System R at IBM, and Ingres (originally INGRES, all uppercase) at the University of California at Berkeley. Unlike System R, Ingres was not originally an SQL system; instead, it supported a language called QUEL ("Query Language"), which was based on relational calculus and in many ways was technically superior to SQL. This paper, which was the first to describe the Ingres prototype, includes a preliminary definition of QUEL.
46. Jim Gray and Andreas Reuter: Transaction Processing: Concepts and Techniques. San Mateo, Calif.: Morgan Kaufmann (1993).

The standard text on transaction management.
47. Lex de Haan and Toon Koppelaars: Applied Mathematics for Database Professionals. Berkeley, Calif.: Apress (2007).

Among other things, this book includes an extensive set of identities (here called rewrite rules) that can be used as in Chapter 11 of the present book to help with the formulation of complex SQL expressions. It also shows how to implement integrity constraints by means of procedural code (if necessary!-see Chapter 8 of the present book). Recommended.
48. Wilfrid Hodges: Logic. London, England: Penguin Books (1977).

A gentle introduction to logic for the uninitiated.
49. International Organization for Standardization (ISO): Database Language SQL, Document ISO/IEC 9075:2008 (2008).

The official SQL standard (2008 version). Note that it is indeed an international standard, not just (as so many seem to think) an American or "ANSI" standard. Note too that although SQL:2008 is the current version of the standard, almost all of the SQL features discussed in the present book were already included in SQL:2003 or SQL:1999; in fact, most of them were included in SQL:1992 or even earlier versions.
50. David McGoveran: "Nothing from Nothing" (in four parts), in C. J. Date, Hugh Darwen, and David McGoveran, Relational Database Writings 1994-1997. Reading, Mass.: Addison-Wesley (1998).

This paper is referenced in Appendix B of the present book.
51. Jim Melton and Alan R. Simon: SQL:1999—Understanding Relational Components; Jim Melton: Advanced SQL:1999—Understanding Object-Relational and Other Advanced Features. San Francisco, Calif.: Morgan Kaufmann (2002 and 2003, respectively).

As mentioned in Chapter 1, the SQL standard has been through several versions over the years-the current version is SQL:2008 [49], the previous version was SQL:2003, the one before that was SQL:1999, and the one before that was SQL:1992. So far as I know, these two books are the only ones available that cover, between them, any version later than SQL:1992. Melton was the editor of the SQL standard for many years.
52. Raghu Ramakrishnan and Johannes Gehrke: Database Management Systems (3rd edition). New York, N.Y.: McGraw-Hill (2003).
53. Avi Silberschatz, Henry F. Korth, and S. Sudarshan: Database System Concepts (5th edition). New York, N.Y.: McGraw-Hill (2005).
54. Robert R. Stoll: Sets, Logic, and Axiomatic Theories. San Francisco, Calif.: W. H. Freeman and Company (1961).

The relational model is solidly founded on logic and set theory. This book provides a fairly formal but not too difficult introduction to these topics. Note: For a less formal introduction, see the book by Hodges [48].
55. Dave Voorhis: Rel. http://db@builder.sourceforge.net/rel.html.

Downloadable code for Rel, a prototype implementation of (a dialect of) Tutorial D.
56. Moshé M. Zloof: "Query-By-Example," Proc. NCC 44, Anaheim, Calif. (May 1975). Montvale, N.J.: AFIPS Press (1977).

Query-By-Example (QBE) is a nice illustration of the fact that it's entirely possible to produce a very "user friendly" language based on relational calculus instead of relational algebra. (In the interest of accuracy, however, I should note that QBE is really based more on the domain calculus than it is on the tuple calculus, which is the version of the calculus discussed in the body of this book.) Zloof was the original inventor and designer of QBE, and this paper was the first of many by Zloof on the subject.

## Index

For alphabetization purposes, (a) differences in fonts and case are ignored; (b) quotation marks are ignored; (c) other punctuation symbols-hyphens, underscores, parentheses, etc.-are treated as blanks; (d) numerals precede letters; (e) blanks precede everything else.
$\stackrel{\text { def }}{=}$ (is defined as), 127,158
| (Sheffer stroke), 397
$\downarrow$ (Peirce arrow), 397
$\in$ (set membership), 57
$\equiv$ (equivalent to), 218
$\Rightarrow$ (implies), 218
$\Leftrightarrow$ (bi-implies, equivalent to), 218
$\subset$ (properly included in), 59
$\supset$ (properly includes), 59
$\subseteq$ (included in), 59
〇 (includes), 59
$\exists$ (existential quantifier), 224
$\forall$ (universal quantifier), 224
!! (image relation reference), 136
$\theta$-join, 122
0-tuple, 52,53
1NF, see first normal form
2VL, see two-valued logic
3VL, see three-valued logic
Abbey, Edward, 97,247
access method, 13
ACID properties, 180
aggregate operators, 140-144,157
empty argument, 143-144
vs. summaries, see summary
algebra, see relational algebra
ALL BUT, 112
ALL or ANY comparison, 42,265-268
alternate key, 93
Anthony, Susan B., 105
Appleby, Sir Humphrey, 273
architecture, 330
argument, 222
Aristotle, 288
arity, 14
assignment
multiple, see multiple assignment
relational, see relational assignment
tuple, 38
Assignment Principle, The, 91,300
SQL violations, 76,96,340,353
associativity, 125
Atkinson, Malcolm, 289
atomicity
scalar value, 31-34
statement, 88
transaction, 10
attribute
attribute-name:type-name pair, 14
extracting value from tuple, 52
pictured as column, 5
relation, 5,55
relvar, 330
tuple, 50
attribute constraint, 173
attribute ordering (left to right)
in SQL tables, 15
not in relations, 15
attribute value, 50
AVGX, 144
axiom (database), 100
bag, 33
Bancilhon, François, 289
base relation, 18
base relvar, 21
see also base relation
base table constraint, 178
bill of materials, 102,159
Billings, Josh, iii
binary relation (mathematics), 287
BNF grammar
SQL, 281-285
Tutorial D, 321-323
body
relation, 14,55
relvar, 330
BOOLEAN
relational model, 25,294
SQL, 40
bound variable, 226
Boyce, Raymond F., 409
Breazu-Tannen, Val, 126
Buneman, Peter, 126
Bush, George W., 224
business rules, 169
calculus, see relational calculus
candidate key, 6,92-94
Cardelli, Luca, 25
cardinality
relation, 14,55
relvar, 330
Carroll, Lewis, 60,406
cartesian product, see product
CASCADE (referential action), 96
Celko, Joe, 201,289
Chamberlin, Don, 84,409
Chaudhuri, Surajit, 409
CHECK option, 205
Chudnovsky, Gregory, 302
Closed World Assumption, The, 98,300, 318-319,357
closure, 8,108-110
CNF, see conjunctive normal form
COALESCE, 78
CODASYL, 288
Codd, E. F., passim
coercion, 29
SQL, 40-42
collation, 42
column, see attribute
column constraint, 179
column naming, 62-63
commalist, 6
common attribute, 106
commutativity, 125
compensatory action, 208
component (tuple), 50
conjunct, 400
conjunction, 400
conjunctive normal form, 400
CONNECT BY (Oracle), 161
connective, 218,348
consistency, 180,185,243
vs. correctness, 185
constant, 196
constraint
attribute, 173
database, see database constraint
relvar, see relvar constraint
state, 187
transition, 187
tuple, 174
type, see type constraint
vs. performance, 188
vs. predicate, 185
contradiction, 82
contrapositive, 221
correlated subquery, see subquery
CORRESPONDING, 116
correctness, see consistency
correlation name (SQL), 275
cost based optimizing, 123
CREATE ASSERTION, 178
cursor, 81
CWA, see Closed World Assumption, The,
D_INSERT, 89
expansion, 89

D_UNION, 89,117,314
$n$-adic, 117-118
da Vinci, Leonardo, iii,4
Darwen, Hugh, passim
data independence, 13
logical, 211-212
physical, 13
data model, 12,14
two meanings, 12,14
data type, see type
database, passim
collection of propositions, 180
logical system, 100
vs. DBMS, 27
database administrator, 28
database constraint, 174-179
checked immediately, 180-182
SQL, 178-179
the (total) database constraint, 186
database management system, 27
vs. database, 27
database statistics, 123
database variable, 297,298-299
Date, C. J., passim
DBA, 28
DBMS, 27
dbvar, 297,298-299
DCO, 27-28
De Haan, Lex, iii,179,188,417
De Morgan's laws, 249
declarative, 22
decomposition (missing information), 307
horizontal, 309-311
vertical, 308-309
DEE, see TABLE_DEE
degree
foreign key, 94
key, 92
relation, 14,55
relvar, 330
tuple, 15,50

DELETE, 88
expansion, 88
via cursor (SQL), 87
Derbyshire, John, 131
derived relation, 18
derived relvar, see snapshot; view see also derived relation
designator, 187
DeWitt, David, 289
difference, 118-119
see also semidifference
disjoint INSERT, see D_INSERT
disjoint union, see D_UNION
disjunct, 400
disjunction, 400
disjunctive normal form, 400
DISTINCT, 73-74
distributive law, 249
distributivity, 124
Dittrich, Klaus, 289
divide, 138-139
DIVIDEBY, 138
DNF, see disjunctive normal form
domain, 5,26,329
SQL, 40
see also type
domain calculus, 230
"domain check override," 27-28
dot qualification, 274-275
double negation law, 248
double underlining, 6,103,358
DUM, see TABLE_DEE
duplicate elimination, 111
duplicates, 17,52.67ff
in SQL tables, 15
not in relations, 15
see also tuple equality
durability, 180
dyadic predicate, 223
Einstein, Albert, 292
Elmasri, Ramez, 416
Emerson, Ralph Waldo, 169
empty argument, see aggregate operator
empty heading, 52
empty key, 102
empty range, 237-238
empty relation, 57
empty set, 52
SQL, 281-282
empty tuple, 52
empty type, 340
encapsulation, 37
entity integrity, 7,195
EQD, see equality dependency
equality, 17,26-31,294
relation, see relation equality
tuple, see tuple equality
equality dependency, 313
equijoin, 122,358
essentiality, 300
Euclid, iii
EXCEPT, 118
exclusive union, see union
existential quantifier, 224
EXISTS
iterated OR, 236
SQL, 229
see also existential quantifier
explicit table, 274
expression transformation, 123
expression vs. statement, 27,340
EXTEND, 133-135,158
Extensible Markup Language, see XML
extension vs. intension, 98
factorial, 50,354
Faroult, Stéphane, 416
FD, see functional dependency
field (SQL), 44,53
first normal form, 16
relvar, 338
fixpoint, 160
"flat relation," 58

FORALL
iterated AND, 237
not in SQL, 229
see also universal quantifier
foreign key, 7,94
not fundamental, 96
free variable, 226
see also bound variable
function (SQL), 2
functional dependency, 94,177

Gehrke, Johannes, 418
generated type, 37
generic type, 338
Golden Rule, The, 186,301
googol, 81
googolplex, 81
Graunt, John, 49
Graves, Robert, 216
Gray, Jim, 180,417
gross requirements, 160
GROUP, 152
GROUP BY redundant, 148

Haldeman, H. R., 193
Hall, Patrick, 417
Hardy, G. H., iii
HAVING redundant, 150
heading
relation, 14,55
relvar, 330
tuple, 50
Held, G. D., 417
Hitchcock, Peter, 417
Hodge, Alan, 216
Hodges, Wilfrid, 417
hold (constraint), 169

I_DELETE, 90
expansion, 90
I_MINUS, 90
idempotence, 129,364
identity projection, 111
identity restriction, 111
image relation, 135-138,144-145,150-151
implementation, 12
vs. model, 12
implementation defined, 273
implementation dependent, 273
implication, see logical implication
implication law, 248
IMS, 377
inflight checking, 88
information equivalence, 394
Information Principle, The, 38,295,300
INSERT, 88

$$
\text { expansion, } 88
$$

instantiation, 98,222
integrity constraint, see constraint
intended interpretation, 97
intension, 97
interpretation, see intended interpretation
intersect, 114,118
introduced name, see WITH
involution law, 248
irreducibility (key), 92
IS_EMPTY, 59
IS_NOT_EMPTY, 59
isolation, 180
Jay, Antony, 237
join, 112-116
$n$-adic, 114
see also equijoin; $\theta$-join
JOIN (SQL), 114-116
joinability, 112
key, 6
for expression, 203
irreducibility, 92
uniqueness, 92
values are tuples, 93
see also candidate key
Koppelaars, Toon, 179,188,417
Korth, Henry F., 418
lateral subquery, see subquery
Lincoln, Abraham, 244
literal, 36,196,335
relation, 56
tuple, 51
see also selector
logical difference, 16
logical implication, 219-220
logical system, 100
Lorentzos, Nikos A., 302,416
Lynn, Jonathan, 237
Magritte, René, 16
Maier, David, 289
Marx, Groucho, iii
MATCHING, 132-133
materialization (vs. substitution), 199
"materialized view," see snapshot
MAXX, see AVGX
MAYBE, 82
McGoveran, David, 416,418
Melton, Jim, 418
method (SQL), 3
MINUS, see difference
MINX, see AVGX
missing information (without nulls), 307-320
model, 12,14
vs. implementation, 12
modus ponens, 223
modus tollens, 223
monadic predicate, 223
Muir, John, 294
multidimensional database, 58
multirelvar constraint, 178
multiple assignment, 183-184
multiset, 33
$n$-adic predicate, 223
$n$-ary relation, 5
$n$-ary tuple, see tuple
$n$-place predicate, 223
n-tuple, see tuple

Nagel, Ernest, 216
NAND, 397
natural join, see join
Navathe, Shamkant, 416
Newton, Isaac, 292
NO CASCADE (referential action), 355
NO PAD (collation), 42
nonscalar, 37
NOR, 397
normalized, see first normal form
NOT MATCHING, 133
NOT NULL, 77
null, 7,74-80
not a value, 7
not in relational model, 7
not in relations, 77
not in tuples, 51
object/relational, 33
Ohori, Atsushi, 126, 33
Open World Assumption, The, 102,357
operator, passim
SQL, 2
optimizer, 67
ORDER, 163
ORDER BY, 163
not in views, 163
ordinal position (SQL columns), 63-64
orthogonality (language design), 30
outer join, 79,153-155
OWA, see Open World Assumption, The,

PAD SPACE (collation), 42
parameter, 222
parameterized type, 338
part explosion, 161
part implosion, 161
performance
not a model issue, 12
vs. constraint, 188
Peirce arrow, 397
physical representation hidden, 28
physical storage
not in relational model, 18
Pietarinen, Lauri, 216
pipelining, 108
placeholder, 222
PNF, see prenex normal form
pointer, 45
not in relational model, 45
SQL, 45
polymorphic type, 338
possible representation, 170
possibly nondeterministic, 43,280-281
positioned update, 101
"possrep," 170
predicate, 222
compound, 223
dyadic, 223
monadic, 223
$n$-adic, 223
n-place, 223
relvar, see relvar predicate
simple, 223
vs. constraint, 185
predicate calculus, see predicate logic
predicate logic, 223
"predicate transitive closure," 364
prenex normal form, 230
primary key, 6
primitive operators, 119,302
principle, 4
Principle of Identity of Indiscernibles, The, 301,346
Principle of Interchangeability, The, 178,181, 195,300
procedural, 22
procedure (SQL), 2
product (cartesian), 113
project, 111
proper subset, see subset
proper superkey, see superkey
proper superset, see superset
proposition, 97,217
compound, 218
simple, 218
proto tuple, 228
pseudovariable, 299
public table, 126,214

QBE, see Query-By-Example
quantification law, 249
quantifier, 224
don't need both, 234,249
existential, 224
other kinds, 237-238
sequence, 225
universal, 224
see also EXISTS; FORALL; UNIQUE
Quarles, Francis, 1
QUEL, 215
Query-By-Example, 215
query rewrite, 68
quota query, 270,378

Ramakrishnan, Raghu, 418
range variable, 228,230-231,275-277
RANK, 378
rational number, 26
recommendations summarized, 325-328
recursion, 159-162
REF type, 45
referenced relvar, 94
referencing relvar, 94
referential action, 96
referential constraint, see foreign key
referential integrity, 7,329
metaconstraint, 187
refresh, see snapshot
Rel, 418
relation, 5,19,55
n-dimensional, 58
pictured as table, 58
vs. relvar, 329
vs. type, 99-101
see also relvar
relation constant, 196
relation equality, 58
relation selector, 56,345
relation type, 56
inference, 108-109
name, 56
RELATION $\{H\}, 56$
RELATION type generator, 38
relation value, see relation
relation valued attribute, 33,152-157
relation variable, see relvar
relational algebra, 8
generic, 105
purpose, 297-298
read-only, 105
relational assignment, 8,20,88-91
not in SQL, 20
relational calculus, 215,227-234
relational comparisons, 58-59
relational completeness, 242,297
SQL, 401-402
relational database, see The Information Principle
relational inclusion, 59
relational model, passim
formal definition, 293-298
informal definition, 5-11
objectives, 299-300
vs. other models, 288-291
relations, tuples, and attributes, 2
relcon, 197
relvar, 19
base vs. stored, 18-19
constraint, see relvar constraint
predicate, see relvar predicate
virtual, see view
vs. file, 97
vs. relation, 329
see also relation
relvar constraint, 178
the (total) relvar constraint, 186
relvar predicate, 97-98,121-122,317-320
relvar reference, 106
RENAME, 109
repeating group, 32
representation vs. type, 28
restrict, 110
restriction condition, 110
Reuter, Andreas, 180,417
Riemann, Bernhard, 292
Robson, Peter, 416
routine (SQL), 2
row, see tuple
row constraint, 174
row ID, 3,196
row type (SQL), 53
row type constructor (SQL), 53
row value constructor (SQL), 53
row variable (SQL), 44,53
rules of inference, 223 relation types, see relation type
Russell, Bertrand, 224,293
RVA, see relation valued attribute
Sagan, Carl, 292
"same as," see equality
satisfies (constraint), 169
scalar, 37
SELECT *, 273
SELECT - FROM - WHERE, 122
semantics, 122
too rigid, 134
selector, 29,36,171-172,395
relation, 56,345
scalar, 29,36,171-172
tuple, 51
self-referencing relvar, 102
semantic optimization, 181
semidifference, see NOT MATCHING
semijoin, see MATCHING
semistructured model, 288
SEQUEL, 409
"set function," 147
set level operations, 86
set membership, 57
Shakespeare, William, 97
Sheffer stroke, 397

Silberschatz, Avi, 418
Simon, Alan R., 418
single relvar constraint, 178
snapshot, 212
SQL, passim
departures from relational model, 305-306
expression evaluation, 122
legacy, 304
means the SQL standard, xiii
not the same as the relational model, 2
pronunciation, xiii
vs. Tutorial D, 106
SQL:1992, 3
SQL:1999, 3
SQL:2003, 3
SQL:2008, 3
state constraint, 187
statement (two meanings), 98-99
statement vs. expression, 23,340
Stoll, Robert R., 418
Stonebraker, Mike, 290,417
strong typing, 30
subject to (constraint), 169
subkey, 354
subquery, 277-280,407
correlated, 254,278
lateral, 278-279
row, 42,44,277
scalar, 42,277-278
table, 277
subset, 17
proper, 17
substitution procedure, 198-199
Sudarshan, S., 418
summarize, 146-152
SUMMARIZE BY, 146
SUMMARIZE PER, 146
summary, 146
superkey, 94
proper, 94
superset, 17
proper, 17
symmetric difference, 132
table, see relation; relvar SQL, 60-61
TABLE_DEE, 59,111,114,130,197,318-319,344
and TABLE_DUM, 59
identity with respect to join, 114
TABLE_DUM, see TABLE_DEE
table value constructor (SQL), 60
"tables and views," 19
tables, rows, and columns, 2
target key, 94
target relvar, 94
tautology, 82
TCLOSE, 160,401
THE_ operator, 30,36,171-172,336
theorem (database), 100
theory, 291
Third Manifesto, The, 20,290,301
three-valued logic, 74,229
truth tables, 74
TIMES, 113,114
Todd, Stephen, 417
transaction, 180
transition constraint, 187
transitive closure, 159
"trigger," 87
triggered action, 87
truth functional completeness, 82,220
truth table, 214,348
truth vs. consistency, 185
tuple, 5,49-50
extracting attribute value from, 52
pictured as row, 49
relvar, 330
tuple calculus, 230
tuple constraint, 174
tuple equality, 52,93
TUPLE FROM, 38
tuple selector, 51
tuple type, 50
name, 50-51
TUPLE $\{H\}, 50$
TUPLE type generator, 38
tuple value, see tuple
tuple variable, 38
not in relational model, 174
Tutorial D, passim
used in The Third Manifesto, 20
vs. SQL, 106
two-valued logic, 74,229
truth tables, 218
type, 34
vs. physical representation, 28
vs. possible representation, see possible
representation
type constraint, 169-174
checked during selector invocation, 172
not in SQL, 173-174
type constructor, 43,338
type error, 29
type generator, 38
RELATION, 38
TUPLE, 38
type template, 338
"typed table," 45
Uhren, Thomas, xv
UNGROUP, 152
union, 116
disjoint, see D_UNION
exclusive, 131-132
$n$-adic, 118
SQL 27 varieties, 296
with coercion, 42
UNIQUE (quantifier), 238,263
UNIQUE (SQL), 176,263-264
uniqueness (key), 92
universal quantifier, 224
universal relation, 374
UNKNOWN, 74,229
UPDATE
expansion, 158
via cursor (SQL), 87
update vs. UPDATE, 8
"updating attributes," 87
"updating tuples," 87
updating views, see view
value, 21
can't be updated, 21
vs. variable, 21
value unknown, 7,74
VALUES, 60
variable, 21
can be updated, 21
vs. value, 21
view, 194
constraint, 199-203
"materialized," see snapshot
predicate, 197
retrieval, 198-199
two different purposes, 211
updating, 203-211
view defining expression, 194
virtual relvar, see view
Voorhis, Dave, 418
Wegner, Peter, 25
Weikum, Gerhard, 409
"what if," 157-158
"where used," 161
WITH, 119,257
SQL, 120
Wittgenstein, Ludwig, 16,215
Wong, E., 417
Wright, Andrew, 336
XML, 21,296,335,377
XPath, 35
XQuery, 35
XUNION, see union
Zdonik, Stanley, 289
Zloof, Moshé, 418


[^0]:    ${ }^{1}$ For reasons that aren't important here, I've replaced a few of the questions in this list by new ones.

[^1]:    ${ }^{2}$ In this connection, I'd like to acknowledge the contribution of a reader of the first edition, Thomas Uhren, who found an embarrassingly large number of errors. I'll try harder in future. I promise.

[^2]:    ${ }^{1}$ There's at least one pundit who doesn't. The following is a direct quote from a document purporting (like this book!) to offer advice to SQL users: "Don't use joins ... Oracle and SQL Server have fundamentally different approaches to the concept ... You can end up with unexpected result sets ... You should understand the basic types of join clauses ... Equijoins are formed by retrieving all the data from two separate sources and combining it into one, large table ... Inner joins are joined on the inner columns of two tables. Outer joins are joined on the outer columns of two tables. Left joins are joined on the left columns of two tables. Right joins are joined on the right columns of two tables."

[^3]:    ${ }^{2}$ The standard has been through several versions, or editions, over the years. The version current at the time of writing is SQL:2008 (a formal reference for which can be found in Appendix G); the previous version was SQL:2003, the one before that was SQL:1999, and the one before that was SQL:1992. Most of the SQL features discussed in this book were present in SQL:1992, and often in even earlier versions.
    ${ }^{3}$ These particular limitations were added in SQL:2003; they didn't apply to SQL:1992, which is where explicit JOIN invocations were first introduced, nor to SQL:1999.

[^4]:    ${ }^{4}$ Strictly speaking, this sentence should read "Every relvar has at least one candidate key" (see the section "Relations vs. Relvars," later). Note: Actually, a similar remark applies elsewhere in this chapter as well. Exercise 1.1 at the end of the chapter addresses this issue.
    ${ }^{5}$ See Exercise 5.27 in Chapter 5 for further explanation of this convention.

[^5]:    ${ }^{6}$ This definition is deliberately somewhat simplified. A better definition can be found in Chapter 5.

[^6]:    ${ }^{7}$ Except that Codd additionally defined an operator called divide. I'll explain in Chapter 7 why I omit that operator here.

[^7]:    ${ }^{8}$ More than one reviewer observed that this sentence didn't make sense (how can a system be used as a method?). Well, if you're too young to be familiar with the term access method, then I envy you; but the fact is, that term, inappropriate though it certainly was (and is), was widely used for many years to mean a simple record level I/O facility, of one kind or another.

[^8]:    ${ }^{9}$ It's also used in connection with keys (see Chapter 5).

[^9]:    10 "First" normal form because, as I'm sure you know, it's possible to define a series of "higher" normal forms-second normal form, third normal form, and so on-that are relevant to the discipline of database design.

[^10]:    ${ }^{11}$ What I've described in this paragraph is the standard mathematical convention; however, you might have encountered a different convention in less formal contexts. To be specific, some people use " $B$ is a subset of $A$ " to mean what I mean when I say $B$ is a proper subset of $A$, and use " $B$ is a subset of or equal to $A$ " to mean what I mean when I say $B$ is a subset of $A$. Similarly for supersets, of course, mutatis mutandis.

[^11]:    ${ }^{12}$ You might be thinking this claim can't be 100 percent true for update operations. If so, you might be right as far as today's SQL products are concerned; nevertheless, I still claim it's true in principle. See the section "Update Operations" in Chapter 9 for further discussion.

[^12]:    ${ }^{13}$ I say this knowing full well that the majority of today's SQL products do provide a variety of options for hashing, partitioning, indexing, clustering, and otherwise organizing the data as stored on the disk. Despite this state of affairs, I still consider the mapping from base tables to physical storage in those products to be fairly direct.

[^13]:    ${ }^{14}$ Several reviewers complained about this fact-that is, they felt I should be using SQL itself, not some nonstandard language like Tutorial D, in order to illustrate relational ideas. (One even suggested the book be renamed "Tutorial $\mathbf{D}$ and Relational Theory"!) But SQL as such was never intended to be a vehicle for illustrating relational ideas, while Tutorial $\mathbf{D}$ explicitly was; and in any case, SQL simply isn't adequate to the task. Indeed, if it were, a book like this one wouldn't be necessary in the first place.

[^14]:    ${ }^{15}$ SQL makes the same mistake, of course, because it too has just one term, table, that has to be understood as sometimes meaning a table value and sometimes a table variable.

[^15]:    ${ }^{1}$ DBMS $=$ database management system. Note that there's a logical difference between a DBMS and a database! Unfortunately, the industry very commonly uses the term database when it means either some DBMS product, such as Oracle, or the particular copy of such a product that happens to be installed on a particular computer. I do not follow this usage in this book. The problem is, if you call the DBMS a database, what do you call the database?

[^16]:    ${ }^{2} \mathrm{DBA}=$ database administrator.

[^17]:    ${ }^{3}$ This observation is valid regardless of whether we're in an SQL context (as in the present discussion) or otherwise-but I should make it clear that selectors in SQL aren't as straightforward as they might be, and selector as such isn't an SQL term. I should also make it clear that selectors have nothing to do with the SQL SELECT operator.

[^18]:    ${ }^{4}$ Again this observation is valid regardless of whether we're in an SQL context or some other context-though (as with selectors) THE_operators in SQL aren't as straightforward as they might be, and "THE_operator" as such isn't an SQL term. I note too that some types might have more than one associated THE_operator. See Chapter 8 for further discussion.

[^19]:    ${ }^{5}$ Observe that I don't claim it's well designed-indeed, it probably isn't-but that's not the point. I'm concerned here with what's legal, not with questions of good design. The design of R3 is legal.
    ${ }^{6}$ In case you're wondering, the difference is that sets in general can contain anything, but relations contain tuples. Note, however, that a relation certainly resembles a general set inasmuch as it too can be regarded as a single value.

[^20]:    ${ }^{7}$ The individual elements in an SQL multiset don't have to be rows but can be values of any available SQL type-for example, integers. The same goes for arrays as well.

[^21]:    ${ }^{8}$ Throughout this book I treat declared and defined as synonymous.

[^22]:    ${ }^{9}$ The logical difference between type and representation is important here. To spell the matter out, the operators associated with type $T$ are the operators associated with type $T$-not the operators associated with the representation of type $T$. For example, just because the representation for type SNO happens to be CHAR (say), it doesn't follow that we can concatenate two supplier numbers; we can do that only if concatenation is an operator that's defined for type SNO. (In fact I did mention exactly this example in passing in the section "Equality Comparisons," as you might recall.)

[^23]:    ${ }^{10}$ This paragraph touches on another important logical difference, incidentally: namely, that between arguments and parameters (see Exercise 2.5 at the end of the chapter). Note too that the POINT selector, unlike the SNO and PNO selectors discussed earlier, takes two arguments (because points are represented by pairs of values, not just by a single value).
    ${ }^{11}$ The concept might be familiar, but it seems to be quite difficult to find a good definition for it in the literature! See Exercise 2.2.

[^24]:    ${ }^{12}$ This sentence is only an approximation to the truth. A more accurate statement would be: Nongenerated types-see later in the present section-are scalar; generated types (e.g., relation types) are typically nonscalar, but don't have to be. An example of a scalar generated type is the SQL type CHAR(25) (see the next section).

[^25]:    ${ }^{13}$ Note that it does make sense to talk about the heading of a tuple-tuples have headings just as relations do (as will be explained in more detail in the next chapter).

[^26]:    ${ }^{14}$ True bit string types-BIT $(n)$ and BIT VARYING $(n)$, where $n$ was the length in bits-were introduced in SQL:1992 but dropped again in SQL:2003.
    ${ }^{15}$ SQL also supports a ROW type generator, as we know. In fact, it also supports ARRAY, MULTISET, and REF (but, oddly enough, not TABLE) as type generators.

[^27]:    ${ }^{16}$ Unfortunately that support is severely flawed, however. First of all, SQL supports coercions (see later), with the consequence that "=" can give TRUE even when the comparands are of different types. Second, in the case of character string types, it's possible for " $=$ " to give TRUE even when the comparands are of the same type but clearly distinct (see the section "Collations in SQL"). And it's also possible-for all types, not just character string types-for "=" not to give TRUE even when the comparands aren't distinguishable; in particular, this happens when (but not only when) the comparands are both null. Also, for certain types not discussed in detail in this book, including type XML and certain user defined types, "=" isn't defined at all.

[^28]:    ${ }^{17}$ In connection with this example, one reviewer suggested that the "strangeness" of the union might not matter in practice, since at least no information has been lost in the result. Well, that observation might be valid, in this particular example. But if the SQL language designers want to define an operator that manifestly doesn't behave like the union operator of the relational model (or set theory, come that), then it seems to me that, first, it doesn't help the cause of understanding to call that operator "union"; second (and rather more important), it isn't incumbent on me to show such a "union" can sometimes cause problems-rather, it's incumbent on those language designers to show it can't.

[^29]:    ${ }^{18}$ As a historical note, I remark that in the original (i.e., IBM) version of SQL, the only available collation-which was based on the internal coding scheme, of course-supported PAD SPACE only, and did that only tacitly. The reason for this state of affairs was a desire to conform to the corresponding rules for $\mathrm{PL} / \mathrm{I}$.

[^30]:    ${ }^{19}$ Oddly enough, SQL refers to the components of row types produced by invocation of the ROW type constructor (and to the components of rows of such types) not as columns but as fields.

[^31]:    ${ }^{20}$ Perhaps I should elaborate briefly on what I mean by the term pointer. A pointer is a value (an address, essentially) for which certain special operators-notably referencing and dereferencing operators-are, and in fact must be, defined. Here are rough definitions of those operators: (a) Given a variable $V$, the referencing operator applied to $V$ returns the address of $V$; (b) given a value $v$ of type pointer (i.e., an address), the dereferencing operator applied to $v$ returns the variable that $v$ points to (i.e., the variable located at the given address).
    ${ }^{21}$ As a matter of fact, the target table can't either.

[^32]:    ${ }^{1}$ More generally, the expression $n$ ! (which is read as either " $n$ factorial" or "factorial $n$ " and is often pronounced " $n$ bang") is defined as the product $n *(n-1) * \ldots * 2 * 1$.

[^33]:    ${ }^{2}$ Strictly speaking, I shouldn't be talking about assignments of any kind in this chapter, because assignment has to do with variables and this chapter is concerned with values, not variables. But it's convenient to include at least this brief mention of SQL row assignment here.

[^34]:    ${ }^{3} A$ remark on notation: In mathematics, the symbol $H$ enclosed in braces, as in " $\{H\}$ " here, would denote a set containing a single element $H$. And so it does in this definition too, of course-but that symbol $H$ in turn must be understood as denoting a composite object (viz., a commalist of attributes, in the case at hand). Now, if I were to say, in mathematics, that the set $\{X\}$ is a subset of the set $\{Y\}$, I could only mean $X$ and $Y$ were identical. But when I say the same thing in the present book (and when $X$ and $Y$ do indeed denote composite objects), then I mean the set of items constituting $X$ is a subset of the set of items constituting $Y$. I hope that's clear! Note: The picture is perhaps muddied slightly by the fact that I don't always use notation of the form $\{X\}$ to denote sets of attributes. For example, my definition of the term key in Chapter 5 begins: "Let $K$ [i.e., not $\{K\}$ ] be a subset of the heading of relvar $R$." In general, in fact, I'll feel free to include braces or exclude them according to what suits my purpose best at the time. I hope this state of affairs won't confuse you.

[^35]:    ${ }^{4}$ But see Exercise 3.15 .

[^36]:    ${ }^{5}$ Why exactly is the definite article correct here ("the" row)?
    ${ }^{6}$ Indeed, I think it could be argued that one reason we hear so much about the need for "multidimensional databases" (for decision support applications in particular) is precisely because so many people fail to realize that relations are multidimensional already.

[^37]:    ${ }^{7}$ The Tutorial D expression $r\{A, B, \ldots, C\}$ denotes the projection of relation $r$ on attributes $A, B, \ldots, C$. See Chapter 6 for further discussion.

[^38]:    ${ }^{8}$ Perhaps I should say a little more about the pet names TABLE_DUM and TABLE_DEE. First, for the benefit of non English speakers, I should explain that they're basically just wordplay on Tweedledum and Tweedledee, who were originally characters in a children's nursery rhyme and were subsequently incorporated into Lewis Carroll's Through the Looking-Glass. Second, the names are perhaps a little unfortunate, given that these two relations are precisely the ones that can't reasonably be depicted as tables! But we've been using those names for so long now in the relational world that we're probably not going to change them.

[^39]:    ${ }^{9}$ The odd thing is, it does have direct support for equality testing on "multisets"-including, therefore, multisets of rows in particular. (It also has direct support for equality testing on arrays.) Here's a quote from the standard: "Two multisets $A$ and $B$ are distinct if there exists a value $V$ in the element type of $A$ and $B$, including the null value [sic], such that the number of elements in $A$ that are not distinct from $V$ does not equal the number of elements in $B$ that are not distinct from $V$." (I hope that's perfectly clear! Note that the extract quoted does indeed define what it means for two multisets to be equal, because-simplifying slightly-if $A$ and $B$ aren't distinct, then they're equal.) As noted in Chapter 2, however, a multiset of rows in SQL isn't the same thing as a table, because it can't be operated upon by means of SQL's regular table operators.

[^40]:    ${ }^{10}$ It's certainly true in this latter case that SQL fails to enforce the rule against duplicate column names. However, it's not quite true to say it fails to enforce the rule against anonymous columns: If some column would otherwise have no name, the implementation is supposed to give that column a name that's unique within its containing table but is otherwise implementation dependent. In practical terms, however, there's no real difference between saying something is implementation dependent and saying it's undefined (see Chapter 12). Calling such columns anonymous is thus not too far from the truth.

[^41]:    ${ }^{11}$ For this reason, in fact, I always show the keyword explicitly, even when it's not required. It can be hard to remember when keywords are optional in SQL and when they're mandatory. And in any case it would surely seem strange, in the case of AS in particular, to talk about something being an "AS clause" or "AS specification" if there isn't any AS.

[^42]:    ${ }^{1}$ One reviewer felt strongly that an even more powerful practical argument (in fact, the most practical argument of all) is simply that duplicates don't match reality-a database that permits duplicates just hasn't been designed properly and can't be, as I put it in Chapter 1, "a faithful model of reality." I'm very sympathetic to this position. But this book isn't about database design, and duplicates aren't just a design issue in any case. Thus, what I'm trying to do here is show the problems duplicates can cause, regardless of whether they're due to bad design. A detailed analysis of this whole issue, design aspects included, can be found in the paper "Double Trouble, Double Trouble" (see Appendix G).
    ${ }^{2}$ Here's as good a place as any to stress the point that-contrary to common commercial practice, perhaps-my use of the unqualified term "optimization" (and related terms) in this book always refers to something the DBMS is responsible for, not something the user has to do. In other words, I'm not talking about what's sometimes called "hand optimization."

[^43]:    ${ }^{3}$ I once quoted this line in a seminar, and an attendee said "You can say that again!" To which I replied "Yes一there's a logical difference between logic and rhetoric."
    ${ }^{4}$ I note in passing, therefore, that duplicates violate one of the original objectives of the relational model, which was explicitness-the meaning of the data should be as obvious and explicit as possible, since databases are supposed to be suitable for sharing among a variety of disparate users. The presence of duplicates strongly suggests that part of that meaning is hidden instead of being explicit. In fact, duplicates violate one of the most fundamental relational principles of all: viz., The Information Principle (discussed further in Appendix A).

[^44]:    ${ }^{5}$ The implication is that SELECT DISTINCT might take longer to execute than SELECT ALL, even if the DISTINCT is effectively a "no op." Well, that might be so; I don't want to labor the point; I'll just observe that the reason those implementations typically can't optimize away unnecessary DISTINCTs is that they don't understand how key inference works (i.e., they can't figure out the keys that apply to the result of an arbitrary table expression). This latter issue is explored in depth in a paper by Hugh Darwen, "The Role of Functional Dependence in Query Decomposition" (see Appendix G).

[^45]:    ${ }^{6}$ A more accurate statement is: If the boolean expression in a WHERE clause evaluates to UNKNOWN, that UNKNOWN gets coerced to FALSE. Incidentally, it's interesting to note that, by contrast, if the boolean expression in a CHECK clause-see Chapter 8-evaluates to UNKNOWN, that UNKNOWN gets coerced not to FALSE but to TRUE! This state of affairs (this inconsistency, rather) might reasonably be regarded as yet another nail in the nulls coffin. See the answer to Exercise 8.21 g in Appendix F for further discussion.
    ${ }^{7}$ I'm relying here on the fact that (as noted earlier) the intended interpretation of null is value unknown, from which it follows that the fact that "the CITY is null" for part P1 means part P1 does have some city, but we don't know what it is. (In fact, if part P1 had no city at all-i.e., if the property of having a city didn't apply to part P1—then that part shouldn't have been mentioned in the table in the first place. See the discussion of relvar predicates in Chapter 5.)

[^46]:    ${ }^{8}$ SQL's UNION JOIN operator, which was a flawed attempt to support an already flawed operator called outer union, was introduced in SQL:1992 and dropped in SQL:2003.

[^47]:    ${ }^{9}$ Useful, that is, if we buy into the notion that 3 VL as such is useful, which of course I don't.

[^48]:    ${ }^{10}$ If $\{$ SNO,PNO $\}$ is the primary key for shipments, then columns SP.SNO and SP.PNO couldn't permit nulls without violating the entity integrity rule. So in case such a possibility bothers you (it doesn't bother me, because I don't believe in that rule anyway), let me change the example slightly; let me introduce another column, SPNO (shipment number), into the shipments table, and let me make \{SPNO\} the primary key. Then $\{\mathrm{SNO}, \mathrm{PNO}\}$ will still be a key, but it won't be the primary key, and the entity integrity rule therefore won't apply. (Incidentally, the very fact that the entity integrity rule is supposed to apply only to primary keys, not to candidate keys in general, seems to me to be another reason to regard that rule with suspicion. Not to mention the fact that it's also supposed to apply only to base tables, not to tables in general, which I think makes it more suspect still.)
    ${ }^{11}$ Note that the standard uses True, Unknown, and False in prose discussions but TRUE, UNKNOWN, and FALSE in its SQL grammar.

[^49]:    ${ }^{1}$ There's at least one that doesn't (at least, not 100 percent), because it does what it calls inflight checking. See Chapter 8 for further discussion.

[^50]:    ${ }^{2}$ The common special case "DELETE $R$ WHERE $b x$;" can be thought of as shorthand for "DELETE $R$ ( $R$ WHERE $b x$ );".

[^51]:    ${ }^{3}$ Even though this tactic-i.e., specifying the option-does fix the problem at hand, I'd like to inject here a comment that Hugh Darwen once made to me (in a private communication): "The syntax of a language should in all places be in the spirit of that language. Then it's easier to learn, because people get to know what to expect. A proper relational language attaches no significance to column ordering. Not anywhere."

[^52]:    ${ }^{4}$ On the other hand, it does make sense to say of some relation that it either does or does not satisfy some key constraint. We might even go further and say, a trifle sloppily, that a relation that satisfies a given key constraint actually "has" the key in question-though such a manner of speaking is likely to cause confusion, and I wouldn't recommend it.

[^53]:    ${ }^{5}$ Also known as functional dependence. The terms dependence and dependency are used interchangeably in the literature (and in this book), in contexts such as the one under discussion.

[^54]:    ${ }^{6}$ Columns $A 1, A 2, \ldots, A n$ must be the columns named in some UNIQUE or PRIMARY KEY specification in the definition of table $T 1$, but they don't have to appear in that UNIQUE or PRIMARY KEY specification in the same sequence as they do in the FOREIGN KEY specification for table $T 2$. Moreover, they, and the parentheses surrounding them, can be omitted entirely from this latter specification-but if so, then they must appear in a PRIMARY KEY specification, not a UNIQUE specification, for table $T 1$, and they must appear in that specification in the appropriate sequence.

[^55]:    ${ }^{7}$ In case you're wondering about the SQL terminology here, ON DELETE CASCADE is a "referential triggered action" and CASCADE by itself is a "referential action."

[^56]:    ${ }^{8}$ When I say types and relations are necessary and sufficient, I am of course talking only about the logical level. Obviously other constructs (pointers, for example) are needed at the physical level, as we all know-but that's because the design goals are different at that level. The physical level is beyond the purview of the relational model, deliberately.

[^57]:    ${ }^{9}$ Or the relational calculus. Either way, it comes to the same thing (see Chapter 10).

[^58]:    ${ }^{1}$ This is true in the algebra but not necessarily true in SQL. For example, if $T 1$ and $T 2$ are SQL table names, we typically can't write things like $T 1$ UNION $T 2$-we have to write something like SELECT * FROM $T 1$ UNION SELECT * FROM $T 2$ instead.
    ${ }^{2}$ Again, this is true in the algebra but not necessarily true in SQL. See the BNF grammar for SQL table expressions in Chapter 12.
    ${ }^{3}$ Here's a test of your SQL knowledge: For which of these formulations do corresponding columns have to be of the same type?

[^59]:    ${ }^{4}$ With one slight exception: Some writers regard relational inclusion (" $\subseteq$ ") as a relational operation-more specifically, as part of the relational algebra-even though it produces a result that's a truth value, not a relation. The point isn't very important, however; certainly it's not worth fighting over here.
    ${ }^{5}$ I assume for the sake of the example that the comparison PNAME > SNAME is a sensible one-though if it is, then attributes PNAME and SNAME must presumably represent "the same kind of information," and in accordance with my own recommendations in Chapter 3 I ought perhaps to have given them the same name.

[^60]:    ${ }^{6}$ In other words, any restriction in which $b x$ is a tautology (see Exercise 4.12 in Chapter 4).
    ${ }^{7}$ I remark in passing out that the phrase "duplicate elimination," which is used almost universally (not just in SQL contexts), would more accurately be duplication elimination.

[^61]:    ${ }^{8}$ A relvar, as opposed to a relation, of such a high degree is unlikely, since it would almost certainly be in violation of the principles of normalization. But such violations aren't exactly unknown in practice.

[^62]:    ${ }^{9}$ For usability reasons, Tutorial D also supports $n$-adic versions of INTERSECT and TIMES. I'll skip the details here.
    ${ }^{10}$ As noted in Chapter 4, the SQL "set function" SUM yields null, not zero, if it's invoked on a set of no numbers. But this is just a logical mistake on the part of SQL-it has no bearing on the present discussion.

[^63]:    ${ }^{11}$ Perhaps I should inject a small note of caution here. In practice, it's very common for SQL tables to have some kind of "comments" column; thus, there's a risk that NATURAL JOIN might produce unexpected results, unless some appropriate naming discipline is followed (or some appropriate renaming is done) in connection with such columns.
    ${ }^{12}$ Greater-than join is a special case of what's called $\theta$-join, which I'll be discussing later in this chapter.

[^64]:    ${ }^{13}$ Another SQL question for you: Does SQL in fact allow those corresponding columns to be of different types?
    ${ }^{14}$ I omitted CORRESPONDING from examples in earlier chapters because at the time it would only have been distracting.
    ${ }^{15}$ In the interest of completeness, I note that omitting the BY option is actually equivalent to specifying BY $(A, B, \ldots, C)$, where $A, B, \ldots, C$ are all of the common columns, in the left to right order in which they appear in the first operand table.

[^65]:    ${ }^{16}$ Note, incidentally, that the condition $\mathrm{SA}<\mathrm{SB}$ wouldn't be legal if supplier numbers were of some user defined type (SNO, say) and the operator " $<$ " hadn't been defined in connection with that type.

[^66]:    ${ }^{17}$ Except that the expression in question mustn't be such that it relies on (syntactic) context for its evaluation; in other words, it must be what's called a closed expression. For example, "S WHERE STATUS $=20$ " is closed, but "STATUS $=20$ " isn't. Of course, a similar rule applies in SQL also.

[^67]:    ${ }^{18}$ One reviewer asked why CITY is mentioned in the predicate at all, since it isn't part of the result of the projection. This is an important question! A short answer is: Because that result is obtained by projecting away the CITY attribute specifically, nothing more and nothing less. A much longer answer can be found in my book Logic and Databases: The Roots of Relational Theory (Trafford, 2007), pages 387-391 (see Appendix G). Further relevant discussion can be found in the book Normal Forms and All That Jazz: A Database Professional's Guide to Database Design Theory (again, see Appendix G).

[^68]:    ${ }^{19}$ Cost based optimizing is beyond the scope of this book because it has to do with how the data is physically stored, which isn't a relational issue by definition. But I should at least note that such optimizing is possible in the first place only because (as we saw in Chapter 1) the relational model insists on a sharp and rigid distinction between the logical and physical levels of the system, which has the effect among other things of keeping access strategies out of applications.

[^69]:    ${ }^{20}$ Strictly speaking, the SQL analogs of these operators aren't commutative, because-among other things-the left to right column order of the result depends on which operand is specified first. Indeed, the disciplines recommended in this book in connection with these operators are designed, in part, precisely to avoid such problems. More generally, the possibility of such problems occurring is one reason out of many why you're recommended never to write SQL code that relies on column positioning.

[^70]:    ${ }^{21}$ Actually such an error might not occur in SQL, because SQL permits coercions; but Tutorial D doesn't, and the observation is certainly true of Tutorial D.
    ${ }^{22}$ From Atsushi Ohori, Peter Buneman, and Val Breazu-Tannen: "Database Programming in Machiavelli-A Polymorphic Language with Static Type Inference," Proc. ACM SIGMOD International Conference on Management of Data, Portland, Ore. (June 1989).

[^71]:    ${ }^{1}$ Also known, a trifle inappropriately, as antijoin

[^72]:    ${ }^{2}$ And/or ungroup (see later in this chapter).

[^73]:    ${ }^{3}$ More accurately, it defines a corresponding range variable. See Chapter 12 for further explanation.

[^74]:    ${ }^{4}$ As noted elsewhere in this book, in mathematics the expression " $n$ !" ( $n$ factorial) is often pronounced " $n$ bang"; hence my choice of pronunciation for the symbol "!!".

[^75]:    ${ }^{5}$ Tutorial D doesn't directly support this operator, and $r 1$ DIVIDEBY $r 2$ is thus not valid Tutorial D syntax.

[^76]:    ${ }^{6}$ If you're wondering what the logical difference is here, consider the slightly different query "Get suppliers who supply all purple parts" (the point being, of course, that there are no purple parts). If there aren't any purple parts, then every supplier supplies all of them!-even supplier S5, who supplies no parts at all, and is thus not represented in relvar SP, and so can't be returned by an analogous DIVIDEBY expression. And if you're still wondering, then see the further discussion of this example in Chapter 11.
    ${ }^{7}$ Nonscalar aggregate operators can be defined too, as we'll see in the section "GROUP, UNGROUP, and Relation Valued Attributes."

[^77]:    ${ }^{8}$ AND, OR, and XOR apply to aggregates of boolean values specifically. AND in particular can be useful in connection with integrity constraints (see Chapter 8 for further discussion). Note: SQL's counterparts to AND and OR are called EVERY and SOME, respectively (there's no counterpart to XOR). SOME can alternatively be spelled ANY; likewise, in ALL or ANY comparisons (see Chapter 12), ANY can alternatively be spelled SOME. Oddly enough, however, the SQL set function EVERY can't alternatively be spelled ALL, and in ALL or ANY comparisons ALL can't alternatively be spelled EVERY.

[^78]:    ${ }^{9}$ It might be claimed, somewhat more reasonably, that the COUNT invocations within those expressions are SQL aggregate operator invocations. But the whole point about such invocations is that they can't appear as "stand alone" expressions in SQL; rather, they can only appear as part of some table expression, because they rely on that expression to identify the table over which the aggregation is to be done. For example, a statement like "SET X = COUNT(*);" would be meaningless in SQL, since it fails to mention the table whose rows are to be counted.
    ${ }^{10}$ The lack of doubly underlined columns in these tables is not an error.

[^79]:    ${ }^{11}$ By contrast, as noted in Chapter 4, the SQL analogs of these operators all return null if their argument is empty (except for COUNT and $\operatorname{COUNT}(*)$, which do correctly return zero).

[^80]:    ${ }^{12}$ More accurately, it would be understood as defining a corresponding range variable (see Chapter 12).

[^81]:    ${ }^{13}$ Here's another test of your SQL knowledge: In the example under discussion, would it be possible to save some keystrokes by using WITH to introduce a name for the common subexpression "(SELECT SUM(SPY.QTY) ...)"?
    ${ }^{14}$ Not to mention the fact that SUMMARIZE involves a syntactic construct that looks a bit like an aggregate operator invocation but isn't one-which as pointed out earlier is a good reason why it might be desirable to dispense with SUMMARIZE altogether.

[^82]:    ${ }^{15}$ The double enclosing parentheses, both here and in the UNGROUP example, are necessary-the argument expression within the outer parentheses is a subquery, which requires parentheses of its own.

[^83]:    ${ }^{16}$ Note, therefore, that the input relation isn't exactly being "extended" in the usual sense, so it might be nice to find a better keyword than EXTEND for the purpose.

[^84]:    ${ }^{17}$ Nothing to do with the closure property of the relational algebra.

[^85]:    ${ }^{18}$ Actually Tutorial D goes beyond the relational algebra as conventionally understood in that it provides TCLOSE as a built in operator. I show it as a user defined operator here just to show how recursive operators can be defined in Tutorial D.

[^86]:    ${ }^{19}$ In particular, therefore, it can't appear in a view definition-despite the fact that at least one well known product allows it to! Note: It's sometimes suggested-and, sadly, the SQL standard now explicitly supports the idea-that ORDER BY is needed in connection with what are called quota queries, but this is a popular misconception (see Exercise 7.14).
    ${ }^{20}$ I suppose SQL might claim it is defined for rows, as opposed to tuples (again, see Chapter 3).

[^87]:    ${ }^{1}$ As we saw in Chapter 5, constraints constrain updates and updates apply to variables, not values, so it makes sense to talk of a constraint "applying to" some variable.

[^88]:    ${ }^{2}$ There are some minor exceptions to this rule that need not concern us here.

[^89]:    ${ }^{3}$ I would much have preferred to use the more formal term object in this sentence in place of the very vague term thing, but object has become a loaded word in computing contexts.

[^90]:    ${ }^{4}$ In SQL, that shorthand would involve a specification of the form UNIQUE(SNO) as part of the CREATE TABLE for table S. The semantics of such a specification are explained by the standard as follows (I've adapted the standard's own generic phrasing to apply to the specific case at hand): "The constraint UNIQUE(SNO) is not satisfied if and only if EXISTS(SELECT * FROM S WHERE NOT(UNIQUE(SELECT SNO FROM S))) is true." I hope that's perfectly clear.
    ${ }^{5}$ But is this SQL formulation valid? As you can see, it involves an equality comparison in which the comparands are denoted by subqueries. Since subqueries evaluate to tables, it appears we're trying to test two tables for equality-yet we saw in Chapter 3 that SQL doesn't support table comparisons. See Exercise 12.5 in Chapter 12.

[^91]:    ${ }^{6}$ Except that (as you'll recall from Chapter 2) SQL constraints are supposed not to contain "possibly nondeterministic expressions," a rule that could cause serious problems in practice if true. See Chapter 12 for further discussion.

[^92]:    ${ }^{7}$ In this connection, I'd like to recommend the book Applied Mathematics for Database Professionals, by Lex de Haan and Toon Koppelaars (see Appendix G)-especially Chapter 11 of that book.

[^93]:    ${ }^{8}$ The standard reference-highly recommended, by the way-is Transaction Processing: Concepts and Techniques, by Jim Gray and Andreas Reuter (see Appendix G).

[^94]:    ${ }^{9}$ The constraint must be stated declaratively, however; obviously there's no way the optimizer can "understand" and exploit constraints that have been specified procedurally (and so we have here another strong reason for requiring declarative constraint support).

[^95]:    ${ }^{10}$ In case you're wondering how the deferring is done, I should explain that in general-there are some exceptions that don't need to concern us here-every SQL constraint is defined at compile time to be (a) either DEFERRABLE or NOT DEFERRABLE and (b) if DEFERRABLE, either INITIALLY DEFERRED or INITIALLY IMMEDIATE. Then, at run time, the statement SET CONSTRAINTS <constraint name commalist> <option>, where <option> is either DEFERRED or IMMEDIATE, sets the "mode" of the specified constraint(s) accordingly. (Of course, the constraint(s) in question must have been defined to be DEFERRABLE.) COMMIT forces all DEFERRABLE constraints into immediate mode; if some integrity check then fails, the COMMIT fails, and the transaction is rolled back.

[^96]:    ${ }^{11}$ I' m told, however, that this functionality is likely to be provided in some future version of the standard.

[^97]:    ${ }^{12}$ Or that supplier S1 used to be located in London, or that supplier S1 has an office in London, or that supplier S1 doesn't have an office in London, or any of an infinite number of other possible interpretations (corresponding, of course, to an infinite number of possible relvar predicates).

[^98]:    ${ }^{13}$ "Unlikely" is right; every relvar is supposed to be subject to a key constraint at the very least.

[^99]:    ${ }^{14}$ I don't mean to suggest here that system enforcement of constraints implies bad performance; in fact, I think it ought to improve performance. (Not to mention the fact that user enforcement is highly nontrivial, and very likely to be incorrect! The book by Lex de Haan and Toon Koppelaars, Applied Mathematics for Database Professionals, mentioned in an earlier footnote, gives a good idea of what's involved in such enforcement.) All I mean is, there tends to be a huge emphasis in vendor development effort on performance issues, to the exclusion of other matters such as data integrity.

[^100]:    ${ }^{1}$ In the standard, that feature goes by the name of REF types and reference values (see Chapter 2).

[^101]:    ${ }^{2}$ According to the standard, in the example under discussion, the substitution procedure now yields an expression along the following lines:
    SELECT CITY FROM S WHERE (SELECT AST FROM (SELECT CITY, SUM(STATUS) AS AST FROM S GROUP BY CITY)) > 25.

[^102]:    ${ }^{3}$ Any table, that is, so long as the definition of the table in question doesn't involve a possibly nondeterministic expression, a complication I choose to ignore for now. See Chapter 12 for further discussion.
    ${ }^{4}$ A more accurate statement is: The specification $\mathrm{KEY}\{\mathrm{SNO}\}$ applies to view LS as a logical consequence of the fact that it applies to base relvar S. Note, however, that the two specifications don't mean quite the same thing-the one for view LS means suppliers numbers are unique with respect to London suppliers, the one for base relvar S means they're unique with respect to all suppliers.

[^103]:    ${ }^{5}$ The tables are rather obviously not very well designed, and thus you might think I'm "stacking the deck" in an attempt to make my point seem more convincing than it really is. So I'd like to say the example isn't in fact mine at all; rather, it's a lightly edited version of one from Joe Celko's article "Back to the Future" (Database Programming \& Design 4, No. 12, December 1991).

[^104]:    ${ }^{6}$ If you look carefully, you'll see I'm not exactly replacing those references to V by the defining expression for V . The reason is that (as we saw in Chapter 6) SQL requires an explicit JOIN invocation like FDH NATURAL JOIN DFGP to have a "SELECT * FROM" prefix if it appears at the outermost level of nesting but allows it not to have such a prefix otherwise.

[^105]:    ${ }^{7}$ And then only if the view defining expression isn't possibly nondeterministic (see Chapter 12). Incidentally, note the implication here that SQL allows updates on "possibly nondeterministic views," and the further implication that SQL is apparently quite willing to allow certain updates to have unpredictable results! This state of affairs strikes me as odd, given that (as far as I know) the rationale for not allowing possibly nondeterministic expressions in constraints was precisely to avoid updates having unpredictable results.

[^106]:    ${ }^{8}$ The asymmetry here is intuitively odd. An argument might be made-not by me!-that you can do DELETEs but not INSERTs on a union view, because the delete rule is obvious (delete from both operands) but the insert rule isn't (do we insert into both operands or just one-and if just one, which?). But an exactly analogous argument would surely say you can do INSERTs but not DELETEs on an intersection view (?). In other words, if some views are "deletable from but not insertable into," then surely others must be "insertable into but not deletable from."

[^107]:    ${ }^{9}$ Cascade delete in particular is usually thought of as applying to foreign key constraints specifically, but the concept of compensatory actions in general is applicable to constraints of all kinds. Also, don't get the idea that such actions must always take the form of simple "cascades"; while all of the examples examined in the present subsection do happen to take that form, more complicated cases are likely to require actions of some less straightforward form.

[^108]:    ${ }^{10}$ Despite the fact that, as we saw earlier, there's at least one product on the market that does materialize them at least some of the time.

[^109]:    ${ }^{1}$ Thanks to Lauri Pietarinen of Relational Consulting Oy, Helsinki, Finland, for drawing this splendid example to my attention. The example is included and analyzed in detail in Ernest Nagel: "Symbolic Notation, Haddocks' Eyes, and the Dog-Walking Ordinance," in James Newman (ed.), The World of Mathematics, Vol. 3. Mineola, N.Y.: Dover Publications (2000).

[^110]:    ${ }^{2}$ Conventional logic (i.e., so called two valued logic, 2VL) is thus truth functionally complete. In general, a logic is truth functionally complete if and only if every possible connective can be defined in terms of the given ones. Truth functional completeness is an extremely important property; a logic that didn't satisfy it would be like an arithmetic that was missing certain operations (the operation of addition, say) and would thus be of extremely limited utility.

[^111]:    ${ }^{3}$ Easier, yes, but it's still necessary to appeal to certain transformation laws that I haven't yet defined (though they're intuitively obvious). See Chapter 11 for further discussion.
    ${ }^{4}$ Recall from Chapter 5 that statements in logic aren't the same as statements in a programming language; in some respects, in fact, a statement in logic is more like a programming language expression, at least inasmuch as it denotes a value (a truth value, of course). In logic contexts, therefore, I'll use the terms statement and expression (both in this chapter and the next) more or less interchangeably-and I apologize if this usage on my part leads to any confusion.

[^112]:    ${ }^{5}$ Existentially just to be definite. Quantifying universally instead would make no difference to the point I'm making here.

[^113]:    ${ }^{6}$ Even, sometimes, in logic textbooks, where the practice really ought to be deprecated.

[^114]:    ${ }^{7}$ It might help to point out that SQL's EXISTS is rather similar to Tutorial D's IS_NOT_EMPTY (see Chapter 3). See the section "Some Equivalences," later.
    ${ }^{8}$ To be a little more precise about the matter: Suppose $t x$ denotes a nonempty restriction of some table $T$ and the restriction condition evaluates to UNKNOWN for every row in $T$; then EXISTS $(t x)$ ought logically to return UNKNOWN but will actually return TRUE, in SQL.

[^115]:    ${ }^{9}$ In practice, the term tuple calculus is used mainly to distinguish the version of the relational calculus discussed in the present chapter from the domain calculus, which is a version of the relational calculus in which the variables range over domains-i.e., types-instead of relations. But there's no need to discuss the domain calculus in this book; if you want to know more, you can find a detailed explanation in my book $A n$ Introduction to Database Systems (see Appendix G).

[^116]:    ${ }^{10}$ To elaborate: Consider by way of example the proposition EXISTS $x(p(x))$, where $p$ is a predicate with just one parameter, $x$. If $x$ ranges over an infinite set, then any attempt to use an "iterated OR" algorithm for evaluating the proposition will inevitably be flawed, since the algorithm might never terminate (it might never find the one value of $x$ that satisfies $p$ ). Likewise, any attempt to use an "iterated AND" algorithm for FORALL $x(p(x)$ ) will also inevitably be flawed, since again the algorithm might never terminate (it might never find the one value of $x$ that fails to satisfy $p$ ).
    ${ }^{11}$ A note on syntax: Recall from Chapter 7 that Tutorial D supports the aggregate operators AND and OR, thereby allowing us to write, e.g., $\mathrm{AND}(\mathrm{SP}, \mathrm{QTY}>0)$, to express the fact that QTY values in relvar SP must be greater than zero. The discussions of the present section suggest that more "user friendly" names for these operators might well be FORALL and EXISTS, respectively; for example, the expression FORALL(SP, QTY $>0$ ) does read quite well from an intuitive point of view. Likewise, EXISTS(SP, QTY $>250$ ) seems to be an intuitively pleasing way of expressing the fact that at least one QTY value in relvar SP must be greater than 250 .

[^117]:    ${ }^{12}$ By contrast, the UNIQUE quantifier gives FALSE if its range is empty.
    ${ }^{13}$ It follows that AT_MOST_ONE (or perhaps NO_DUPS) would be a better name for the SQL operator than UNIQUE, at least in a context like the one under discussion. (Come to that, AT_LEAST_ONE might be a better name than EXISTS, too, both for the existential quantifier as such and for SQL's analog of that quantifier.)

[^118]:    ${ }^{14}$ To repeat something I said (in a footnote) in Chapter 8, what the standard actually says in this connection is as follows (more or less): "The constraint UNIQUE (SNO) is not satisfied if and only if EXISTS (SELECT * FROM S WHERE NOT (UNIQUE (SELECT SNO FROM S))) evaluates to TRUE." Well, it seems to me this definition could surely be simplified, thus: "The constraint UNIQUE (SNO) is satisfied if and only if UNIQUE (SELECT SNO FROM S) evaluates to TRUE." Now, I dare say there's a good reason for the standard's circumlocution here, but whatever it is certainly escapes me. Perhaps it has to do with nulls, in which case I'm not interested.
    ${ }^{15}$ Note that in this context, by contrast, UNIQUE does mean exactly one.

[^119]:    ${ }^{16}$ This might not be true. I have in mind here the fact that COUNT is often used in an algebraic context in formulating constraints to the effect that some functional dependency is in effect; for example, to express the fact that the FD $\{A\} \rightarrow\{B\}$ holds in relvar $R$, we can write $\operatorname{COUNT}(R\{A\})=\operatorname{COUNT}(R\{A, B\})$. But-assuming for simplicity that $R$ has no attributes other than $A$ and $B$-the following will also do the trick: IS_EMPTY ( $(R$ JOIN $(R$ RENAME $\{B$ AS $C\}))$ WHERE $B \neq C)$.

[^120]:    ${ }^{17}$ Don't confuse relational completeness with any other kind of completeness: in particular, with truth functional completeness (mentioned in an earlier footnote).

[^121]:    ${ }^{18}$ The term valid is something of a loaded word in logical contexts. I'm using it here to mean the statement in question is true, regardless of what values are assigned to any variables involved (in other words, the statement in question is a tautology).

[^122]:    ${ }^{1}$ I remark, for what it's worth, that both the relational model and SQL are essentially products of California.

[^123]:    ${ }^{2}$ What would happen if we omitted that ELSE TRUE?
    ${ }^{3}$ I'm being sloppy here. The phrase "range over table P " ought really to be "range over the table value that's the current value of the table variable called P" (and similarly for "range over table S," of course). But SQL has no explicit notion of table values vs. table variables.

[^124]:    ${ }^{4}$ All query results shown in this chapter are based on the usual sample data values, of course. Note: According to reviewers, at least two SQL products gave the same result here regardless of whether or not DISTINCT was specified. If so, then the products in question would seem to have a bug in this area.

[^125]:    ${ }^{5}$ It's worth pointing out in passing that the tactic of introducing names for subexpressions is reminiscent, somewhat, of the use of WITH in simplifying complex expressions as discussed in Chapter 6. But there's a difference: For WITH, the subexpressions in question are required to be closed, whereas no such requirement applies in the present context. Indeed, all we're doing in the present context is, in effect, simple text substitution, which is not what happens with WITH.

[^126]:    ${ }^{6}$ Is it obvious that this version is equivalent to the previous one?

[^127]:    ${ }^{7}$ Actually we could employ the same trick in mapping EXISTS-i.e., we could define EXISTS SX ( $b x$ ) as mapping to EXISTS (SELECT $k$ FROM S AS SX WHERE $(s b x)$ ), instead of EXISTS (SELECT * ... WHERE ( $s b x$ ) ). For symmetry I've done exactly this in the formulation of constraint CX6 that follows.

[^128]:    ${ }^{8}$ Given that $\{\mathrm{SNO}\}$ is a key for S , it would be possible to omit that portion of constraint CX 6 that requires there to be at most one matching supplier. Of course, this fact doesn't affect the overall message of the present section.

[^129]:    ${ }^{9}$ And that TRUE is logically correct! This behavior is certainly a little surprising, given that SQL's EVERY "set function" incorrectly returns null, not TRUE, if its argument is empty. (EVERY is, of course, the "set function" analog of ALL in the context under discussion.) The reason for the inconsistency is that-as perhaps you've guessed-SQL's ALL or ANY comparisons were defined before nulls were added to the language. (Is there a moral here?) Analogous remarks apply to ANY comparisons also.

[^130]:    ${ }^{10}$ Or is it? What if supplier or part cities could be null?

[^131]:    ${ }^{11}$ Note that both expressions involve some coercion. As a slightly nontrivial exercise, you might like to try figuring out exactly what coercions are involved in each case.

[^132]:    ${ }^{1}$ Here I might admit if pressed to a sneaking sympathy with a remark an old friend once made to me in connection with this very point: "You mathematicians are all alike-you spend hours agonizing over things that are perfectly obvious to everybody else."
    ${ }^{2}$ The standard term is table reference, but that term is hardly very apt. In most languages, a variable reference is a special case of an expression; syntactically, it's just a variable name, used to denote the value of the variable in question. But an SQL "table reference" isn't a special case of a table expression-not in the sense in which the latter term is used in this book, and (perhaps more to the point) not in the sense in which it's used in SQL, either.

[^133]:    ${ }^{3}$ As the example suggests, the column name commalist in a range variable definition is required, somewhat annoyingly, to be exhaustive-there's no way to rename just some of the columns concerned and not others. Also, note the need here to be fully cognizant of SQL's rules regarding left to right column ordering in the result of the explicit JOIN!
    ${ }^{4}$ I'll omit them from most of my own examples in the remainder of this chapter, however, because (a) using explicit range variables might distract from the main point I'm trying to make with those examples and (b) those examples are all fairly simple, anyway.
    ${ }^{5}$ It was legal in SQL:1992 but became illegal in SQL:2003.

[^134]:    ${ }^{6}$ Nor in view definitions, if the CHECK option is specified.
    ${ }^{7}$ What follows represents my own understanding and paraphrasing of the pertinent text from SQL:1992 (except that I've taken into account certain minor revisions made in subsequent versions of the standard). More important, I follow SQL:1992 here in talking about character string types only. The rules have since been extended to include as possibly nondeterministic (a) expressions involving data of certain user defined types and (b) expressions involving invocations of certain user defined operators (routines, to use the standard's term). Further details are beyond the scope of this book.

[^135]:    ${ }^{8}$ Appendix D gives a BNF grammar for relational expressions (and assignments) in Tutorial D.

[^136]:    ${ }^{1}$ One obvious objection is that there are clearly many nonrelational databases in existence already. True enough-but (unlike modern databases) those existing databases were never meant to be general purpose and application neutral; rather, they were typically built to serve some specific application. As a consequence, they don't and can't provide all of the functionality we've come to expect from a modern database (ad hoc query, view support, full data independence, flexible security and integrity controls, and so forth). In other words, I regard those older databases as nothing more than application specific data stores, and I would frankly prefer not to call them databases at all.

[^137]:    ${ }^{2}$ Which is why I set them all in quotation marks. I'll drop those quotation marks from this point forward because I know how annoying they can be, but you should think of them as still being there in some virtual kind of sense.
    ${ }^{3}$ My own opinion (for what it's worth) is that the semistructured model and the object model are, respectively, just the old hierarchic model warmed over and the old network model warmed over.
    ${ }^{4}$ Actually I think these remarks are rather charitable; in my opinion, those other models are really little more than slightly abstract, but otherwise ad hoc, storage structures that have been elevated above their station and will not stand the test of time.

[^138]:    ${ }^{5}$ I'll have a little more to say about those noneuclidean geometries in the next section.

[^139]:    ${ }^{6}$ "I see relational theory as simply a body of theory to which many people are contributing in different ways" (E. F. Codd, in an interview in Data Base Newsletter 10, No. 2, March 1982).

[^140]:    ${ }^{7}$ As explained in Chapter 2, the relational model doesn't rely on the scalar vs. nonscalar distinction in any formal sense. I appeal to it here (as elsewhere in this book) merely as an aid to intuition.
    ${ }^{8}$ I've already said it's misleading to think of the relational model as structure plus integrity plus manipulation because all three aspects are inextricably intertwined; well, of course, the same goes for the five components described in the following subsections, to some extent.

[^141]:    ${ }^{9}$ It goes without saying that object databases, XML databases, and more generally nonrelational databases of any kind, do all violate it, necessarily.
    ${ }^{10}$ We'd also have to choose which data we wanted to represent as relations and which as arrays, probably without any good guidelines to help us in making such choices. And what about the catalog? Would it contain relations, or arrays? Or a mixture?
    ${ }^{11}$ To all of the foregoing I'd like to add a comment Hugh Darwen once made to me (in a private communication): "UNION CORRESPONDING was added to SQL in 1992, presumably to fill some perceived gap in functionality. Suppose it had been part of the language as originally defined; when if ever would the need have emerged to introduce a UNION based on left to right column ordering instead?" I note too that questions like this one apply to a whole host of constructs that have been added to SQL since it was first defined.

[^142]:    ${ }^{12}$ The concept of essentiality is closely related to The Information Principle. To elaborate briefly: Let DM be a data model in the first sense of that term (see Chapter 1) and let $D S$ be a data structure provided by $D M$. Let $d m$ be a data model in the second sense of that term (again, see Chapter 1), created using the facilities of $D M$, and let $d m$ include an occurrence $d s$ of $D S$. Let $d b$ be a database conforming to $d m$. If removal from $d b$ of the data corresponding to $d s$ would cause a loss of information from $d b$, then $d s$ is essential in $d m$ (and, loosely, $D S$ is essential in $D M$ ). Clearly, then, relational systems provide just one essential data construct, viz., the relation itself. By contrast, nonrelational systems provide numerous different ways of representing information essentially, including (e.g.) pointers, record ordering, repeating groups, and so forth.

[^143]:    ${ }^{13}$ So here we have another reason-a somewhat philosophical reason, perhaps-for rejecting the notion of duplicates.
    ${ }^{14}$ In this connection, we'd also like to see an implementation that's more sophisticated in certain respects than most current SQL implementations typically are. More specifically, we'd like to see an implementation based on what's called The TransRelational ${ }^{\text {tm }}$ Model (see Appendix G).

[^144]:    ${ }^{15}$ As a matter of fact, Darwen and I show in our Manifesto book that every algebraic operator discussed in this book-with the sole exception of TCLOSE-can be expressed in terms of just two primitives, remove (which is basically "project over all attributes but one") and either nand or nor (which are basically algebraic analogs of the logical operators with the same names-see the answer to Exercise 10.4 in Appendix F).

[^145]:    ${ }^{16}$ Yes, it really was thought of in such terms. Here's a quote from the very first paper on the language we now know as SQL (see Appendix G):
    "Examples of such users are accountants, engineers, architects, and urban planners. It is for this class of users that [SQL] is intended."

[^146]:    ${ }^{1}$ I remind you too (one more time) that All logical differences are big differences.

[^147]:    ${ }^{1}$ In SQL, by contrast, it's considered to be of every type. To quote the standard: "Every data type includes a special value, called the null value $\ldots$ [that] is neither equal to any other value nor not equal to any other value."

[^148]:    ${ }^{2}$ They can be expressed in SQL, too, though not quite so easily (exercise for the reader). In fact, they're both examples of what are called-for obvious reasons-equality dependencies (EQDs). Note that if an EQD is in effect, and if that EQD spans two or more tables, then certain updates on just one of those tables will necessarily cause that EQD to be violated. See the discussion of multiple assignment and related matters in Chapter 8.

[^149]:    ${ }^{3}$ The section is based on material from Chapter 4 ("The Closed World Assumption") from my book Logic and Databases: The Roots of Relational Theory (see Appendix G).

[^150]:    ${ }^{4}$ I have to use Tutorial D here, not SQL, because the example under discussion is a yes/no query; as we'll see in a moment, therefore, it relies on the special relations TABLE_DEE and TABLE_DUM (the only relations of degree zero-see Chapter 3), and SQL doesn't support those relations.
    ${ }^{5}$ Even the terms know and knowledge might be a little strong in contexts such as those at hand (the terms believe and beliefs might be better)-but I'll stay with know and knowledge for the purposes of the present discussion.

[^151]:    ${ }^{1}$ Note that tuple ordering does indeed constitute a way of representing information-namely, by position; that is, the fact that a given tuple appears here and not there certainly does represent information of some kind.

[^152]:    ${ }^{2}$ A much more extensive discussion of the logical difference between foreign keys and pointers can be found in the paper "Inclusion Dependencies and Foreign Keys" (see Appendix G).

[^153]:    ${ }^{3}$ Another useful informal characterization is this: Relations $r 1$ and $r 2$ represent the same information if and only if, for any query $q 1$ that can be addressed to $r 1$, there's a corresponding query $q 2$ that can be addressed to $r 2$ that produces the same result (and vice versa).

[^154]:    ${ }^{4}$ Note that the dyadic tables are shown here in a style slightly different from that used in the body of the chapter. Both styles are acceptable, but (as I'll mention again in Chapter 10) sometimes one style is more convenient, sometimes the other is.

[^155]:    ${ }^{5}$ The phrase "The null value of the boolean data type" is rather strange in itself, since there's just a single null and that null, since it isn't a value, actually has no type at all.

[^156]:    ${ }^{6}$ Details of Boyce/Codd normal form and other normal forms higher than 1NF are beyond the scope of this book. However, I'm sure you know something about them anyway, so I'll feel free to mention them from time to time in the present appendix without further apology. For a detailed tutorial treatment of such topics, see the book Normal Forms and All That Jazz (referenced in Appendix G).
    ${ }^{7}$ Recall from Chapter 3 that the expression $n$ ! (which is read as either " $n$ factorial" or "factorial $n$ " and is often pronounced " $n$ bang") is defined as the product $n *(n-1) * \ldots * 2 * 1$.

[^157]:    ${ }^{8}$ Or even CITY is the name of somebody's favorite teddy bear. There's nothing in the relvar definition to say that CITY has to denote a city.

[^158]:    ${ }^{9}$ It doesn't really have a column called S.SNO, either (it has a column called SNO, unqualified, instead); however, there's a bizarre syntax rule to the effect that the column can be referred to by that qualified name anyway, as in the case at hand. (When I say the rule is bizarre, I mean it's extremely difficult to state precisely, as well as being both counterintuitive and logically incorrect.)
    ${ }^{10}$ In other words, although a column reference of the form "S.SNO" would be legal in the SELECT clause here-see part e. of the exercise-the expanded form of the expression "S.*" in that same context includes no such reference!

[^159]:    ${ }^{11}$ The example thus points up an important difference between RVAs in a relational system and hierarchies in a system like IMS (or XML?). In IMS, the hierarchies are "hardwired into the database," as it were; in other words, we're stuck with whatever hierarchies the database designer has seen fit to give us. In a relational system, by contrast, we can dynamically construct whatever hierarchies we want, by means of appropriate operators of the relational algebra.

[^160]:    ${ }^{12}$ No such rule exists in SQL, however. What's more, any implementation that tried to impose such a rule would be in violation of the standard!-i.e., the SQL standard explicitly permits "keys" to be declared that the user and the system both know to be proper superkeys. The "justification"-such as it is-for this state of affairs is beyond the scope of this book.

[^161]:    ${ }^{13}$ Indeed, logical data independence is a strong argument in favor of allowing constraints in general to be defined for views as well as base relvars.

[^162]:    ${ }^{14}$ In other words, $x$ here is a bound variable, and the sentence could be rephrased thus: "There exists some city, say $x$, such that supplier S2 is located in city $x$."

[^163]:    ${ }^{15}$ Except for TCLOSE-but TCLOSE wasn't included in the original definition of what it meant to be relationally complete, anyway. Note: You might be wondering about some of the other operators from Chapter 7 (e.g., EXTEND, SUMMARIZE, and GROUP and UNGROUP). In fact, Hugh Darwen and I show in our book Databases, Types, and the Relational Model: The Third Manifesto (see Appendix G) that these operators can indeed be defined in terms of restrict, project, etc.
    ${ }^{16}$ To state the matter a little more precisely: An SQL analog of the algebraic expression $R$ RENAME $\{A$ AS $B\}$ is the (extremely inconvenient!) SQL expression SELECT $X, Y, \ldots, Z, A$ AS $B$ FROM $R$ (where $X, Y, \ldots, Z$ are all of the columns of $R$ apart from $A$, and I choose to overlook the fact that the SQL expression results in a table with a left to right ordering to its columns).
    ${ }^{17}$ I choose to overlook the fact that SQL would actually require such a table expression, when used in any of the contexts under discussion, to be accompanied by a pointless range variable definition.

